ex)

$I = [0,1] \times [0,2]$

$$f(x,y) = \begin{cases} x^3 y & x^2 < y < 2x^2 \\ 0 & \end{cases}$$

$$\int_I f = \int_0^1 \int_0^2 f(x,y)\, dx\, dy = \int_0^1 \left(\int_0^{x^2} 0\, dy + \int_{x^2}^{2x^2} x^3 y\, dy + \int_{2x^2}^{2} 0\, dy \right) dx =$$

$$= \int_0^1 \left(0 + \frac{x^3 (2x^2)^2}{2} - \frac{x^3 (x^2)^2}{2} + 0 \right) dx =$$

$$= \int_0^1 \frac{3x^7}{2} dx = \left[\frac{3 \cdot x^8}{2 \cdot 8} \right]_0^1 = \frac{3}{16} \text{ //}$$

$$A = \{ (x,y,z) \in \mathbb{R}^3 : x+y+z \leq 1 ,\ x \geq 0 ,\ y \geq 0 ,\ z \geq 0 \}$$

$z = 1 - x - y$

$y = 1 - x$

$$\int_A 1\, dx\, dy\, dz$$

$$\hookrightarrow \int_A 1 = \int_0^1 \int_0^{1-x} \int_0^{1-x-y} 1\, dz\, dy\, dx = \int_0^1 \int_0^{1-x} 1 - x - y\, dy\, dx =$$

$$= \int_0^1 \left[y - xy - \frac{y^2}{2} \right]_0^{1-x} dx = \int_0^1 1 - x - x(1-x) - \frac{(1-x)^2}{2} dx =$$

$$= \left[x - \frac{x^2}{2} - \frac{x^2}{2} + \frac{x^3}{3} + \frac{(1-x)^3}{6} \right]_0^1 = 1 - \frac{1}{2} - \frac{1}{2} + \frac{1}{3} + 0 - \frac{1}{6} = \frac{1}{6}$$