1. Calcule a derivada $D(f \circ g)(1,1)$ em que

$$g(x,y) = (e^{x-y}, x-y); \quad f(u,v) = (u + \arctan v, 2e^v + u, \ln(u+2v)).$$

2. Considere as funções $\gamma(t) = (\operatorname{sen} t, t^2, \cos t)$, $F(x,y,z) = x^2+y^2+z^2+1$ e $\sigma(t) = F(\gamma(t))$. Calcule a derivada $\sigma'(t)$.

3. Considere a função $f(x,y,z) = ye^x + xz^2$ e seja $g: \mathbb{R}^2 \to \mathbb{R}^3$ uma função de classe C^1 tal que $g(0,0) = (0,1,2)$ e

$$Dg(0,0) = \begin{bmatrix} 0 & 1 \\ 2 & 3 \\ 4 & 0 \end{bmatrix}.$$

Calcule a derivada $D_v(f \circ g)(0,0)$ em que $\vec{v} = (1,2)$.

1) $D(f \circ g)(1,1) = Df(g(1,1)) \cdot Dg(1,1)$

$Dg(x,y) = \begin{bmatrix} e^{x-y} & -e^{x-y} \\ 1 & -1 \end{bmatrix}$ $\quad Df(u,v) = \begin{bmatrix} 1 & \frac{1}{1+v^2} \\ 1 & 2e^v \\ \frac{1}{u+2v} & \frac{2}{u+2v} \end{bmatrix}$

$g(1,1) = (1,0)$

$x, y \to 1, 1$ $\quad \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix}$ $\quad \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 2 \end{bmatrix}$

$\begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 2 & -2 \\ 3 & -3 \\ 3 & -3 \end{bmatrix}$

Ou

$Df(g(x,y)) = \begin{bmatrix} e^{x-y} + \frac{1}{1+(x-y)^2} & -e^{x-y} + \frac{-1}{1+(x-y)^2} \\ 3e^{x-y} & -3e^{x-y} \\ \frac{e^{x-y}+2}{e^{x-y}+2x-2y} & \frac{-e^{x-y}-2}{e^{x-y}+2x-2y} \end{bmatrix} \xrightarrow{x,y=1,1} \begin{bmatrix} 2 & -2 \\ 3 & -3 \\ 3 & -3 \end{bmatrix}$

2. $F(\gamma(t))$

$D\gamma(t) = \begin{bmatrix} \cos(t) \\ 2t \\ -\sin(t) \end{bmatrix}$ $\quad DF(x,y,z) = [2x \quad 2y \quad 2z]$

$(\sin(t), t^2, \cos(t)) \to [2\sin(t) \quad 2t^2 \quad 2\cos(t)]$

$[2\sin(t) \quad 2t^2 \quad 2\cos(t)] \cdot \begin{bmatrix} \cos(t) \\ 2t \\ -\sin(t) \end{bmatrix} = 2\cancel{\sin(t)\cos(t)} + 4t^3 - 2\cancel{\cos(t)\sin(t)} = 4t^3$

3.) $D_v(f \circ g)(0,0) = D(f \circ g)\begin{bmatrix} 1 \\ 2 \end{bmatrix} = D(f(g(0,0)) \cdot Dg(0,0) \cdot \begin{bmatrix} 1 \\ 2 \end{bmatrix} =$

$= Df(0,1,2) \cdot \begin{bmatrix} 0 & 1 \\ 2 & 3 \\ 4 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 2 \end{bmatrix} = [5 \ 1 \ 0] \cdot \begin{bmatrix} 0 & 1 \\ 2 & 3 \\ 4 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 2 \end{bmatrix} = [2 \ 8] \cdot \begin{bmatrix} 1 \\ 2 \end{bmatrix} = 2 + 16 = 18$

$Df = [ye^x + z^2 \quad e^x \quad 2zx]$

$\downarrow (0,1,2)$

$[5 \quad 1 \quad 0]$

4. Considere a função $\sigma(x) = f(\operatorname{sen} x, x + e^x)$ em que $f: \mathbb{R}^2 \to \mathbb{R}^3$ é de classe C^1 e tal que

$$Df(0,1) = \begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 3 & 2 \end{bmatrix}.$$

Calcule a derivada $\sigma'(0)$.

$D\sigma(x) = D(f \circ g)(x) = Df(g(x)) \cdot Dg(x) = \begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 3 & 2 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ 7 \end{bmatrix}$

$g(x) = (\sin(x), x + e^x)$

$Dg(x) = \begin{bmatrix} \cos(x) \\ 1 + e^x \end{bmatrix} \xrightarrow{x=0} \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

5. Seja $f : \mathbb{R}^3 \to \mathbb{R}^3$ dada por

$$f(x, y, z) = (x^2 + y^2 + z^2, x + y - z, xye^z)$$

e $g : \mathbb{R}^3 \to \mathbb{R}$ uma função diferenciável.

a) Calcule $\frac{\partial}{\partial y}(g \circ f)(1, 1, 0)$, sabendo que $\nabla g(2, 2, 1) = (-1, 0, 3)$.

b) Para $g(u, v, w) = u^2 - v^2 + e^w$, calcule $\frac{\partial}{\partial z}(g \circ f)(0, 1, 0)$.

5 a) $\frac{\partial g}{\partial f} \cdot \frac{\partial f}{\partial y} = \frac{\partial g}{\partial f} \cdot \begin{bmatrix} 2y \\ 1 \\ xe^z \end{bmatrix} =$ $\quad f(1,1,0) = (2,2,1)$

logo $\frac{\partial g}{\partial f} = \nabla g(2,2,1) = (-1,0,3)$

$= \begin{bmatrix} -1 & 0 & 3 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} = -2 + 3 = 1$ //

b) $\frac{\partial}{\partial z}(g \circ f)(0,1,0)$

$\frac{\partial g}{\partial f} \cdot \frac{\partial f}{\partial z} = \frac{\partial g}{\partial f} \cdot \begin{bmatrix} 2z \\ -1 \\ xye^z \end{bmatrix} = \frac{\partial g}{\partial f} \cdot \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 & -2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} = 2$ //

$Dg = \begin{bmatrix} 2u & -2v & e^w \end{bmatrix}$

$(u,v,w) = f(0,1,0) = (1,1,0)$

$\begin{bmatrix} 2 & -2 & 1 \end{bmatrix}$

6. Seja $g : \mathbb{R}^3 \to \mathbb{R}$ uma função diferenciável. Determine

$$\frac{\partial}{\partial x}(g(g(x^2, xy, x + y) + e^x, xy, g(x, x, x)))$$

em função das derivadas parciais de g.

$\frac{\partial}{\partial x} g(\overbrace{g(x^2, xy, x+z) + e^x}^{u}, \overbrace{xy}^{v}, \overbrace{g(x,x,x)}^{w}) =$

$\frac{\partial g}{\partial u} \cdot \frac{\partial u}{\partial x} + \frac{\partial g}{\partial v} \cdot y + \frac{\partial g}{\partial w} \cdot \frac{\partial w}{\partial x} =$

$= \frac{\partial g}{\partial u} \cdot \left[\overbrace{2x \frac{\partial g}{\partial u} + y \frac{\partial g}{\partial v} + \frac{\partial g}{\partial w}}^{\text{em } g(x^2, xy, x+y)} + e^x \right] + y \frac{\partial g}{\partial v} + \frac{\partial g}{\partial w} \cdot \left[\overbrace{\frac{\partial g}{\partial u} + \frac{\partial g}{\partial v} + \frac{\partial g}{\partial w}}^{\text{em } g(x,x,x)} \right]$

$\text{em } g(x,x,x)$

$\frac{\partial w}{\partial x} = \frac{\partial g(x,x,x)}{\partial x} = \frac{\partial g}{\partial u} + \frac{\partial g}{\partial v} + \frac{\partial g}{\partial w}$

$\frac{\partial u}{\partial x} = \frac{\partial (g(x^2, xy, x+y) + e^x)}{\partial x} =$

$= \overbrace{2x \frac{\partial g}{\partial u} + y \frac{\partial g}{\partial v} + \frac{\partial g}{\partial w}}^{\text{em } g(x^2, xy, x+y)} + e^x$

7. Sejam $F : \mathbb{R}^3 \to \mathbb{R}$ e $g : \mathbb{R}^2 \to \mathbb{R}$ funções de classe C^1 e tais que se verifica a equação $F(x, y, g(x, y)) = 0$. Supondo que $\frac{\partial F}{\partial z}(x, y, z) \neq 0$ calcule a derivada $Dg(x, y)$.

7)

$F(x, y, g(x,y)) = 0$

Seja $h(x,y) = (x, y, g(x,y))$

Logo $F(h(x,y)) = 0$

$DF(h(x,y)) = 0$

$\hookrightarrow DF(h(x,y)) \cdot Dh(x,y) = 0 \Leftrightarrow$

$$\Leftrightarrow \begin{bmatrix} \frac{\partial F}{\partial x} & \frac{\partial F}{\partial y} & \frac{\partial F}{\partial z} \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ \frac{\partial g}{\partial x} & \frac{\partial g}{\partial y} \end{bmatrix} = 0 \Leftrightarrow \begin{bmatrix} \frac{\partial F}{\partial x} + \frac{\partial F}{\partial z}\frac{\partial g}{\partial x} & \frac{\partial F}{\partial y} + \frac{\partial F}{\partial z}\frac{\partial g}{\partial y} \end{bmatrix} = 0$$

$\hookrightarrow$ Logo

$$\begin{cases} \frac{\partial F}{\partial x} + \frac{\partial F}{\partial z}\frac{\partial g}{\partial x} = 0 \\ \frac{\partial F}{\partial y} + \frac{\partial F}{\partial z}\frac{\partial g}{\partial y} = 0 \end{cases} \Leftrightarrow$$

$$\begin{cases} \frac{\partial g}{\partial x} = -\frac{\frac{\partial F}{\partial x}}{\frac{\partial F}{\partial z}} \\ \frac{\partial g}{\partial y} = -\frac{\frac{\partial F}{\partial y}}{\frac{\partial F}{\partial z}} \end{cases}$$

$$D_g = \begin{bmatrix} -\frac{\frac{\partial F}{\partial x}}{\frac{\partial F}{\partial z}} & -\frac{\frac{\partial F}{\partial y}}{\frac{\partial F}{\partial z}} \end{bmatrix} //$$

8. Determine a recta tangente e o plano normal à linha definida por

$$\{(e^t, \cos t, \operatorname{sen} t) \; ; \; -\pi < t < \pi\}$$

no ponto $(1, 1, 0)$.

9. Determine a recta normal e o plano tangente ao parabolóide

$$P = \{(x, y, z) \in \mathbb{R}^3 : z = 1 - x^2 - y^2\}$$

no ponto $(0, 1, 0)$.

8) $f(t) = (e^t, \cos(t), \operatorname{sen}(t))$

$e^t = 1 \Leftrightarrow t = 0$

$\frac{df}{dt} = (e^t, -\operatorname{sen}(t), \cos(t))$

$t=0 \quad = (1, 0, 1)$

Recta: $(1,1,0) + \lambda(1,0,1) \quad \lambda \in \mathbb{R}$

Plano $ax + by + cz + d = 0$

$\Leftrightarrow 1\times1 + 1\times0 + 1\times1 = -d \Leftrightarrow$

$\Leftrightarrow d = -1$

$x + z - 1 = 0$

9) $x^2 + y^2 + z = 1$

$f(x, y, z) = x^2 + y^2 + z$

$S_1(f)$

$\nabla f = (2x, 2y, 1)$

$(0,1,0) \downarrow$

$(0, 2, 1)$

Recta: $(0,1,0) + \lambda(0,2,1) \quad \lambda \in \mathbb{R}$

Plano: $0\times0 + 1\times2 + 0\times0 = -d \Leftrightarrow$

$-2 = d$

$2y + z = 2$