Aula 36

$$\int_M f = \int_V f \circ g \sqrt{\det Dg^T Dg} \qquad \text{com } k=1 \quad \sqrt{\det Dg^T Dg} = \|g'(t)\|$$

$$-\text{vol-}k(M) = \int_M 1 \qquad - \text{Massa}(M) = \int_M \alpha \qquad \text{etc...}$$

ex) $\gamma:]a,b[\to \mathbb{R}^n$ classe C^1

$$\gamma(t) = (\gamma_1(t), \dots, \gamma_n(t)) \qquad L_\gamma = \int_C 1 = \int_a^b \|\gamma'(t)\| \, dt$$

$$\gamma'(t) = (\gamma_1'(t), \dots, \gamma_n'(t))$$

ex) circunferência de raio R no plano

$$\gamma(\theta) = (R\cos(\theta), R\sin(\theta)) \quad \theta \in]0,2\pi[$$

$$\gamma'(\theta) = (-R\sin(\theta), R\cos(\theta))$$

$$\|\gamma'(\theta)\| = R \qquad L_C = \int_0^{2\pi} \|\gamma'(\theta)\| \, d\theta = 2\pi R$$

x) Superfície esférica de raio R

$$\sqrt{\det Dg^T Dg} = \left\| \frac{\partial g}{\partial \theta} \times \frac{\partial g}{\partial \phi} \right\| = R^2 \sin\phi$$

$$g(\theta,\phi) = (R\cos(\theta)\sin(\phi), R\sin(\theta)\sin(\phi), R\cos(\phi))$$

$$\theta \in]0,2\pi[, \ \phi \in]0,\pi[$$

$$\int_G 1 = \int_0^{2\pi} \int_0^{\pi} R^2 \sin(\phi) \, d\phi \, d\theta = 4\pi R^2$$

ex) $\begin{cases} z = x^2+y^2 \\ 1 < x^2+y^2 < 4 \end{cases} \Rightarrow g(\rho,\theta) = (\rho\cos(\theta), \rho\sin(\theta), \rho^2) \quad \begin{matrix} \rho \in]1,2[\\ \theta \in]0,2\pi[\end{matrix}$

$$Dg = \begin{bmatrix} \cos(\theta) & -\rho\sin(\theta) \\ \sin(\theta) & \rho\cos(\theta) \\ 2\rho & 0 \end{bmatrix}$$

$$\left\| \frac{\partial g}{\partial \rho} \times \frac{\partial g}{\partial \theta} \right\| = \left\| \det \begin{bmatrix} e_1 & e_2 & e_3 \\ \cos(\theta) & \sin(\theta) & 2\rho \\ -\rho\sin(\theta) & \rho\cos(\theta) & 0 \end{bmatrix} \right\| = \|(-2\rho^2\cos(\theta), -2\rho^2\sin(\theta), \rho)\| =$$

$$= \sqrt{4\rho^4\cos^2(\theta) + 4\rho^4(\sin^2(\theta)) + \rho^2} =$$

$$= \rho\sqrt{1+4\rho^2}$$

$$\int_M 1 = \int_0^{2\pi} \int_1^2 \rho\sqrt{1+4\rho^2} \, d\rho \, d\theta \qquad \int_M \alpha = \int_0^{2\pi} \int_1^2 \underbrace{\rho^2}_{\alpha \circ g(\rho,\theta)} \rho\sqrt{1+4\rho^2} \, d\rho \, d\theta$$

$$\frac{1}{\text{Massa}} \int_M \alpha \cdot z = \int_0^{2\pi} \int_1^2 \rho^2 \cdot \rho^2 \cdot \rho\sqrt{1+4\rho^2} \, d\rho \, d\theta \qquad \int \alpha \cdot d_x^2 = \int_0^{2\pi} \int_1^2 \rho^2 \cdot (\rho^2\sin(\theta)^2 + \rho^4)\, \rho\sqrt{1+4\rho^2} \, d\rho \Big] \, d\theta$$