1. Mostre que cada um dos conjuntos seguintes é uma variedade, determine a respectiva dimensão e descreva-o parametricamente:

   a) $\{(x,y)\in\mathbb{R}^2:\ y=x^3;\ -\infty<x<+\infty\}$.
   b) $\{(x,y)\in\mathbb{R}^2:\ 4x^2+9y^2=1\}$.
   c) $\{(x,y,z)\in\mathbb{R}^3:\ y^2+z^2=1;\ y>0;\ z>0;\ |x|<1\}$.
   d) $\{(x,y,z)\in\mathbb{R}^3:\ x^2+y^2=z<1\}$.
   e) $\{(x,y,z)\in\mathbb{R}^3:\ (\sqrt{x^2+y^2}-3)^2+z^2=1;\ z>0;\ x>0;\ y>0\}$.
   f) $\{(x,y,z)\in\mathbb{R}^3:\ x^2+y^2+z^2=1;\ z>0;\ y>|x|\}$.
   g) $\{(x,y,z)\in\mathbb{R}^3:\ x^2+z^2=y^2+1;\ |y|<1;\ x>0\}$.
   h) $\{(x,y,z)\in\mathbb{R}^3:\ z=x^2+y^2<1;\ x+y=1;\ x>0;\ y>0\}$.

a) Dim 1 $\quad -\infty<x<+\infty$

$g(x)=(x,x^3)$

b) Dim 1

$(2x)^2+(3y)^2=1 \qquad 0<\theta<2\pi$

$2x=R\cos(\theta)$
$3y=R\sin(\theta)$

$g(\theta)=\left(\frac{\cos(\theta)}{2},\frac{\sin(\theta)}{3}\right)$

c) Dim 2

$\begin{cases} y^2+z^2=1\\ y>0\\ z>0\\ |x|<1\end{cases}$

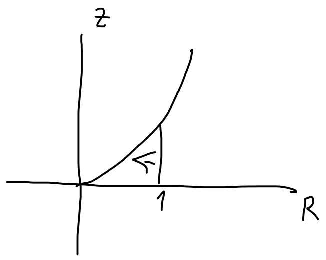
$g(x,\theta)=(x,\cos(\theta),\sin(\theta))$
$-1<x<1$
$0<t<\frac{\pi}{2}$

Dim 2

d) $x^2+y^2=z<1$

$R^2=x^2+y^2$

$x=R\cos(\theta)$
$y=R\sin(\theta)$



$g(\theta,z)=(z\cos(\theta),z\sin(\theta),z)$
$0<z<1$
$0<\theta<2\pi$

e) $\begin{cases}(\sqrt{x^2+y^2}-3)^2+z^2=1\\ z>0\\ x>0\\ y>0\end{cases}$

$g(\theta,\phi)=(\cos(\theta)(3+\cos(\phi)),\sin(\theta)(3+\cos(\phi)),\sin(\phi))$

$0<\theta<\frac{\pi}{2}$
$0<\phi<\pi$

f) $\begin{cases}x^2+y^2+z^2=1\\ z>0\\ y>|x|\end{cases}$

$g(\theta,\phi)=(\sin(\phi)\cos(\theta),\sin(\phi)\sin(\theta),\cos(\phi))$

$\frac{\pi}{4}<\theta<\frac{3\pi}{4} \qquad 0<\phi<\frac{\pi}{2}$

g) $\sqrt{x^2+z^2-1}=y$ $\quad$ dim 2

$g(x,z)=(x,\sqrt{x^2+z^2-1},z)$

$x>0$

$x^2+z^2<4$

h) $\begin{cases} z=x^2+y^2<1\\ x+y=1\\ x>0\\ y>0\end{cases}$

$\begin{cases} x^2+y^2<1\\ x+y=1\end{cases} \Leftrightarrow \begin{matrix}(1-y)^2+y^2<1\\ x=1-y\end{matrix}$

Dim 1

$g(y)=(1-y,\ y,\ (1-y)^2+y^2)$

$0<y<1$

2. Determine o espaço tangente e o espaço normal à variedade

$$A=\{(x,y,z)\in\mathbb{R}^3: z=x^2+y^2, x=y\}$$

no ponto $(1,1,2)$.

3. Determine o espaço tangente e o espaço normal à variedade

$$S=\{(x,y,z)\in\mathbb{R}^3:\ x^2+y^2=z^2+1\},$$

no ponto $(0,1,0)$.

2.) $g(x)=(x,x,2x^2)$

$Dg=\begin{bmatrix}1\\1\\4x\end{bmatrix}\to\begin{bmatrix}1\\1\\4\end{bmatrix}$

$T_gA=\{\lambda(1,1,4);\ \lambda\in\mathbb{R}\}$

$T_gA^{\perp}=\{a(1,0,-\frac{1}{4})+b(0,1,-\frac{1}{4});\ a,b\in\mathbb{R}\}$

3.) $g(x,z)=(x,\sqrt{z^2-x^2+1},z)$

$Dg\begin{bmatrix}1 & 0\\ \frac{x}{\sqrt{z^2-x^2+1}} & \frac{z}{\sqrt{z^2-x^2+1}}\\ 0 & 1\end{bmatrix}\to\begin{bmatrix}1&0\\0&0\\0&1\end{bmatrix}$

$T_gS=\{a(1,0,0)+b(0,0,1);\ b,a\in\mathbb{R}\}$

$T_gS^{\perp}=\{\lambda(0,1,0);\ \lambda\in\mathbb{R}\}$

4. Determine a recta tangente e o plano normal à linha definida por

$$\{(\cos t,\operatorname{sen} t,\operatorname{sen}(2t));\ t\in\mathbb{R}\},$$

no ponto $(1,0,0)$.

5. Determine a recta normal e o plano tangente à superfície definida por

$$\{(x,y,xy);\ x,y\in\mathbb{R}\},$$

no ponto $(1,1,1)$.

4) $g(t)=(\cos(t),\sin(t),\sin(2t))$ $\quad t=0$

$Dg=\begin{bmatrix}-\sin(t)\\ \cos(t)\\ 2\cos(2t)\end{bmatrix}\to\begin{bmatrix}0\\1\\2\end{bmatrix}$

$R:(0,1,2)k,\ k\in\mathbb{R}$

Plano: $(1,-1,0)a+(2,0,-1)b$ $\quad a,b\in\mathbb{R}$

5) $g(x,y)=(x,y,xy)$

$Dg=\begin{bmatrix}1&0\\0&1\\y&x\end{bmatrix}\to\begin{bmatrix}1&0\\0&1\\1&1\end{bmatrix}$

$R: a(1,0,1)+b(0,1,1)$ $\quad a,b\in\mathbb{R}$

Plano: $\lambda(-1,-1,1)$ $\quad \lambda\in\mathbb{R}$