ex) $f(x,y,z) = (xyz, x^2y + \cos(z))$

$$Df(x,y,z) = \begin{bmatrix} yz & xz & xy \\ 2xy & x^2 & \sin(z) \end{bmatrix} \xrightarrow[\text{ponto}(1,2,\frac{\pi}{2})]{\text{Vector}(1,4,3)} \begin{bmatrix} \pi & \frac{\pi}{2} & 2 \\ 4 & 1 & -1 \end{bmatrix} \times \begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix} = \begin{bmatrix} 3\pi + 6 \\ 5 \end{bmatrix}$$

• Se $f: D \subset \mathbb{R}^n \to \mathbb{R}$ é um campo escalar diferenciável, D aberto, $Df(a)$ é uma matriz linha $1 \times n$

$$Df(a) = \left[\frac{\partial f}{\partial x_1}(a) \dots \frac{\partial f}{\partial x_n}(a) \right] \quad a \in D$$

Associada a esta derivada existe um campo vectorial: O gradiente de f

$$\nabla f(a): D \subset \mathbb{R}^n \to \mathbb{R}$$

$$\nabla f(a) = \left(\frac{\partial f}{\partial x_1}(a), \dots, \frac{\partial f}{\partial x_n}(a) \right)$$

Dado um campo escalar $f: D \subset \mathbb{R}^n \to \mathbb{R}$, diferenciável, $a \in D$, $v \in \mathbb{R}^n$:

$$\frac{\partial f}{\partial v}(a) = Df(a) \cdot v = \left[\frac{\partial f}{\partial x_1}(a) \dots \frac{\partial f}{\partial x_n}(a) \right] \cdot \begin{bmatrix} v_1 \\ \vdots \\ v_n \end{bmatrix} = \underbrace{\nabla f(a) \cdot v}_{\text{Produto interno}} = \|\nabla f(a)\| \cdot \|v\| \cdot \cos(\theta)$$

$\nabla f(a)$ dá-nos a direção e sentido de maior crescimento de f!

$-\nabla f(a)$ " " decrescimento de f!

Se $v \perp \nabla f(a)$ então $\frac{\partial f}{\partial v}(a) = 0$

ex) $f(x,y) = 2x^2 + 3y^2$

$\nabla f(x,y) = (4x, 6y)$

$\nabla f(2,1) = (8,6)$

$(2,1)$ $\longrightarrow$ $(8,6) = \nabla f(2,1)$

A direção e sentido de crescimento máximo de f em (2,1) é

$$\frac{v}{\|v\|} = \frac{8,6}{\sqrt{8^2+6^2}} = \left(\frac{4}{5}, \frac{3}{5} \right)$$

Se $f: D \subset \mathbb{R}^n \to \mathbb{R}^m$, $a \in D$, e f é dif

$$Df(a) = \begin{bmatrix} \frac{\partial f_1}{\partial x_1}(a) & \dots & \frac{\partial f_1}{\partial x_n}(a) \\ \vdots & & \vdots \\ \frac{\partial f_m}{\partial x_1}(a) & \dots & \frac{\partial f_m}{\partial x_n}(a) \end{bmatrix} = \begin{bmatrix} - \nabla f_1(a) - \\ \vdots \\ - \nabla f_m(a) - \end{bmatrix}$$