$n_x > 0$

$f(x,y,z) = (2x, -y, -z)$

$S: \begin{cases} x^2 = y^2 + z^2 \\ 1 < y^2 + z^2 < 4 \end{cases}, x > 0$

Div

$\int_V \operatorname{div} f = \int_{\partial V} f \cdot n_{ext} = \int_A f \cdot n_A + \int_B f \cdot n_B - \int_S f \cdot n \qquad \operatorname{div} f = 2 - 1 - 1 = 0$

$\int_S f \cdot n = \int_A f \cdot n_A + \int_B f \cdot n_B - \int_V \operatorname{div} f \Leftrightarrow$

$\Leftrightarrow \int_S f \cdot n = \int_A f \cdot n_A + \int_B f \cdot n_B = 16\pi - 2\pi = 14\pi$

$\boxed{A}$

$n_A (1,0,0)$

$\int_A f \cdot n_A = \int_A 2x = 4 \times \text{Área}(A) = 16\pi$

$\boxed{B}$

$n_B (-1,0,0)$

$\int_B f \cdot n_B = \int_B -2x = -2 \times \text{Área}(A) = -2\pi$

Definição

$\int_S f \cdot n$

$g(\rho,\theta) = (\rho, \rho\cos(\theta), \rho\sin(\theta))$

$\rho \in]1,2[$
$\theta \in]0,2\pi[$

$Dg = \begin{bmatrix} 1 & 0 \\ \cos(\theta) & -\rho\sin(\theta) \\ \sin(\theta) & \rho\cos(\theta) \end{bmatrix} \quad n = \pm \dfrac{\frac{\partial g}{\partial \rho} \times \frac{\partial g}{\partial \theta}}{\left\| \frac{\partial g}{\partial \rho} \times \frac{\partial g}{\partial \theta} \right\|}$

tem de ser $n_x > 0$

$\dfrac{\partial g}{\partial \rho} \times \dfrac{\partial g}{\partial \theta} = \begin{vmatrix} e_1 & e_2 & e_3 \\ 1 & \cos(\theta) & \sin(\theta) \\ 0 & -\rho\sin(\theta) & \rho\cos(\theta) \end{vmatrix} = (\rho, -\rho\cos(\theta), -\rho\sin(\theta))$

$\int_S f \cdot n = \int_1^2 \int_0^{2\pi} (2\rho, -\rho\cos(\theta), -\rho\sin(\theta)) \cdot \dfrac{(\rho, -\rho\cos(\theta), -\rho\sin(\theta))}{\cancel{\left\| \frac{\partial g}{\partial \rho} \times \frac{\partial g}{\partial \theta} \right\|}} \times \cancel{\left\| \frac{\partial g}{\partial \rho} \times \frac{\partial g}{\partial \theta} \right\|} \, d\theta \, d\rho =$

$= 2\pi \int_1^2 3\rho^2 \, d\rho = 14\pi$

c) Stokes

$f(x,y,z) = (2x, -y, -z)$

Encontrar h tal que $\operatorname{rot} h = f$

div = 0 p.v.

$\begin{cases} \partial_2 h_3 - \partial_3 h_2 = 2x \\ \partial_3 h_1 - \partial_1 h_3 = -y \\ \partial_1 h_2 - \partial_2 h_1 = -z \end{cases}$

$\dfrac{\partial h_3}{\partial x} = y \leadsto h_3 = yx + \alpha(y,z)$

$\dfrac{\partial h_2}{\partial x} = -z \leadsto h_2 = -zx + \beta(y,z)$

$x + \dfrac{\partial \alpha}{\partial y} + x + \dfrac{\partial \beta}{\partial z} = 2x \Leftrightarrow 2x = 2x$

$\beta = 0$ e $\alpha = 0$

$h: (0, -zx, yx)$

$\int_S f \cdot n = \int_S \operatorname{rot} h \cdot n = \int_{\partial S} h$

$\partial S = A \cup B \quad A\, g_A(2, 2\cos(\theta), 2\sin(\theta)) \quad g'_A(0, -2\sin(\theta), 2\cos(\theta))$

$B\, g_B(1, \cos(\theta), -\sin(\theta)) \quad g'_B(0, -\sin(\theta), -\cos(\theta))$

$\int_0^{2\pi} (0, -4\sin(\theta), 4\cos(\theta)) \cdot (0, -2\sin(\theta), 2\cos(\theta)) \, d\alpha = \int_0^{2\pi} 8 \, d\alpha = 16\pi$

$\int_0^{2\pi} (0, \sin(\theta), \cos(\theta)) \cdot (0, -\sin(\theta), -\cos(\theta)) \, d\alpha = \int_0^{2\pi} -1 \, d\alpha = -2\pi$

$16\pi - 2\pi = 14\pi$