1. Calcule o integral da função indicada no rectângulo $\{(x,y)\in\mathbb{R}^2:\ : 0\le x\le 2,\ 0\le y\le 1\}$.

a) $f(x,y)=xy^3$.

b) $f(x,y)=x\cos(xy)$.

1 a) $\int_0^2\int_0^1 xy^3\,dy\,dx = \int_0^2\left[\frac{xy^4}{4}\right]_{y=0}^{y=1}dx = \int_0^2\frac{x}{4}\,dx = \left[\frac{x^2}{8}\right]_0^2 = \frac12$

b) $\int_0^2\int_0^1 x\cos(xy)\,dy\,dx = \int_0^2[\sin(xy)]_0^1\,dx = \int_0^2\sin(x)\,dx =$

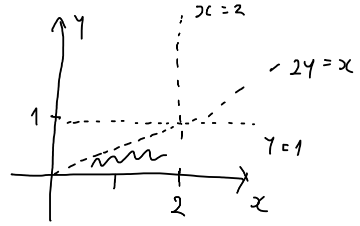
$= [-\cos(x)]_0^2 = 1-\cos(2)$

2. Invertendo a ordem de integração, calcule:

a) $\int_0^1\left(\int_{2y}^2\cos(x^2)\,dx\right)dy$.

b) $\int_0^1\left(\int_{\operatorname{arcsen} y}^{\pi/2} y\operatorname{sen}x\,dx\right)dy$.

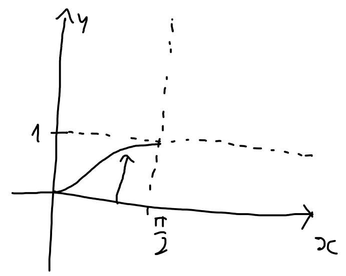
2 a) $2=x$
$2y=x$
$1=y$
$0=y$



$\int_0^2\int_0^{\frac{x}{2}}\cos(x^2)\,dy\,dx = \int_0^2\underbrace{\frac{x}{2}}_{\frac14 u'}\cos(\underbrace{x^2}_{u})\,dx = \frac14\int_0^4\cos(u)\,du =$

$= \left[\frac{\sin(u)}{4}\right]_0^4 = \frac{\sin(4)}{4}$ //

b)
$y=1$
$y=0$
$x=\frac{\pi}{2}$
$x=\arcsin(y)$



$\int_0^{\frac{\pi}{2}}\int_0^{\sin(x)} y\sin(x)\,dy\,dx = \int_0^{\frac{\pi}{2}}\frac{\sin(x)^2}{2}\cdot\sin(x)\,dx = \int_0^{\frac{\pi}{2}}\frac{\sin(x)^3}{2}\,dx = -\frac12\left(\frac13-1\right) = \frac13$ //

$\frac12 P(\sin(x)\cdot\sin(x)^2) = \frac12 P((1-\underbrace{\cos^2(x)}_{u})(\underbrace{\sin(x)}_{u'})) = \frac12 P(u^2-1) = \frac12\left(\frac{\cos(x)^3}{3}-\cos(x)\right)$

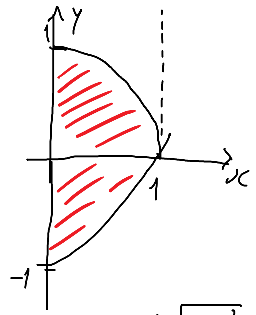
3. Inverta a ordem de integração dos seguintes integrais duplos:

a) $\int_0^1\left(\int_{x^2-1}^{\sqrt{1-x^2}} f(x,y)dy\right)dx$.

b) $\int_0^1\left(\int_{\sqrt{1-x^2}}^{2-x} f(x,y)dy\right)dx$.

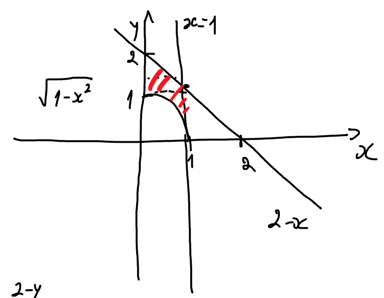
c) $\int_0^{2\pi}\left(\int_{-1}^{\operatorname{sen} y} f(x,y)dx\right)dy$.

a)
$x=1$
$x=0$
$y=\sqrt{1-x^2}$
$y=x^2-1$
$\sqrt{-(y^2-1)}=x$
$\sqrt{y+1}=x$



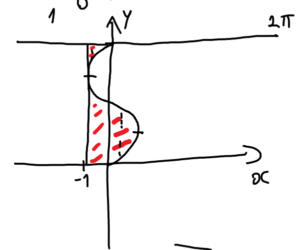
$\int_{-1}^0\int_0^{\sqrt{y+1}} f(x,y)\,dx\,dy + \int_0^1\int_0^{\sqrt{1-y^2}} f(x,y)\,dx\,dy$

b)
$x=1$
$x=0$
$y=2-x$
$y=\sqrt{1-x^2}$



$\int_0^1\int_{\sqrt{1-y^2}}^1 f(x,y)\,dx\,dy + \int_1^2\int_0^{2-y} f(x,y)\,dx\,dy$

c)
$y=2\pi$
$y=0$
$x=\sin(y)$
$x=-1$

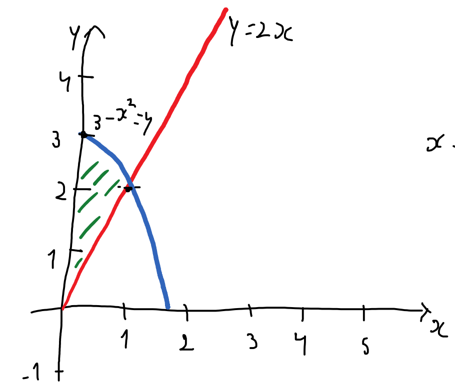


$\int_{-1}^0\left[\int_0^{\pi-\arcsin(x)} f(x,y)\,dy + \int_{2\pi+\arcsin(x)}^{2\pi} f(x,y)\,dy\right]dx \qquad \int_0^1\int_{\arcsin(x)}^{\pi-\arcsin(x)} f(x,y)\,dy\,dx$

4. Calcule a área da região

$$D=\{(x,y)\in\mathbb{R}^2: 0<2x<y<3-x^2\},$$

usando um integral iterado da forma $\int(\int dx)dy$. Calcule ainda (usando a ordem de integração que entender) a coordenada x do centróide.



$x=\sqrt{3-y}$

$\int_0^2\int_0^{\frac{y}{2}} 1\,dx\,dy + \int_2^3\int_0^{\sqrt{3-y}} 1\,dx\,dy =$

$= \int_0^2\frac{y^2}{4}\,dy + \int_2^3\sqrt{3-y}\,dy = 1 + \left[-\frac{2\sqrt{(3-y)}^3}{3}\right]_2^3 = 1+\frac23 = \frac53$ //

$-\int -\sqrt{3-y}\,dy = -\int\sqrt{u}\,du = -\int u^{\frac12}\,du = -\frac{u^{\frac32}}{\frac12+1} = -\frac{2\sqrt{3-y}^3}{3}$

$\int_0^2\int_0^{\frac{y}{2}} x\,dx\,dy + \int_2^3\int_0^{\sqrt{3-y}} x\,dx\,dy =$

$= \int_0^2\frac{y^2}{8}\,dy + \int_2^3\frac{3-y}{2}\,dy = \left[\frac{y^3}{24}\right]_0^2 + \left[\frac32 y - \frac{y^2}{4}\right]_2^3$

$= \frac13 + \frac92 - \frac94 - \frac62 + 1 = \frac{7}{12}$

$\frac{\frac{7}{12}}{\frac{5}{3}} = \frac{7}{12}\times\frac{3}{5} = \frac{7}{4\times5} = \frac{7}{20}$

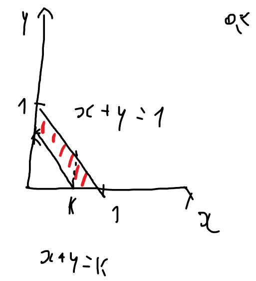
5. Escreva expressões para o volume de V na ordem indicada.

a) $V=\{(x,y,z)\in\mathbb{R}^3: x\geq 0, y\geq 0, x+y\leq 1, 0\leq z\leq x+y\}$ nas ordens $\int\left(\int\left(\int dz\right)dx\right)dy$ e $\int\left(\int\left(\int dy\right)dx\right)dz$.

b) $V=\{(x,y,z)\in\mathbb{R}^3 : x^2+y^2\leq 1\ ;\ y^2+z^2\leq 1\}$ nas ordens $\int\left(\int\left(\int dz\right)dx\right)dy$ e $\int\left(\int\left(\int dz\right)dy\right)dx$.

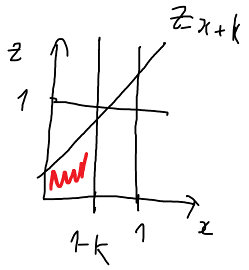
c) $V=\{(x,y,z)\in\mathbb{R}^3 : \frac{x}{2}\leq y\leq x\ ;\ 0\leq z\leq x\ ;\ x\leq 1\}$ nas ordens $\int\left(\int\left(\int dx\right)dz\right)dy$ e $\int\left(\int\left(\int dx\right)dy\right)dz$.

a) Cortes segundo $z=k$, $0\leq k\leq 1$



$$\int_0^1\left[\int_0^{z}\int_{z-x}^{1-x} dy\,dx+\int_z^1\int_0^{1-x} dy\,dx\right]dz$$

Corte segundo $y=k$, $0\leq k\leq 1$

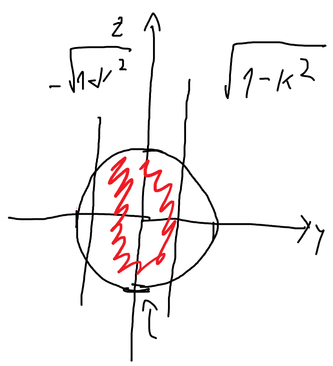


$x=1-k$

$z=x+k$

$$\int_0^1\int_0^{1-y}\int_0^{x+y} dz\,dx\,dy$$

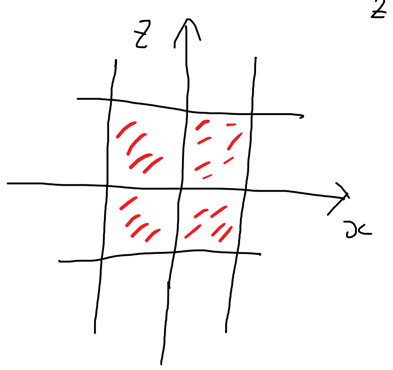
b) cortes com $x=k$



$y\leq\sqrt{1-k^2}$

$$\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} dz\,dy\,dx$$

cortes com $y=k$



$x\leq\pm\sqrt{1-k^2}$

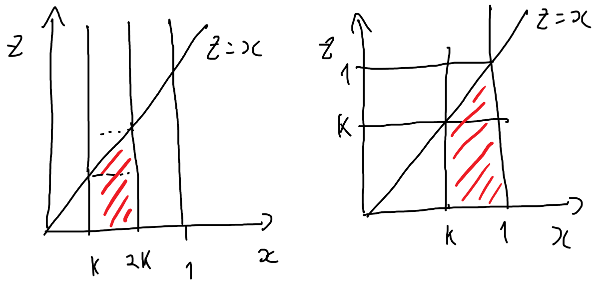
$z\leq\pm\sqrt{1-k^2}$

$$\int_{-1}^{1}\int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}}\int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} dz\,dx\,dy$$

c) cortes a $y=k$

$k\leq x\leq 2k$ $\qquad$ $k\leq x\leq 1$

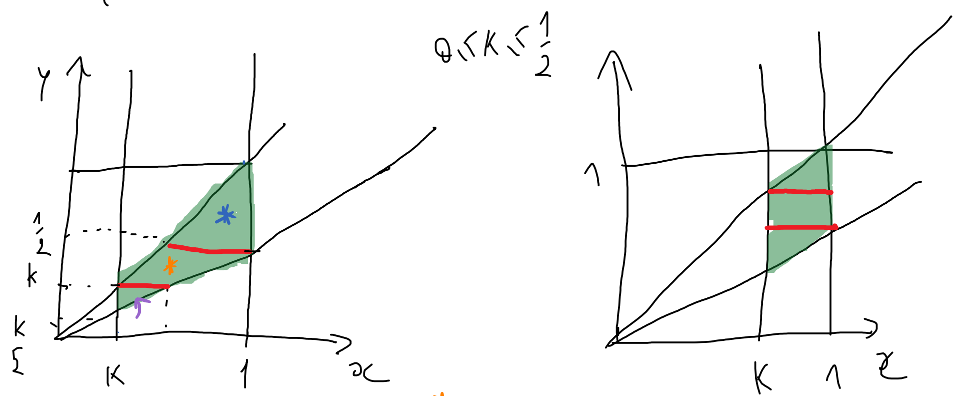
Para $[0;\frac{1}{2}]$ $\qquad$ Para $[\frac{1}{2};1]$



$$\int_0^{1/2}\left[\int_0^{y}\int_y^{2y} dx\,dz+\int_y^{2y}\int_z^{2y} dx\,dz\right]dy+\int_{1/2}^{1}\left[\int_0^{y}\int_y^{1} dx\,dz+\int_y^{1}\int_z^{1} dx\,dz\right]$$

Corte para $z=k$ $\qquad$ $0\leq k\leq x$

$0\leq k\leq\frac{1}{2}$



$$\int_0^{1/2}\left[\int_{z/2}^{z}\int_z^{2y} dx\,dy+\int_z^{1/2}\int_y^{2y} dx\,dy+\int_{1/2}^{1}\int_y^{1} dx\,dy\right]+$$

$$\int_{1/2}^{1}\left[\int_{z/2}^{z}\int_z^{2y} dx\,dy+\int_z^{1/2}\int_z^{1} dx\,dy+\int_z^{1}\int_y^{1} dx\,dy\right]$$

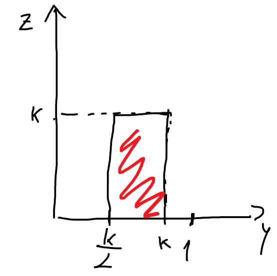
6. Para cada um dos conjuntos seguintes escreva uma expressão para o respectivo volume, usando um único integral triplo:

a) $A=\{(x,y,z)\in\mathbb{R}^3 : \frac{x}{2}\leq y\leq x\ ;\ 0\leq z\leq x\ ;\ x\leq 1\}$,

b) $B=\{(x,y,z)\in\mathbb{R}^3 : x^2+y^2\leq 1\ ;\ 0\leq z\leq x^2-y^2\ ;\ x>0\}$.

a) Cortes a $x=k$ $\qquad$ $0\leq k\leq 1$

$\frac{k}{2}\leq y\leq k$



$$\int_0^1\int_0^{x}\int_{x/2}^{x} dy\,dz\,dx$$

b) $z=k$

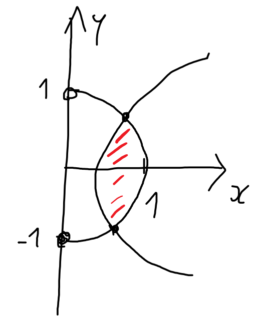
$k=x^2-y^2$

$x^2+y^2=1$

$k+y^2=x^2$

$k+2y^2=1$

$y=\sqrt{\frac{1-k}{2}}$



$$\int_0^1\int_{-\sqrt{\frac{1-z}{2}}}^{\sqrt{\frac{1-z}{2}}}\int_{\sqrt{z+y^2}}^{\sqrt{1-y^2}} dx\,dy\,dz$$

7. Considere a região

$$V=\{(x,y,z)\in\mathbb{R}^3 : x+y+2z\le 1\ ;\ x+y-2z\le 1\ ;\ x\ge 0\ ;\ y\ge 0\}.$$

Calcule o volume de V na forma:

a) $\int_{\cdots}^{\cdots}\left(\int_{\cdots}^{\cdots}\left(\int_{\cdots}^{\cdots}\cdots dy\right)dx\right)dz$.

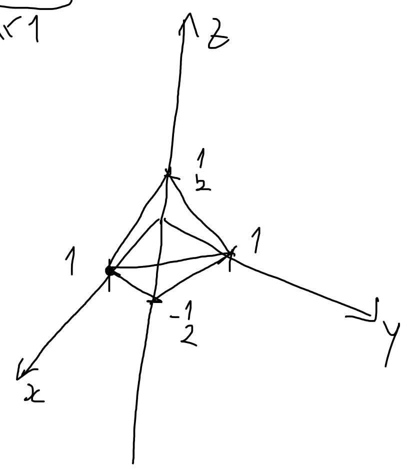
b) $\int_{\cdots}^{\cdots}\left(\int_{\cdots}^{\cdots}\left(\int_{\cdots}^{\cdots}\cdots dz\right)dx\right)dy$.

a) $\boxed{x+y+2z\le 1}$

$x+y-2z\le 1$

$x\ge 0$

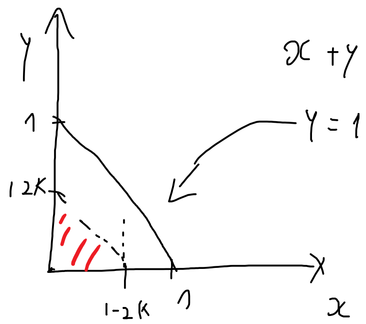
$y\ge 0$



$-\frac{1}{2}\le k\le\frac{1}{2}$

$k=z$

a)



$x+y+2z=1$

$y=1-x$

$$V=\int_0^{1/2}\int_0^{1-2z}\int_0^{1-2z-x} 2\,dy\,dx\,dz=$$

$$=2\times\int_0^{1/2}\int_0^{1-2z} 1-2z-x\,dx\,dz=$$

$$=2\times\int_0^{1/2}\left[x-2zx-\frac{x^2}{2}\right]_0^{1-2z}dz=$$

$$=2\times\int_0^{1/2}\frac{(1-2z)^2}{2}dz=2\times\left[\frac{z}{2}-z^2+\frac{2z^3}{3}\right]_0^{1/2}=2\times\frac{1}{12}=\frac{1}{6}$$

$$1-2z-2z(1-2z)-\frac{(1-2z)^2}{2}=$$

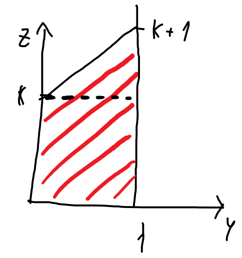
$$=(1-2z)(1-2z)\left(1-\frac{1}{2}\right)$$

$$=\frac{(1-2z)^2}{2}$$

8. Calcule $\int_V f$ sendo $f:\mathbb{R}^3\to\mathbb{R}$ a função definida por $f(x,y,z)=z$ e V o sólido limitado pelos planos $x=0$, $x=1$, $y=0$, $y=1$, $z=0$ e $z=x+y$.

$x=k$

$0\le k\le 1$



$z=k+y$

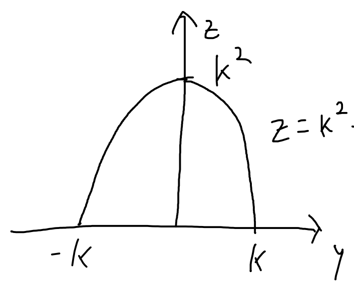
$$\int_0^1\int_0^1\int_0^{x+y} z\,dz\,dy\,dx=\int_0^1\int_0^1\frac{(x+y)^2}{2}dy\,dx=\int_0^1\left[\frac{x^2}{2}y+\frac{xy^2}{2}+\frac{y^3}{6}\right]_{y_0}^{y=1}dx=$$

$$=\int_0^1\frac{x^2}{2}+\frac{x}{2}+\frac{1}{6}\,dx=\left[\frac{x^3}{6}+\frac{x^2}{4}+\frac{x}{6}\right]_0^1=\frac{1}{6}+\frac{1}{4}+\frac{1}{6}=\frac{7}{12}$$

9. Calcule a primeira coordenada do centróide do sólido limitado pela superfície $z=x^2-y^2$, o plano xy e os planos $x=0$ e $x=1$.

$x=k$

$0\le k\le 1$



$z=k^2-y^2$

$$\int_0^1\int_{-x}^{x}\int_0^{x^2-y^2}1\,dz\,dy\,dx=$$

$$=\int_0^1\int_{-x}^{x}x^2-y^2\,dy\,dx=$$

$$=\int_0^1\frac{4}{3}x^3\,dx=$$

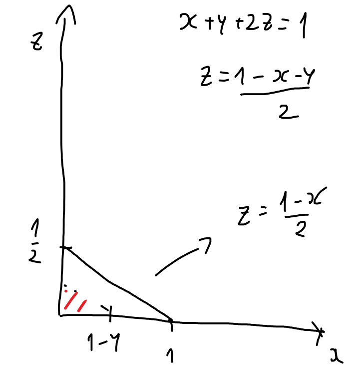
$$=\frac{4}{3}\times\frac{1}{4}=\frac{1}{3}$$

$$\int_0^1\int_{-x}^{x}\int_0^{x^2-y^2}x\,dz\,dy\,dx=\int_0^1\int_{-x}^{x}x(x^2-y^2)\,dy\,dx=\int_0^1\frac{4}{3}x^4\,dx=\frac{4}{15}$$

$$\frac{\frac{4}{15}}{\frac{1}{3}}=\frac{4}{15}\times 3=\frac{4}{5}$$

7 b)

$0\le y\le 1$



$x+y+2z=1$

$z=\frac{1-x-y}{2}$

$z=\frac{1-x}{2}$

$$V=\int_0^1\int_0^{1-y}\int_0^{\frac{1-x-y}{2}}2\,dz\,dx\,dy=$$

$$=\int_0^1\int_0^{1-y}1-x-y\,dx\,dy=\int_0^1\left[x-\frac{x^2}{2}-yx\right]_0^{1-y}dy=$$

$$\int_0^1 1-y-\frac{(1-y)^2}{2}-y(1-y)\,dy=\int_0^1\frac{1-y^2}{2}dy=$$

$$\left[\frac{1}{2}y-\frac{y^2}{2}+\frac{y^3}{6}\right]_0^1=\frac{1}{2}-\frac{1}{2}+\frac{1}{6}=\frac{1}{6}$$

C.A.

$$1-y-\frac{(1-y)^2}{2}-y(1-y)=(1-y)\left(1-y-\frac{(1-y)}{2}\right)=$$

$$=(1-y)^2\left(1-\frac{1}{2}\right)=\frac{1-y^2}{2}$$