$g(V)=S$; $g(\partial V)=\partial S \to$ sentido compatível

sentido anti-horário

$$\int_S \text{Rot}\, f \cdot n = \int_V (\dots) \times \left(\pm \frac{\partial g}{\partial t_1} \times \frac{\partial g}{\partial t_2} \right) dt_1\, dt_2 = \int_V \left(\frac{\partial}{\partial t_1}(\dots) - \frac{\partial}{\partial t_2}(\dots) \right) dt_1\, dt_2 = \oint_{\partial V} (h,u) =$$

$$= \oint_{\partial g} f$$

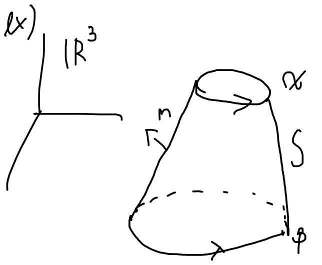
Teorema de Stokes

$S \subset \mathbb{R}^3$ var-2 orientável com bordo ∂S uma var-1 compacta

Seja n uma orientação de S e oriente-se ∂S de forma compatível

$$\int_S \text{Rot}\, f \cdot n = \oint_{\partial S} f$$

ex)



$\partial S = \alpha \cup \beta$ Seja f um campo fechado i.e. Rot $f = 0$

$\alpha \sim \beta$

$$\int_S \text{Rot}\, f \cdot n = 0 = \oint_{\partial S} f = -\oint f\, d\alpha + \oint f\, d\beta \Rightarrow \oint f\, d\alpha = \oint f\, d\beta$$

ex) $S: \begin{cases} x^2+y^2+z^2=1 \\ z>0 \end{cases}$ c/ normal n tal que $n_z>0$

$f(x,y,z) = (-y, x, z)$

$\int_S \text{Rot}\, f \cdot n$;

Rot $f = (0,0,2)$

$g(\theta,\phi) = (\cos(\theta)\sin(\phi), \sin(\theta)\sin(\phi), \cos(\phi))$

$\theta \in]0,2\pi[$, $\phi \in]0,\frac{\pi}{2}[$

$$Dg(\theta,\phi) = \begin{bmatrix} -\sin\theta \sin\phi & \cos\theta\cos\phi \\ \cos\theta\sin\phi & \sin\theta\cos\phi \\ 0 & -\sin\phi \end{bmatrix} \quad n = \pm \frac{\frac{\partial g}{\partial\theta} \times \frac{\partial g}{\partial\phi}}{\left\|\frac{\partial g}{\partial\theta} \times \frac{\partial g}{\partial\phi}\right\|}$$

$$\frac{\partial g}{\partial\theta} \times \frac{\partial g}{\partial\phi} \; (-\cos\theta\sin^2\phi, -\sin\theta\sin^2\phi, -\sin(\phi)\cos(\phi))$$

$$\int_S \text{Rot}\, f \cdot n =$$

$$\int_0^{2\pi}\int_0^{\pi/2} (0,0,2) \cdot \frac{(\cos\theta\sin^2\phi, \sin(\theta)\sin^2\phi, \sin\phi\cos\theta)}{\cancel{\left\|\frac{\partial g}{\partial\theta} \times \frac{\partial g}{\partial\phi}\right\|}} \cdot \cancel{\left\|\frac{\partial g}{\partial\theta} \times \frac{\partial g}{\partial\phi}\right\|}$$

$= 2\pi$