$U \subset \mathbb{R}^n$ aberto

$f: U \to \mathbb{R}^n$ classe C^1

f conservativo em U $\iff$ $\exists \phi: U \to \mathbb{R}$ classe C^2 c/ $f = \nabla\phi$ $\iff$ $\oint f\,dg = 0$ $\forall g$ caminho fechado em U

f conservativo em U $\Rightarrow$ f é fechado $\dfrac{\partial f_i}{\partial x_j} = \dfrac{\partial f_j}{\partial x_i}$

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ex) $f(x,y) = \left(\dfrac{x}{\sqrt{x^2+y^2}}, \dfrac{y}{\sqrt{x^2+y^2}}\right)$

$\oint_C f = 0$, $f \perp C$

f é fechado: $\dfrac{\partial f_2}{\partial x} = \dfrac{\partial f_1}{\partial y} = \dfrac{-xy}{(x^2+y^2)^{3/2}}$

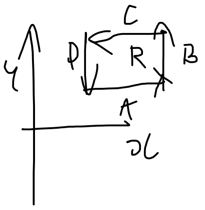
$\dfrac{\partial \phi}{\partial x} = f_1 = \dfrac{x}{\sqrt{x^2+y^2}} \Rightarrow \phi(x,y) = \sqrt{x^2+y^2} + C$

$\phi \in C^1$ logo f é conservativa

$\int f\,dg = \phi(g(b)) - \phi(g(a)) \Rightarrow \int_a^b f(g(\theta)) \cdot g'(\theta)\,d\theta$

Teorema de Green

∂R = fronteira de $R = A \cup B \cup C \cup D$



$$\oint_{\partial R} f = \int_{x_0}^{x_1} f_1(x,y_0)\,dx + \int_{y_0}^{y_1} f_2(x_1,y)\,dy - \int_{x_0}^{x_1} f_1(x,y_1)\,dx - \int_{y_0}^{y_1} f_2(x_0,y)\,dy =$$

$$= -\int_{x_0}^{x_1}\int_{y_0}^{y_1} \frac{\partial f_1}{\partial y}\,dy\,dx + \int_{y_0}^{y_1}\int_{x_0}^{x_1} \frac{\partial f_2}{\partial x}(x,y)\,dx\,dy = \iint_R \left(\frac{\partial f_2}{\partial x} - \frac{\partial f_1}{\partial y}\right) dx\,dy$$

$\leftarrow$ válido para regiões simples: simultaneamente x-simples e y-simples

$\Downarrow$

(x varia entre gráficos de funções de y)

$$\oint_{\partial R} f = \iint_R \left(\frac{\partial f_2}{\partial x} - \frac{\partial f_1}{\partial y}\right) dx\,dy$$

f classe C^1 em D contínuo em $D \cup \partial D$