1 -

$K = 2\ W\,m^{-1}K^{-1}$ $\quad L = 10\ cm$ $\quad T_h = 60°C$ $\quad J = 80\ W\,m^{-2}$ $\quad A = \infty$

$$J = -K\frac{\partial T}{\partial x} \Rightarrow \frac{dT}{dx} = -\frac{J}{K} \Rightarrow T(x) = -\frac{J}{K}x + \alpha$$

$$T_h = T(0) = \alpha = 60°C \rightarrow T(x) = -\frac{J}{K}x + 60°C$$

$$T_c = T(L) = -\frac{J}{K}L + 60°C = 56°C$$

2 -

$R = 20\ cm$

$T_{int} = 500K$

$K = 0,5\ W\,m^{-1}K^{-1}$

$e = 2\ cm$

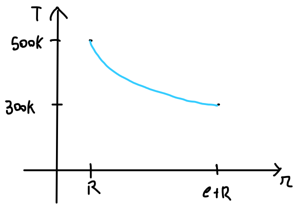
a) $$\frac{I}{ds} = -K\frac{dT}{dr} \Leftrightarrow -\frac{dT}{I} = \frac{1}{K}\cdot\frac{1}{2\pi L r}dr$$

$$ds = r\,d\theta\,dz = 2\pi r L$$

$$R_{Th} = \frac{\Delta T}{I} = \frac{\int -dT}{I} = \int_R^{R+e}\frac{1}{K}\cdot\frac{1}{2\pi L r}dr = \frac{1}{2\pi K L}\int_R^{R+e}\frac{1}{r}dr = \frac{\ln\left(1+\frac{e}{R}\right)}{2\pi K L}$$

b) $T_{ext} = 300K$ $$P_{Th} = \frac{\Delta T}{R_{Th}} = \frac{T_{int}-T_{ext}}{R_{Th}} \Leftrightarrow \frac{P_{Th}}{L} = \frac{2\pi K(T_{int}-T_{ext})}{\ln\left(1+\frac{e}{R}\right)} \approx 6.6\ kW\,m^{-1}$$

c)



$$T(r) = T_{int} - R_{Th}(r)\,I$$

3 -

$R_{int} = 10\ mm$

$T_{int} = 500K$

$K = 0,1\ W\,m^{-1}K^{-1}$

$e = 5\ mm$

$h_{ext} = 5\ W\,m^{-2}K^{-1}$

a) $$\frac{I}{ds} = -K\frac{dT}{dr} \Leftrightarrow -\frac{dT}{I} = \frac{1}{K}\cdot\frac{1}{2\pi L r}dr$$

$$ds = r\,d\theta\,dz = 2\pi r L$$

$$J = \frac{I}{ds} = -h\,dT \Leftrightarrow \frac{\Delta T}{I} = \frac{1}{2\pi h R_{ext} L} = R_{conv}$$

$$R_{cond} = \frac{\Delta T}{I} = \frac{\int -dT}{I} = \int_R^{R+e}\frac{1}{K}\cdot\frac{1}{2\pi L r}dr = \frac{1}{2\pi K L}\int_{R_{int}}^{R_{ext}}\frac{1}{r}dr = \frac{\ln\left(\frac{R_{ext}}{R_{int}}\right)}{2\pi K L}$$

$$R_{Th} = R_{cond} + R_{conv} = \frac{\ln\left(\frac{R_{ext}}{R_{int}}\right)}{2\pi K L} + \frac{1}{2\pi h R_{ext} L}$$

b) $$P_{Th} = \frac{\Delta T}{R_{Th}} = \frac{T_{int}-T_{ext}}{R_{Th}} \Leftrightarrow \frac{P_{Th}}{L} = \frac{T_{int}-T_{ext}}{\frac{\ln\left(\frac{R_{ext}}{R_{int}}\right)}{2\pi K L} + \frac{1}{2\pi h R_{ext} L}} = 72\ W\,m^{-1}$$

c)

$$R_{ext} = R_{int} + e \Rightarrow R_{Th}(e) = \frac{\ln\left(1+\frac{e}{R_{int}}\right)}{2\pi K L} + \frac{1}{2\pi h L(R_{int}+e)}$$

$$\frac{dR_{Th}}{de} = 0 \Leftrightarrow \frac{1}{K}\frac{1}{R_{int}+e} = \frac{1}{h}\frac{1}{(R_{int}+e)^2} \Leftrightarrow e = \frac{K}{h} - R_{int} = \frac{0.1}{5} - 10^{-2} \approx 1\ cm$$

4 - $T_{in} = 22^\circ C$ $\quad L_v = 4\,mm$
$T_{ex} = 12^\circ C$ $\quad L_a = 1\,cm$

$K_v = 0.8\ W m^{-1} K^{-1}$ $\quad h_{ar} = 7\ W m^{-2} K^{-1}$

$h_{in} = 8\ W m^{-2} K^{-1}$ $\quad h_{ex} = 25\ W m^{-2} K^{-1}$

a) $A = 1\,m^2$

a1) $P_{Th} = \frac{\Delta T}{R_{Th}} = 31.46\ W$

$R_{Th} = R_{conv_{ex}} + R_{cond_V} + R_{conv_{ar}} + R_{cond_V} + R_{conv_{in}}$

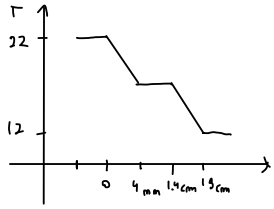
$R_{convV_{ex}} = \frac{1}{h_{ex} A}$ $\quad R_{convV_{in}} = \frac{1}{h_{in} A}$

$R_{cond_V} = \frac{L_V}{K_V A}$ $\quad R_{conv_{ar}} = \frac{1}{h_{ar} A}$

$R_{Th} = \frac{1}{h_{ex} A} + \frac{1}{h_{in} A} + \frac{2 L_V}{K_V A} + \frac{1}{h_{ar} A} =$

$= 0.318$

a2)



b1) $L_c = 2{,}5\,cm$

$A_c = 0.1\,m^2$

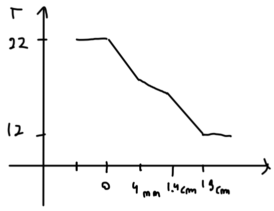
$K_c = 5\ W m^{-1} K^{-1}$

$R_{Th_c} = \frac{L_c}{A_c K_c} + \frac{1}{A_c h_{in}} + \frac{1}{A_c h_{ext}} = 0{,}05 + 1{,}25 + 0{,}4$

$R'_{Th} = R_{Th_c} // R_{Th}$

$P'_{Th} = \frac{\Delta T}{R_{Th}} + \frac{\Delta T}{R_{Th_c}} = 37.3\ W$

b2)



5 - $N_T = 10$ $\quad r_{in} = 2\,cm$ $\quad T_{in} = 29^\circ C$
$r_{ext} = 3\,cm$ $\quad T_{ex} = 28.99^\circ C$
$L = 1\,m$

$I_{Th} = 100\ W$

$h_{ex} = 10\ W m^{-2} K^{-1}$
$h_{in} = 1000\ W m^{-2} K^{-1}$

a) $I_{tubo} = \frac{I_{Th}}{N_T} = 10\ W$

$R_{conv_{ex}} = \frac{1}{h_{ex}\, 2\pi r_{ex} L} \simeq 0.53\ K W^{-1}$

$T_{ex} - T_S = R_{conv_{ex}} I_{tubo} \Leftrightarrow T_S = T_{ex} - R_{convV_{ex}} I_{tubo} = 23{,}68^\circ C$

b) $T_{água} - T_{in} = R_{conv_{in}} I_{tubo}$

$\hookrightarrow T_{água} = T_{in} + \frac{1}{h_{in}\, 2\pi r_{in} L} I_{tubo} = 29.08^\circ C$

$R_{conv_{in}} = \frac{1}{h_{in}\, 2\pi r_{in} L}$

c) $\begin{cases} J = -K \frac{dT}{dr} \\ J = \frac{I_{cond}}{2\pi r L} \end{cases} \Rightarrow -dT = \frac{I_{cond}}{2\pi L K} \frac{dr}{r} \Rightarrow \Delta T = \ln\left(\frac{r_{ex}}{r_{in}}\right) \frac{I_{cond}}{2\pi L K}$

$R_{Th_{cond}} = \ln\left(\frac{r_{ex}}{r_{in}}\right) \cdot \frac{1}{2\pi L K} \Rightarrow K = \ln\left(\frac{r_{ex}}{r_{in}}\right) \cdot \frac{1}{2\pi L R_{Th}} = 64.5\ W m^{-1} K^{-1}$

6 - $T_Q = 400\,K$ $\quad L_m = 2\,mm$ $\quad h_{ar} = 10^2\ W m^{-2} K^{-1}$
$K_m = 20\ W m^{-1} K^{-1}$
$T_F = 300\,K$ $\quad A_m = 1\,m^2$ $\quad h_{água} = 10^4\ W m^{-2} K^{-1}$

a) $R_{Th} = R_{conv_{água}} + R_{conv_{ar}} + R_{cond} = \frac{1}{A_m h_{água}} + \frac{1}{A_m h_{ar}} + \frac{L_m}{K_m A_m} = 0.0102$

b) $T_{FQ} = T_Q + R_{Th} \cdot |\dot{Q}_Q| = 420.4\,K$

$T_{FF} = T_F - R_{Th} \cdot |\dot{Q}_F| = 284.7\,K$

c) $d\dot{S} = \frac{d\dot{Q}}{T} \Rightarrow \Delta\dot{S} = \int \frac{1}{T} d\dot{Q} = -\frac{|\dot{Q}_Q|}{T_{FQ}} + \frac{|\dot{Q}_F|}{T_{FF}} = 0.5\ W K^{-1}$ //

d) Diminuir R_{Th} //

7 -

$$F(v) = \left(\frac{m}{2\pi K_B T}\right)^{3/2} e^{-\frac{1}{2}\frac{m|v|^2}{K_B T}}$$

a) $F(v)d^3v \Rightarrow$ Probabilidade de uma partícula ter $v \in [v, v+dv]$

$f(v)dv = 4\pi^2 v^2 F(v)\,dv \Rightarrow$ Probabilidade de uma partícula ter $|v| \in [v, v+dv]$

b) b 1) $\vec{\bar{v}} = \int d^3v\, F(v)\vec{v} \rightarrow \bar{v}_i = \int_{-\infty}^{+\infty} dv_x \int_{-\infty}^{+\infty} dv_y \int_{-\infty}^{+\infty} dv_z\, F(v) v_1 = 0$

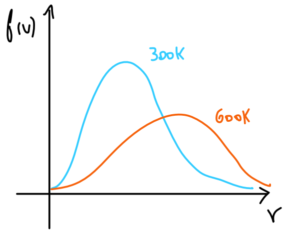
b 2) $v_{avg} = \sqrt{\frac{8 K_B T}{\pi m}} = 398.5$ m/s

b 3) $v_{rms} = \sqrt{\frac{3 K_B T}{m}} = 432\ 5$ m/s

b 4) $v_{mp} = \sqrt{\frac{2 K_B T}{m}} = 353\ 1$ m/s

c) $\bar{E}_{cin} = \int_0^{+\infty} f(v)\, E_{cin}\, dv = \frac{1}{2} m\bar{v}^2 = \frac{3}{2} K_B T \rightsquigarrow$ 3 graus de liberdade

d)



8 - N partículas $\mu_1 = 0$

2 níveis $\mu_2 = \varepsilon$

a) $\bar{N}_i = \frac{N \exp(-\beta \mu_i)}{\sum_j \exp(-\beta \mu_j)}$

$\bar{N}_1 = \frac{N}{1 + \exp(-\beta\varepsilon)}$

$\bar{N}_2 = \frac{N \exp(-\beta\varepsilon)}{1 + \exp(-\beta\varepsilon)}$

$\beta = \frac{1}{K_B T}$

b) $U = \bar{E} = \sum_i \bar{N}_i \mu_i = \frac{\varepsilon N \exp(-\beta\varepsilon)}{1 + \exp(-\beta\varepsilon)}$

$T \to 0$ $\quad \bar{N}_1 = N$ e $\bar{N}_2 = 0$

$T \to \infty$ $\quad \bar{N}_1 = \frac{N}{2}$ e $\bar{N}_2 = \frac{N}{2}$ //

9.)

$r = 2$ cm
$l = 2$ cm
$k = 2\ W\,m^{-1}K^{-1}$
$T_{ar} = 21^oC$
$T_p = 17^oC$
$T_c = 27^oC$
$e_c = 0.75$
$A_c = 20\ m^2$
$e_p = 0$
$h = 10\,W\,m^{-2}K^{-1}$

a) $\dot{Q}_{conv} = h A_c \Delta T = 1200\,W$

b) $\dot{Q}_{rad} = \sigma A_c e_c (T_c^4 - T_p^4) = 8\ 73\,W$

c) $\dot{Q}_{conv} + \dot{Q}_{rad} = \frac{k \cdot A_c}{l} \cdot (T_l - T_c) \Leftrightarrow T_l = 28^o C$

d) $\dot{Q} = \dot{m} C \Delta T = \dot{Q}_{cond} =$

$$\dot{m} = \frac{\frac{k \cdot A_c}{l} \cdot (T_l - T_c)}{C\ \ \Delta T} = \frac{1200 + 873}{1 \cdot 1} = 495\ g/s = 1\ 782\ L h^{-1}$$

10 − $R = 3\ cm$ $\quad T_{ar} = 22°C$ $\quad h_{ex} = 10\ W m^{-2} K^{-1}$

$L = 1\ m$ $\quad T_S = 17°C$

$T_t = 29°C$ $\quad h_{im} = 1000\ W m^{-2} K^{-1}$

a) $\dot{Q}_{conv} = h_{ex} \cdot A \cdot \Delta T = 132\ W$

$A_t = 2\pi R L \Rightarrow A = 10\ A_T$

$\Delta T = 7\ K$

b) $\dot{Q}_{Rad} = \sigma S (T_t^4 - T_s^4) = 52.9\ W$

c) $\dot{Q}_{Rad} + \dot{Q}_{conv} = \dot{Q}'_{conv}$

$132 + 52.9 = h_{im} \cdot A \cdot (T_a - T_t) \Rightarrow T_a = 29.17\ °C$

11 − $A = 1\ m^2$ $\quad e = 1$

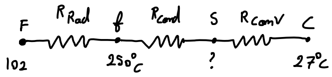
$A_F = 0.05\ m^2$ $\quad e_F = 0.8$

$l = 4\ cm$ $\quad K = 0.04\ W m^{-1} K^{-1}$

$T_{amb} = 27°C$ $\quad h = 2\ W m^{-2} K^{-1}$

$T_F = 102\ °C$ $\quad T = 250°C$

a)



b) $\dot{Q} = \frac{1}{R_{cond} + R_{conv}} \cdot \Delta T = 149\ W$

$R_{cond} = \frac{l}{KA}$ $\quad R_{conv} = \frac{1}{hA}$

c) $\dot{Q} = \frac{1}{R_{cond}} \Delta T \Rightarrow T_S = T_f - \dot{Q} R_{cond} = 101\ W$

d) d1) $\dot{Q} = e_F A_F \sigma (T_f^4 - T_F^4) = 125\ W$

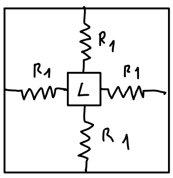
d2) $F_{f,F} = \frac{A_F}{A_f} F_{F,f} = 0.05$ //

12 − $L_p = 5\ mm$ $\quad K_p = 2\ W m^{-1} K^{-1}$ $\quad A_p = 1\ m^2$

$L_a = 2\ mm$ $\quad h_a = 5\ W m^{-2} K^{-1}$

$h_l = 10\ W m^{-2} K^{-1}$ $\quad e_{ex} = 0.8$

a)



b) $R_1 = R_{conv\,l} + R_{cond\,p} + R_{conv\,ar} + R_{cond\,p}$

$\dot{Q}_R = \frac{\Delta T}{R_{Th}} = 4 \frac{\Delta T}{R_1} = 459\ W$

↳ 114 W por parede

$R_{conv\,l} = \frac{1}{h_l A_p}$

$R_{cond\,p} = \frac{L_p}{K_p A_p}$

$R_{conv\,ar} = \frac{1}{h_a A_p}$

c) $\dot{Q}_{Rad} = \sigma \cdot 4 \cdot A_p \cdot e_{ex} \cdot (T^4 - T_0^4) = 192\ W \rightarrow 48\ W$ por parede!

d) $\dot{Q}_{conv} = \dot{Q}_R - \dot{Q}_{Rad} = \frac{\Delta T}{R_{conv}} \Leftrightarrow \frac{Q_{conv}}{\Delta T} = h_{ar} = 4.45\ W m^{-2} K^{-1}$ $\quad R_{conv} = \frac{1}{hA}$

13 − $\lambda_{max} = \frac{2.898 \times 10^{-3}\ m K}{T} = 0.966\ \mu m$

$\frac{P}{A} = \sigma e T^4 = 4.6 \times 10^{-6}\ W m^{-2}$

14 − $\lambda_{max\,Sol} = 483\ nm$

$\lambda_{max\,Tung} = 906\ nm \Rightarrow$ Filtrar de forma a que o max seja o do Sol

15 − $T_{céu} = -5°C = 268\ K$ $\quad \varepsilon_{céu} = 1$

$A_p = 0.9\ m^2$ $\quad \varepsilon_p = 0.9$

a) $\lambda_{max} = \frac{R}{T_p} = 4.4\ \mu m$

b) $I_{Rad} = e_p A_p \sigma (T_p^4 - T_c^4)$

c) $P_{Perdida} = 176\ W$

d) $I_{Rad} = I_p = 50\ W$

$I_p = e_p A_p \sigma (T_p^4 - T_c^4)$

$\sqrt[4]{\frac{I_p}{e_p A_p \sigma} + T_c^4} = T_p \simeq 8°C$

e) $I_p = \frac{T_p - T_c}{R_{cond}} \Rightarrow K = \frac{I_p L}{A(T_p - T_c)} = 0.04\ W m^{-1} K^{-1}$