1. Calcule os seguintes integrais iterados

(a) $\int_0^2 \left[\int_0^1 (xy)^2 \cos(x^3)\, dx\right] dy$

(b) $\int_0^1 \left[\int_{-1}^0 \left[\int_0^2 (3x+2y+z)^2\, dz\right] dy\right] dx$

(c) $\int_0^1 \left[\int_0^1 \left[\int_0^1 e^{x+y+z}\, dz\right] dy\right] dx$

a) $\displaystyle\int_0^2\int_0^1 x^2\cos(x^3)\cdot y^2\, dx\, dy = \int_0^2 \left[\frac{\sin(x^3)}{3}\cdot y^2 x\right]_{x=0}^{x=1} dy = \int_0^2 \frac{\sin(1)}{3}\cdot y^2\, dy$

$\displaystyle= \left[\frac{\sin(1)}{3}\cdot\frac{y^3}{3}\right]_0^2 = \frac{\sin(1)}{3}\cdot\frac{8}{3}$ //

b) $\displaystyle\int_0^1\int_{-1}^0\int_0^2 (3x+2y+z)^2\, dz\, dy\, dx = \int_0^1\int_{-1}^0 \left[\frac{(3x+2y+z)^3}{3}\right]_{z=0}^{z=2} dy\, dx =$

$\displaystyle\int_0^1\int_{-1}^0 \frac{(3x+2y+2)^3}{3} - \frac{(3x+2y)^3}{3}\, dy\, dx =$

$\displaystyle\int_0^1 \left[\frac{(3x+2y+2)^4}{24} - \frac{(3x+2y)^4}{24}\right]_{-1}^0 dx = \int_0^1 \frac{(3x+2)^4}{24} - \frac{(3x)^4}{24} - \frac{(3x)^4}{24} + \frac{(3x-2)^4}{24}\, dx =$

$\displaystyle\left[\frac{(3x+2)^5}{360} - \frac{(3x)^5}{180} + \frac{(3x-2)^5}{360}\right]_0^1 = \frac{5^5}{360} - \frac{3^5}{180} + \frac{1^5}{360} - \frac{2^5}{360} - \frac{(-2)^5}{360} = \frac{22}{3}$ //

c) $\displaystyle\int_0^1\int_0^1\int_0^1 e^{x+y+z}\, dz\, dy\, dx =$

$\displaystyle\int_0^1\int_0^1 \left[e^{x+y+z}\right]_0^1 dy\, dx =$

$\displaystyle\int_0^1\int_0^1 e^{x+y+1} - e^{x+y}\, dy\, dx =$

$\displaystyle\int_0^1 e^{x+2} - e^{x+1} - e^{x+1} + e^{x}\, dx =$

$e^3 - e^2 - e^2 + e^1 - e^2 + e^1 + e^1 - 1 =$

$e^3 - 3e^2 + 3e - 1$ // $= (e-1)^3$

2. Calcule

(a) $\int\int_R [(xy)^2\cos(x^3)]\,dx\,dy$ onde $R=[0,2]\times[0,1]$

(b) $\int\int\int_R [ze^{x+y}]\,dx\,dy\,dz$ onde $R=[0,1]^3$

(c) $\int\int\int_R [\cos(\pi(x+y+z))]\,dx\,dy\,dz$ onde $R=[0,1]\times[1,2]\times[2,3]$

(d) $\int\int\int_R [e^{x+y+z}]\,dx\,dy\,dz$ onde $R=[0,1]^3$

d) = 1 a)

b) $\int_0^1\int_0^1\int_0^1 z e^{x+y}\,dx\,dy\,dz = \int_0^1\int_0^1 \left[z e^{x+y}\right]_0^1 dy\,dz =$

$= \int_0^1\int_0^1 z e^y(e-1)\,dy\,dz = \int_0^1 z(e-1)^2\,dz = \left[\frac{z^2(e-1)^2}{2}\right]_0^1 = \frac{(e-1)^2}{2}$ //

c) $\int_0^1\int_1^2\int_2^3 \cos(\pi(x+y+z))\,dx\,dy\,dz = \int_0^1\int_1^2 \left[\frac{\sin(\pi(x+y+z))}{\pi}\right]_{x=2}^{x=3} dy\,dz =$

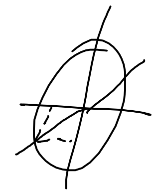
$= \int_0^1\int_1^2 \frac{-2}{\pi}\sin(\pi(y+z))\,dy = \int_0^1 \frac{-2}{\pi^2}\left[-\cos(\pi(y+z))\right]_{y=1}^{y=2} dz =$

$= \int_0^1 \frac{4\cos(\pi z)}{\pi^2}\,dz = \frac{4\sin(\pi)}{\pi^3} - \frac{4\sin(0)}{\pi^3} = 0$ //

d) = 1 c)

$\frac{\sin(3\pi+\pi(y+z)) - \sin(2\pi+\pi(y+z))}{\pi} =$

$\frac{-2\sin(\pi(y+z))}{\pi}$



$-\cos(2\pi+z\pi)+\cos(\pi+z\pi) = -2\cos(z\pi)$