1. Sejam $f : \mathbb{R} \to \mathbb{R}^3$, $g : \mathbb{R}^3 \to \mathbb{R}$ e $h : \mathbb{R}^2 \to R^3$ as funções

$$f(t) = (1+t^3, 1+\operatorname{sen} t, t), \quad g(x,y,z) = xe^{z^2} + zy \quad \text{e} \quad h(x,y) = (x, y^2, xy)$$

Calcule $D(g\circ h)(1,1)$ e $D(f \circ \frac{\partial g}{\partial z})(0, \pi/2, 0)$.

2. Sejam $f : \mathbb{R} \to \mathbb{R}^3$ e $g : \mathbb{R}^3 \to \mathbb{R}$ as funções

$$f(t) = (1+t^3, 1+\operatorname{sen} t, \log(1+t)) \quad \text{e} \quad g(x,y,z) = e^{xyz} - 1.$$

Calcule $D(g\circ f)(0)$ e $D(f\circ g)(1,1,0)$.

$$1)\quad D(g\circ h) = Dg(h(x,y)) \cdot Dh(x,y) = \begin{bmatrix} e & 1 & 2e+1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 2 \\ 1 & 1 \end{bmatrix} =$$

$$Dh = \begin{bmatrix} 1 & 0 \\ 0 & 2y \\ y & x \end{bmatrix} \xrightarrow{x,y=1,1} \begin{bmatrix} 1 & 0 \\ 0 & 2 \\ 1 & 1 \end{bmatrix} \qquad = \begin{bmatrix} 3e+1 & 2e+3 \end{bmatrix}$$

$$Dg = \begin{bmatrix} e^{z^2} & z & 2xze^{z^2}+y \end{bmatrix}$$

$$h(1,1) = (1,1,1) \downarrow$$

$$\begin{bmatrix} e & 1 & 2e+1 \end{bmatrix}$$

$$D(f\circ \frac{\partial g}{\partial z}) = Df(\frac{\partial g}{\partial z}(x,y,z)) \cdot D\frac{\partial g}{\partial z}(x,y,z)$$

$$\frac{\partial g}{\partial z}(x,y,z) = \begin{bmatrix} 2e^{z^2}z & 1 & 4xze^{z^2} \end{bmatrix} \xrightarrow{(0,\frac{\pi}{2},0)} \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}$$

$$\frac{\partial g}{\partial z}(0,\frac{\pi}{2},0) = 2\cdot 0\cdot 0\cdot e^{0^2} + \frac{\pi}{2} = \frac{\pi}{2}$$

$$Df = \begin{bmatrix} 3t^2 \\ \cos(t) \\ 1 \end{bmatrix} \xrightarrow{t=\frac{\pi}{2}} \begin{bmatrix} \frac{3}{4}\pi^2 \\ 0 \\ 1 \end{bmatrix} \qquad \begin{bmatrix} \frac{3}{4}\pi^2 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & \frac{3}{4}\pi^2 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$2)\quad Df = \begin{bmatrix} 3t^2 \\ \cos(t) \\ \frac{1}{1+t} \end{bmatrix} \qquad Dg = \begin{bmatrix} yze^{xyz} & xze^{xyz} & xye^{xyz} \end{bmatrix}$$

$$f(0) = (1,1,0) \qquad g(1,1,0) = 0$$

$$D\,g\circ f(0) = Dg(f(0)) \cdot Df(0) =$$

$$= \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 3\cdot 0 \\ \cos(0) \\ \frac{1}{1+0} \end{bmatrix} = 1$$

$$D\,f\circ g(1,1,0) = Df(g(1,1,0)) \cdot Dg(1,1,0) =$$

$$= \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

3. Seja $f : \mathbb{R}^2 \to \mathbb{R}$ uma função diferenciável e $g : \mathbb{R} \to \mathbb{R}$ a função $g(t) = f(t, t^2)$. Justifique que g é diferenciável e determine $g'(2)$.

4. Seja $f = f(x,y)$ uma função diferenciável e seja $g(u,v) = f(e^u + \operatorname{sen} v, e^u + \cos v)$. Calcule $\frac{\partial g}{\partial u}(0,0)$ e $\frac{\partial g}{\partial v}(0,0)$ usando a seguinte tabela de valores

	f	g	$\frac{\partial f}{\partial x}$	$\frac{\partial f}{\partial y}$
(0,0)	3	6	4	8
(1,2)	6	3	2	5

$$3)\quad g(t) = (f\circ h(t))$$

$$h(t) = (t, t^2) \to \text{Dif.}$$

Como f e h são dif. então g tb é

$$g'(2) = Dg(2) = D\,f\circ h(2) = Df(h(2)) \cdot Dh(2) =$$

$$= Df(h(2)) \cdot \begin{bmatrix} 1 \\ 2\times 2 \end{bmatrix} = Df(2,4) \cdot \begin{bmatrix} 1 \\ 4 \end{bmatrix}$$

$$4.\quad g = f\circ h(u,v) \qquad h(u,v) = (\underbrace{e^u + \operatorname{sen}(v)}_{h_1}, \underbrace{e^u + \cos(v)}_{h_2})$$

$$(u,v) = (0,0)$$

$$\frac{\partial g}{\partial u} = \frac{\partial f}{\partial h_1}\cdot\frac{\partial h_1}{\partial u} + \frac{\partial f}{\partial h_2}\cdot\frac{\partial h_2}{\partial u} = e^u\frac{\partial f}{\partial h_1} + e^u\frac{\partial f}{\partial h_2} = \frac{\partial f}{\partial h_1} + \frac{\partial f}{\partial h_2} = 2+5=7$$

$$h(0,0) = (1,2)$$

$$\frac{\partial g}{\partial v} = \frac{\partial f}{\partial h_1}\cdot\frac{\partial h_1}{\partial v} + \frac{\partial f}{\partial h_2}\cdot\frac{\partial h_2}{\partial v} = \cos(v)\frac{\partial f}{\partial h_1} - \operatorname{sen}(v)\frac{\partial f}{\partial h_2} = \frac{\partial f}{\partial h_1} = 2$$

$$1\times\frac{\partial f(h_1)}{\partial x} - 0\cdot\frac{\partial f(h_2)}{\partial y}$$

5. Seja $u(r,s,t) = f(x(r,s,t), y(r,s,t), z(r,s,t))$ onde $f(x,y,z) = x^4y + y^2z^3$, $x(r,s,t) = rse^t$, $y(r,s,t) = rs^2e^{-t}$ e $z(r,s,t) = r^2s\operatorname{sen}t$. Calcule

$$\frac{\partial u}{\partial s}(2,1,0)$$

$$\frac{\partial u}{\partial s} = \frac{\partial f}{\partial x}\cdot\frac{\partial x}{\partial s} + \frac{\partial f}{\partial y}\cdot\frac{\partial y}{\partial s} + \frac{\partial f}{\partial z}\cdot\frac{\partial z}{\partial s} = \qquad x(2,1,0)=2$$

$$y(2,1,0)=2$$

$$z(2,1,0)=0$$

$$= \frac{\partial f}{\partial x}\cdot re^t + \frac{\partial f}{\partial y}\cdot 2rse^{-t} + \frac{\partial f}{\partial z}\cdot r^2\operatorname{sen}(t) =$$

$$= (4x^3y)\cdot re^t + (x^4 + 2yz^3)\cdot 2rse^{-t} + (3z^2)\cdot r^2\operatorname{sen}(t) =$$

$$= (4\cdot 2^3\cdot 2)\cdot 2\cdot 1 + (2^4 + 2\times 2\cdot 0)\cdot 2\times 2\cdot 1\cdot 1 + 0 =$$

$$= 128 + 64 = 192$$

6. Seja $f : \mathbb{R}^n \to \mathbb{R}$ uma função tal que $f(tx) = t^m f(x)$ para todo $x \in \mathbb{R}^n \setminus \{0\}$ e $t > 0$ (homogeneidade de grau m). Assumindo que f é diferenciável, mostre que para cada $x \in \mathbb{R}^n \setminus \{0\}$ se tem

$$\nabla f(x) \cdot x = m\, f(x).$$

7. Suponha que f é um campo escalar diferenciável numa bola $B(p) \subset \mathbb{R}^N$, $p \in \mathbb{R}^N$. Se $\nabla f(x) = 0$ para cada $x \in B(p)$, prove que f é constante em $B(p)$.

7) $\nabla f(x) = 0 \Rightarrow Df = 0$

se $Df = 0$ então $f = c$ em $B(p)$

6. $f(tx) = t^m f(x)$?

$Df_{(x)} = [- \nabla f(x) -]$