1. Para cada um dos casos seguintes, determine os extremos da função f no conjunto S:
   a) $f(x,y,z) = x+y+z$, $S = \{(x,y,z) \in \mathbb{R}^3 : x^2+y^2+z^2 = 3\}$.
   b) $f(x,y,z) = z$, $S = \{(x,y,z) \in \mathbb{R}^3 : x^2+y^2 = 4 ; x+z = 1\}$.
2. Use o Método dos Multiplicadores de Lagrange para determinar os extremos absolutos da função $f(x,y,z) = z^2 - x - y$ que se encontram na bola $B = \{(x,y,z) \in \mathbb{R}^3 : x^2+y^2+z^2 \leq 2\}$.
3. Determine as dimensões da caixa rectangular com volume igual a 1 m^3 que minimizam a respectiva área.
4. Determine os pontos da linha $\{(\cos t, \operatorname{sen} t, \operatorname{sen}(2t)) ; t \in \mathbb{R}\}$ mais afastados da origem.
5. Determine a massa total do fio $\{(t^2, t\cos t, t \operatorname{sen} t) ; 0 \leq t \leq 2\pi\}$, com densidade de massa por unidade de comprimento $\sigma(x,y,z) = \sqrt{x}$.
6. Calcule o centróide da linha descrita pelas equações $x = y^2+z^2$; $x^2+y^2+z^2 = 2$.

1) a) $\begin{cases} x^2+y^2+z^2-3=0 \\ (1,1,1) = \lambda(2x,2y,2z) \end{cases} \Leftrightarrow \begin{cases} 3y^2 = 3 \\ 1 = \lambda 2y \end{cases} \Leftrightarrow \begin{cases} y = \pm 1 \\ - \end{cases}$ $(-1,-1,-1) = 3$ Max, $(1,1,1) = -3$ Min

b) $\begin{cases} x+z=1 \\ x^2+y^2=4 \\ (0,0,1) = \lambda(2x,2y,0) + \lambda'(1,0,1) \end{cases} \Leftrightarrow \begin{cases} x+z=1 \\ x^2+y^2=4 \\ 0 = \lambda 2x + \lambda' \\ 0 = \lambda 2y \\ 1 = \lambda' \end{cases} \Rightarrow \begin{cases} x+z=1 \\ x^2+y^2=4 \\ (-1,0,0) = \lambda(2x,2y,0) \end{cases} \rightarrow x = \pm 2$

$(2,0,-1)$ e $(-2,0,3)$

$\downarrow$ Max: 3 $\quad \downarrow$ Min: -1

2) Interior: $x^2+y^2+z^2<2$

$Df = [-1 \quad -1 \quad 2z]$ não há

Fronteira:

$\begin{cases} x^2+y^2+z^2-2=0 \\ (-1,-1,2z) = \lambda(2x,2y,2z) \end{cases}$

$\lambda = 1$ e $z \neq 0$ $\quad z = \pm\sqrt{\frac{3}{2}}$

$-\frac{1}{2} = x$

$-\frac{1}{2} = y$

$\left(-\frac{1}{2}\right)^2 + \left(-\frac{1}{2}\right)^2 + z^2 - 2 = 0$

$f\left(-\frac{1}{2}, -\frac{1}{2}, \sqrt{\frac{3}{2}}\right)$, $f\left(-\frac{1}{2}, -\frac{1}{2}, -\sqrt{\frac{3}{2}}\right) = \frac{3}{2}+1 = \frac{5}{2}$ Max //

$z=0$ $\begin{cases} x^2+y^2=2 \\ -1 = 2x\lambda \\ -1 = 2y\lambda \end{cases} \Leftrightarrow \begin{cases} x^2+y^2=2 \\ x = y \end{cases} \Leftrightarrow \begin{cases} x = \pm 1 = y \end{cases}$

$f(1,1,0) = -2$ Min //

$f(-1,-1,0) = 2$

3) $xyz = 1$ $\quad$ A: $f(x,y,z) = 2xy + 2xz + 2yz$

$\begin{cases} xyz - 1 = 0 \\ (2y+2z, 2x+2z, 2x+2y) = \lambda(yz, xz, xy) \end{cases} \Leftrightarrow$

$Df = [2y+2z \quad 2x+2z \quad 2x+2y]$

$D(xyz) = [yz \quad xz \quad xy]$

$\frac{2y+2z}{yz} = \lambda \Leftrightarrow \lambda = \frac{2}{z} + \frac{2}{y}$

$\frac{2x+2z}{xz} = \lambda \Leftrightarrow \lambda = \frac{2}{z} + \frac{2}{x}$

$\frac{2y+2x}{xy} = \lambda \Leftrightarrow \lambda = \frac{2}{x} + \frac{2}{y}$

$\rightarrow \frac{2}{y} = \frac{2}{x}$, $\rightarrow \frac{2}{z} = \frac{2}{y}$ $\Rightarrow x = y = z$

Como $x = y = z$

$y = \sqrt[3]{1} = 1$ $\quad (1,1,1)$

é o cubo de 1m de aresta

4) $d = \sqrt{\cos^2(t) + \operatorname{sen}^2(t) + \operatorname{sen}^2(2t)} = \sqrt{1+\operatorname{sen}^2(2t)}$

$d^2 = 1 + \operatorname{sen}^2(2t)$

$\nabla d^2 = 4\operatorname{sen}(2t)\cos(2t) = 0$

$\operatorname{sen}(2t) = 0 \lor \cos(2t) = 0$

$t = \frac{\pi}{4}k$

$k=0$: $(\cos(0), \operatorname{sen}(0), \operatorname{sen}(0)) = (1,0,0)$ min

$k=1$: $\left(\cos\left(\frac{\pi}{4}\right), \operatorname{sen}\left(\frac{\pi}{4}\right), \operatorname{sen}\left(\frac{\pi}{2}\right)\right) = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 1\right)$ Max

$k=2$: $(0,1,0)$ min $\quad k=5$: $\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}, 1\right)$ Max

$k=3$: $\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 1\right)$ Max $\quad k=7$: $\left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}, -1\right)$ Max

5) $g(t) = (t^2, t\cos(t), t\operatorname{sen}(t))$

$D_g = \begin{bmatrix} 2t \\ \cos(t) - t\operatorname{sen}(t) \\ \operatorname{sen}(t) + t\cos(t) \end{bmatrix}$ $\quad \sqrt{\det D_g^T D_g} = \|D_g(t)\| =$

$= \sqrt{4t^2 + \cos^2(t) - 2\cos(t)t\operatorname{sen}(t) + t^2\operatorname{sen}^2(t) + \operatorname{sen}^2(t) + 2\cos(t)\operatorname{sen}(t)t + t^2\cos^2(t)} =$

$= \sqrt{5t^2+1}$

$\int_0^{2\pi} \sqrt{5t^2+1} \times t \, dt = \int_1^{20\pi^2+1} \sqrt{u} \cdot \frac{1}{10} \, du = \frac{1}{10} \times \left[\frac{2}{3} \times \sqrt[3]{u^2}\right]_1^{20\pi^2+1} =$

seja $u = 5t^2+1$

$u' = 10t$

$= \left((20\pi^2+1)^{\frac{3}{2}} - 1\right) \times \frac{1}{15}$ //

6) $\begin{cases} x = y^2+z^2 \\ x^2+y^2+z^2 = 2 \end{cases} \Leftrightarrow x^2+x = 2 \Leftrightarrow x = -2 \lor x = 1$

$-2 = y^2+z^2$ Imp

logo $y^2+z^2 = 1$

$\rho^2 = y^2+z^2$ $\quad \rho = 1$

$y = \rho\cos(\theta)$

$z = \rho\operatorname{sen}(\theta)$

$g(\theta) = (1, \cos(\theta), \operatorname{sen}(\theta))$ $\quad 0 < \theta < 2\pi$

$\|g'(\theta)\| = 1$

$\int_0^{2\pi} 1 \, d\theta = 2\pi$ $\quad \bar{z} = \frac{1}{2\pi}\int_0^{2\pi} \operatorname{sen}(\theta) \, d\theta = 0$

$\bar{y} = \frac{1}{2\pi}\int_0^{2\pi} \cos(\theta) \, d\theta = 0$

$(1,0,0)$ //

7. Calcule a área de cada uma das superfícies:

a) $A = \{(x,y,z)\in\mathbb{R}^3 : 1+\sqrt{x^2+z^2} = y < 2;\ x>0\}$.

b) $B = \{(x,y,z)\in\mathbb{R}^3 : z = xy;\ x^2+y^2<1\}$.

8. Considere a superfície

$$S = \{(x,y,z)\in\mathbb{R}^3 : x^2+y^2+z^2=a^2;\ z>0\},\ a>0,$$

com densidade de massa igual a um. Calcule o momento de inércia de S relativo ao eixo Oz.

7 $A = \{(x,y,z) \in \mathbb{R}^3 : 1+\sqrt{x^2+z^2} = y < 2 ;\ x>0\}$

$x>0 \Rightarrow 0<\theta<\pi$

$x^2+z^2=\rho^2$

$1+\rho<2 \Leftrightarrow \rho<1$

$$\int_A 1 = \int_0^{\pi}\int_0^1 \sqrt{2}\,\rho \, d\rho \, d\theta = \pi\sqrt{2}\left[\frac{\rho^2}{2}\right]_0^1 = \frac{\pi\sqrt{2}}{2}$$

$g(\theta,\rho) = (\rho\sin(\theta),\ 1+\rho,\ \rho\cos(\theta))$

$$Dg = \begin{bmatrix} \sin(\theta) & \rho\cos(\theta) \\ 1 & 0 \\ \cos(\theta) & -\rho\sin(\theta) \end{bmatrix}$$

$$\sqrt{\det Dg^T Dg} = \left\|\frac{\partial g}{\partial \rho}\times\frac{\partial g}{\partial \theta}\right\| = \left\| \begin{vmatrix} e_1 & e_2 & e_3 \\ \sin(\theta) & 1 & \cos(\theta) \\ \rho\cos(\theta) & 0 & -\rho\sin(\theta)\end{vmatrix} \right\| = \sqrt{(-\rho\sin(\theta))^2 + (-\rho)^2 + (\rho\cos(\theta))^2} = \sqrt{\rho^2+\rho^2} = \sqrt{2}\,\rho$$

b) $\begin{cases} z = xy \\ x^2+y^2<1 \end{cases}$

$x^2+y^2=\rho^2$ $\quad z = \rho^2\cos(\theta)\sin(\theta)$

$0<\rho<1$

$$\int_B 1 = \int_0^{2\pi}\int_0^1 \rho\sqrt{\rho^2+1}\, d\rho\, d\theta = 2\pi\int_0^1 \rho\sqrt{\rho^2+1}\, d\rho =$$

seja $u = \rho^2+1$

$u' = 2\rho$

$$2\pi\int_1^2 \sqrt{u}\times\frac{1}{2}\, du = \pi\int_1^2\sqrt{u}\, du = \pi\left[\frac{u^{\frac{1}{2}+1}}{\frac{1}{2}+1}\right]_1^2 = \pi\,\frac{2^{\frac{3}{2}}-1}{\frac{3}{2}} = \frac{2\pi}{3}(2\sqrt{2}-1)$$

$g(\theta,\rho) = (\rho\cos(\theta),\ \rho\sin(\theta),\ \rho^2\cos(\theta)\sin(\theta))$

$$Dg = \begin{bmatrix} \cos(\theta) & -\rho\sin(\theta) \\ \sin(\theta) & \rho\cos(\theta) \\ 2\rho\cos(\theta)\sin(\theta) & \rho^2(\cos^2(\theta)-\sin^2(\theta)) \end{bmatrix}$$

$$\sqrt{\det Dg^T Dg} = \left\|\frac{\partial g}{\partial\rho}\times\frac{\partial g}{\partial\theta}\right\| = \left\| \begin{vmatrix} e_1 & e_2 & e_3 \\ \cos(\theta) & \sin(\theta) & 2\rho\cos(\theta)\sin(\theta) \\ -\rho\sin(\theta) & \rho\cos(\theta) & \rho^2(\cos^2(\theta)-\sin^2(\theta)) \end{vmatrix} \right\| = \sqrt{\rho^4\sin^2(\theta)+\rho^4\cos^2(\theta)+\rho^2} = \rho\sqrt{\rho^2+1}$$

e_1: $\left(\sin(\theta)\rho^2(\cos^2(\theta)-\sin^2(\theta)) - 2\rho^2\cos^2(\theta)\sin(\theta)\right) = \rho^2(\sin\cos^2 - \sin^3 - 2\cos^2\sin) =$
$= \rho^2\sin(-\cos^2-\sin^2) = -\rho^2\sin(\theta)$

e_2: $\rho^2\cos(\cos^2-\sin^2) + 2\rho^2\cos\sin^2 =$
$= \rho^2\cos(\cos^2-\sin^2+2\sin^2) = \rho^2\cos$

e_3: $\rho\cos^2(\theta) + \rho\sin^2(\theta) = \rho$

8) $S = \{(x,y,z)\in\mathbb{R}^3 : x^2+y^2+z^2=a^2 ;\ z>0\}$ $a>0$

$$I_z(S) = \int_S 1\cdot d_z^2$$

$g(a,\theta,\phi) = (a\cos(\theta)\sin(\phi),\ a\sin(\theta)\sin(\phi),\ a\cos(\phi))$

$\begin{cases} x = a\cos(\theta)\sin(\phi) \\ y = a\sin(\theta)\sin(\phi) \\ z = a\cos(\phi) \end{cases}$ $\quad a>0$, $0<\theta<\pi$, $0<\phi<\frac{\pi}{2}$

$d_z^2 = x^2+y^2 = a^2\sin^2(\phi)$

$$Dg = \begin{bmatrix} \cos(\theta)\sin(\phi) & -a\sin\theta\sin\phi & a\cos\theta\cos\phi \\ \sin(\theta)\sin(\phi) & a\cos\theta\sin\phi & a\sin\theta\cos\phi \\ \cos(\phi) & 0 & -a\sin\phi \end{bmatrix}$$

$$Dg^T Dg = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{a^2-a^2\cos(2\psi)}{2} & 0 \\ 0 & 0 & a^2 \end{bmatrix}$$

$$\sqrt{\det Dg^T Dg} = \sqrt{\frac{a^4-a^4\cos(2\phi)}{2}}$$

$$2\pi\int_0^{\pi/2}\sqrt{\frac{a^4-a^4\cos(2\phi)}{2}}\times a^2\sin^2(\phi)\, d\phi = \pi\,\frac{4}{3}a^4$$

Usem o site: https://www.integral-calculator.com/