$$\nabla(fg) = f\cdot\nabla g + g\cdot\nabla f \quad \rightarrow \quad \frac{\partial(fg)}{\partial x_j} = f\frac{\partial g}{\partial x_j} + g\frac{\partial f}{\partial x_j}$$

Derivada da função composta

$$\frac{d(g\circ f)}{dx}(a) = g'(f(a))\cdot f'(a) \qquad f, g : \mathbb{R} \to \mathbb{R}$$

Em CDI-2

$f: D\subset\mathbb{R}^n \to \mathbb{R}^m \quad g: E\subset\mathbb{R}^m \to \mathbb{R}^k \qquad f(D)\subset E, \ b = f(a)$

D e E aberto

f e g dif.

$g\circ f = D \to \mathbb{R}^k$ é dif. e:

$$D(g\circ f)(a) = Dg(f(a))\cdot Df(a) = D_g(b)\cdot Df(a)$$

ex) $f(x,y) = (xy, x^2, y^2) \quad f(a,b,c) = (\sin b, ac)$

$$Df(x,y) = \begin{bmatrix} y & x \\ 2x & 0 \\ 0 & 2y \end{bmatrix} \qquad D_g(a,b,c) = \begin{bmatrix} 0 & \cos(b) & 0 \\ c & 0 & a \end{bmatrix} \rightarrow \begin{bmatrix} 0 & \cos(x^2) & 0 \\ y^2 & 0 & xy \end{bmatrix}$$

$(a,b,c) \to (xy, x^2, y^2)$

$$\begin{bmatrix} 0 & \cos(x^2) & 0 \\ y^2 & 0 & xy \end{bmatrix} \cdot \begin{bmatrix} y & x \\ 2x & 0 \\ 0 & 2y \end{bmatrix} = \begin{bmatrix} 2x\cos(x^2) & 0 \\ y^3 & 3xy^2 \end{bmatrix}$$

$\boxed{\dfrac{\partial(g\circ f)_p}{\partial x_s}}$ → Mede a T.V. de $(g\circ f)$, em relação a x

g depende de (a,b,c) que depende de x quando $(a,b,c) = (x,y)$

↳ Entrada da linha p e coluna s de $D(g\circ f)(x)$ = Linha p de $Dg(f(x))$ x colunas de $Df(x)$ =

$$\sum_{l=1}^{m} \frac{\partial g_p}{\partial y_l}(f(x))\cdot\frac{\partial f_l}{\partial x_s}(x) \leftarrow \boxed{\text{Regra da Cadeia}}$$