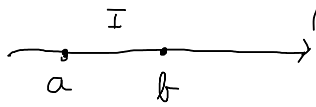
45 ∂V var-1x compacta

$f = (f_1, f_2, f_3)$ em C^1

$$\operatorname{div} f = \frac{\partial f_1}{\partial x} + \frac{\partial f_2}{\partial y} + \frac{\partial f_3}{\partial z}$$

$$\int_V \operatorname{div} f = \int_{\partial V} f \cdot n_{ext}$$

ex) $n=1$

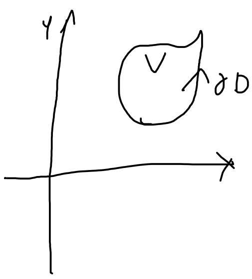


$\partial I = \{a, b\}$

$$\operatorname{div} f = \frac{df}{dx} = f'(x)$$

$$\int_I \operatorname{div} f = \int_a^b f'(x)\,dx = \text{"}\int_{\partial I} f \cdot n_{ext}\text{"} = f(b) - f(a)$$

ex) $n=2$



$f = (f_1, f_2)$ $\quad \tilde{f}(f_2, -f_1)$

$$\iint_V \left(\frac{\partial f_2}{\partial x} - \frac{\partial f_1}{\partial y}\right) dx\,dy = \oint_{\partial V} f$$

$$\|$$

$$\iint_V \operatorname{div} \tilde{f}\, dx\,dy = \int_{\partial V} \tilde{f} \cdot n_{ext}$$

SÓ EM IR^2

$\Downarrow$

$$\oint_{\partial V} f = \int_{\partial V} \tilde{f} \cdot n_{ext}$$

ex)

$D = \{(x, y, z) \in IR^3 : x^2 + y^2 \leq 1 ; 0 \leq z \leq 1\}$

$f(x, y, z) = (x^3, y^3, z)$

$\operatorname{div} f = 3x^2 + 3y^2 + 1$

$$\int_D \operatorname{div} f = \int_0^1 \int_0^1 \int_0^{2\pi} (1 + 3\rho^2)\rho\, d\theta\, dz\, d\rho = \frac{5\pi}{2}$$

ou

corpo do cilindro (C)

$g(\theta, z) = (\cos(\theta), \sin(\theta), z)$

$$D_g = \begin{bmatrix} -\sin(\theta) & 0 \\ \cos(\theta) & 0 \\ 0 & 1 \end{bmatrix} \qquad n = \pm \frac{\frac{\partial g}{\partial \theta} \times \frac{\partial g}{\partial z}}{\left\| \frac{\partial g}{\partial \theta} \times \frac{\partial g}{\partial z} \right\|}$$

$$\frac{\partial g}{\partial \theta} \times \frac{\partial g}{\partial z} = \begin{vmatrix} e_1 & e_2 & e_3 \\ -\sin & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{vmatrix} = (\cos(\theta), \sin(\theta), 0)$$

$$\int_C f \cdot n = \int_0^{2\pi} \int_0^1 (\cos^3, \sin^3, z) \cdot \frac{(\cos, \sin, 0)}{\cancel{\left\| \frac{\partial g}{\partial \theta} \times \frac{\partial g}{\partial z} \right\|}} \cdot \cancel{\left\| \frac{\partial g}{\partial \theta} \times \frac{\partial g}{\partial z} \right\|} =$$

$n = \pm (\cos(\theta), \sin(\theta), 0)$

$$= \frac{3}{2}\pi$$

$n_B = (0, 0, -1)$

$f \cdot n_B = z = 0$

$$\int_B f \cdot n_B = 0$$

$n_A = (0, 0, 1)$

$f \cdot n_A = z = 1$

$$\int_A f \cdot n_A = \int_A 1 = \pi$$

$$\frac{3}{2}\pi + \pi + 0 = \frac{5\pi}{2}$$