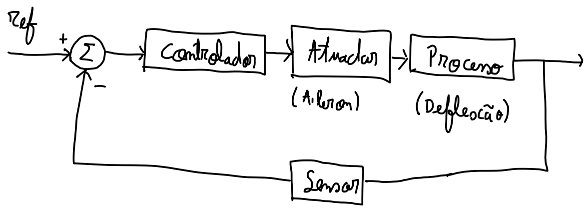
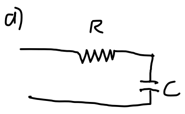
1)



2) a)



$v_i(t) = v_R(t) + v_o(t)$ $\quad i = C\frac{dv_o}{dt}$ $\quad v_R = Ri$

$v_i = RC\dot{v}_o + v_o$

b) $V_i(s) = RCsV_o + V_o$

$H(s) = \frac{V_o}{V_i} = \frac{1}{RCs+1}$

c) $U(s) = \frac{1}{s}$

$V_o(s) = \frac{1/RC}{s + 1/RC} V_i(s)$

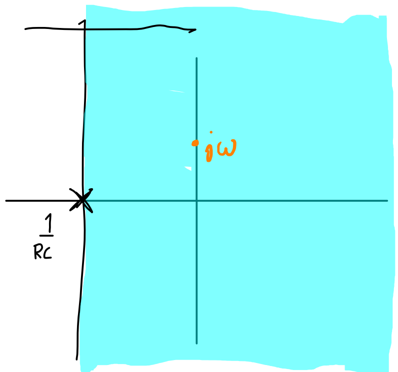
$A = \lim_{s\to 0} s\cdot V_o(s) = 1$

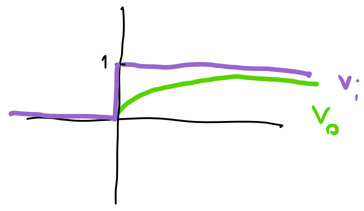
$A = 1$
$B = -1$

$B = \lim_{s \to -\frac{1}{RC}} \left(s + \frac{1}{RC}\right)\cdot V_o(s) = -1$

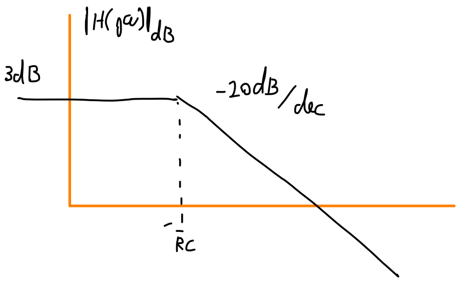
$\frac{1/RC}{s+1/RC}\cdot\frac{1}{s} \to \frac{A}{s} + \frac{B}{s+\frac{1}{RC}} \to v_o = \frac{1}{s} - \frac{1}{s+\frac{1}{RC}} = u(t) - e^{-\frac{1}{RC}t}u(t)$

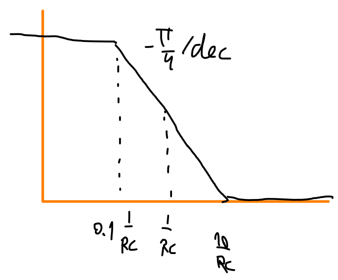
d)





$H(j\omega) = \frac{1/RC}{j\omega + 1/RC}$





3) $s^3 + 4s^2 + 6s + 8$
$s^3 + 3s^2 + 5s + 1$

4) a) $x = \frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 7y$

b) $10x = \frac{d^2y}{dt^2} + 15\frac{dy}{dt} + 5\,6y$

c) $\frac{dx}{dt} + 2x = \frac{d^3y}{dt^3} + 8\frac{d^2y}{dt^2} + 9\frac{dy}{dt} + 15y$

5) a)

$m\ddot{x}_o(t) = -K(x_o(t) - x_i(t)) - b(\dot{x}_o(t) - \dot{x}_i(t))$
$\Leftrightarrow m\ddot{x}_o + b\dot{x}_o + Kx_o = b\dot{x}_i + Kx_i$

$\frac{X_o(s)}{X_i(s)} = \frac{bs+K}{ms^2+bs+K}$

b) $\frac{Y(s)}{X(s)} = \frac{X_o(s)}{X_i(s)} = \frac{bs+k_2}{m_2s^2+bs+k_2}$

Para m_1:

$(m_1+m_2)\ddot{x} = -k(x-u)$
$\Leftrightarrow (m_1+m_2)\ddot{x} + k_1x = k_1u$

$\frac{X(s)}{U(s)} = \frac{k_1}{(m_1+m_2)s^2+k_1}$

$\frac{Y(s)}{U(s)} = \frac{k_1(bs+k_2)}{((m_1+m_2)s^2+k_1)(m_2s^2+bs+k_2)}$

$\lim_{k_1\to\infty} \frac{Y(s)}{U(s)} = \frac{bs+k_2}{m_2s^2+bs+k_2} = \frac{X_o(s)}{X_i(s)}$

6)

$\sum \text{forças} = m\ddot{x}_1$

$m\ddot{x}_1 = -2k(x_1) - b(\dot{x}_1) - m\ddot{x}_2$

$\frac{X_1}{A_{ext}} = \frac{-1}{s^2 + \frac{b}{m}s + \frac{2K}{m}}$

7) a) $\gamma\ddot{\theta}(t) = T_m - Mg\frac{L}{2}\sin(\theta) - D\dot{\theta}(t)$

b) $\gamma\delta\ddot{\theta} = \cancel{T_{m0}} + \delta T_m - \cancel{Mg\frac{L}{2}\sin(\theta)} + \cos(\theta)\delta\theta) - D\delta\dot{\theta}$

$\Leftrightarrow \gamma\delta\ddot{\theta} = \delta T_m - Mg\frac{L}{2}\cos(\theta_0)\delta\theta - D\delta\dot{\theta}$

$\theta_0 = 0$ $\qquad \theta_0 = \frac{\pi}{4}$

$\frac{\Delta\theta(s)}{\Delta T_m(s)} = \frac{1}{\gamma s^2 + Ds + Mg\frac{L}{2}}$ $\qquad \frac{\Delta\theta(s)}{\Delta T_m(s)} = \frac{1}{\gamma s^2 + Ds + \frac{MgL}{2\sqrt{2}}}$

c) Em $s=0$ não há movimento, logo como o coef. de atrito (D) e o coef. de inércia (γ) são parâmetros dinâmicos (apenas contam quando $\dot{x} \neq 0$) não afetam o ganho estático

8) $V(t) = AH(t)$

a) $Q_o(t) = (\beta H(t))^{1/2}$

$\frac{dV(t)}{dt} = Q_i(t) - Q_o(t)$

$A\frac{dH(t)}{dt} = Q_i(t) - \sqrt{\beta H(t)}$

b) $A\delta\dot{H} = \delta Q_i + Q_{i0} - \sqrt{\beta}\left(H_0 + \frac{\delta H}{2\sqrt{H_e}}\right)$
↓ equilíbrio

$A\delta\dot{H} = \delta Q_i - \frac{1}{2}\sqrt{\frac{\beta}{H_e}}\delta H$

$2\delta\dot{H} = \delta Q_i - \frac{1}{6}\delta H$

$\frac{\Delta H(s)}{\Delta Q_i(s)} = \frac{1}{2s+\frac{1}{6}} = \frac{1/2}{s+1/12}$

c) $\tau = RA = \frac{\partial H}{\partial Q_o}\Big|_P A = \frac{2Q_o}{\beta}\Big|_P A = \frac{2\cdot(\beta\cdot H_0)^{1/2}}{\beta}$ $\quad A = 12$ $\longleftrightarrow$ $\tau = \frac{1}{\frac{1}{2A}\sqrt{\frac{\beta}{H_0}}} = \frac{2A\sqrt{H_0}}{\sqrt{\beta}} = 2A\frac{\sqrt{H_0\beta}}{\beta}$

$\tau = \frac{1}{1/12} = 12$ $\quad$ // $\quad$ V

9) a) $m\dot{V} = F - k_1V^2$
$\Leftrightarrow F_0 = k_1V^2 = 0.5N$

b) $m\dot{v} + k_1V^2 = F$ $\qquad m = 1\,kg$ $\quad 2V_0k_1 = 1$

$m\delta\dot{V} + k_1(V_0 + 2V_0\delta V) = \delta F + F_0$

$m\delta\dot{V} + 2V_0k_1\delta V = \delta F$

$\frac{\Delta V}{\Delta F} = \frac{1}{s+1}$

c) $F_0 = K_0 m_0^2 \Leftrightarrow m_0 = \sqrt{\frac{F_0}{K_0}} = 1$ $\qquad \frac{\Delta F(s)}{\Delta N(s)} = 1$

$F = k_0m^2 \Leftrightarrow \delta F + F_0 = K_0(2m_0\delta m + m_0) \Leftrightarrow \delta F = \delta m$

d) $H(s) = \frac{\frac{k_p}{s+1}}{1+\frac{k_p}{s+1}} = \frac{k_p}{s+1+k_p} \Rightarrow k_p > -1$ $\quad$ Pólo no s.p.c.e logo é estável se $k_p > -1$

e) $\frac{\Delta F(s)}{\Delta N(s)} = \sqrt{0.2}$ $\qquad \frac{\Delta V(s)}{\Delta F(s)} = \frac{1}{s+0.2}$

$\frac{\Delta V}{\Delta N} = \frac{\sqrt{0.2}}{s+0.2} \Rightarrow \frac{\Delta V}{\Delta N}k_p$

$H(s) = \frac{\sqrt{0.2}\,k_p}{s+0.2+\sqrt{0.2}\,k_p}$

$0.2 + \sqrt{0.2}\,k_p > 0 \Rightarrow k_p > -\sqrt{0.2}$

Mantém estável para $k_p > 0$

10) $J\ddot{\theta}(t) = \sum$ binários $= F_1(t)\cdot l - m_1 g\cos(\theta(t))l - F_2(t)\cdot l + m_2 g\cos(\theta(t))l \iff$

$\iff J\ddot{\theta}(t) = u(t)l + (m_2-m_1)gl\cos(\theta(t))$

b) $u_0 = -(m_2-m_1)g\cos(\theta_0)$

c) $J\delta\ddot{\theta} = (\cancel{u_0}+\delta u)l + (m_2-m_1)gl(\cancel{\cos(\theta_0)} - \sin(\theta_0)\delta\theta) \iff$

$J\delta\ddot{\theta} = \delta u\, l - (m_2-m_1)gl\sin(\theta_0)\delta\theta \iff$

$\iff J\delta\ddot{\theta} + (m_2-m_1)gl\sin(\theta_0)\delta\theta = \delta u\, l$

$$\frac{\Theta(s)}{U(s)} = \frac{l/J}{s^2 + \frac{(m_2-m_1)gl\sin(\theta_0)}{J}}$$

d) $\theta_0 = \frac{\pi}{6}$

$$\frac{\Theta(s)}{U(s)} = \frac{4}{s^2 + 0.2\cdot 4\sin(\frac{\pi}{6})} = \frac{4}{s^2+0.4}$$

$\alpha = 4$
$\beta = 0.4$

e) $H(s) = \frac{K_p\frac{\alpha}{s^2+\beta}}{1+K_p\frac{\alpha}{s^2+\beta}} = \frac{K_p\alpha}{s^2+\beta+K_p\alpha}$

$s^2+\beta+K_p\alpha = 0$
$\iff s = \pm\sqrt{-K_p\alpha-\beta}$

$-K_p\alpha-\beta \geq 0$ é instável
$-K_p\alpha-\beta < 0$ é estável

$K_p < 0.1 \Rightarrow$ tem pólos negativos: é instável
$K_p = 0.1 \Rightarrow$ tem pólo nulo: é instável
$K_p > 0.1 \Rightarrow$ tem pólos complexos conjugados no s.p.c.e. : é estável

f) Com $K_p = 0.9$ é um sistema estável com T.F. $\Theta(s) = \frac{3.6}{s^2+4}$

$R(s) = \frac{\pi}{9}\cdot\frac{1}{s}\Theta(s) = \frac{0.4\pi}{s(s^2+4)} = 0.4\pi\left(\frac{1}{s(s^2+4)}\right) = 0.4\pi\left(\frac{A}{s}+\frac{B}{s^2+4}\right) =$

$A(s^2+4)+B(s) = 1$
$As^2 + A4 + Bs = 1$
$A4 = 1$
$A = \frac{1}{4}$ $\quad \frac{s^2}{4} \Rightarrow B = -\frac{s}{4}$

$= \frac{0.4\pi}{4}\left(\frac{1}{s} - \frac{s}{s^2+4}\right) = 0.1\pi(u(t) - \cos(2t)u(t)) =$

$= 0.1\pi(1-\cos(2t)u(t))$

h) $m_2 - m_1 = 0 \Rightarrow \beta = 0$

Logo

$$\frac{\Theta(s)}{U(s)} = \frac{4}{s^2}$$

É preciso considerar o erro

Logo antes o controlador adiciona-se $D(s)$, um desvio, tal que $D(s) = \frac{u_0}{s}$

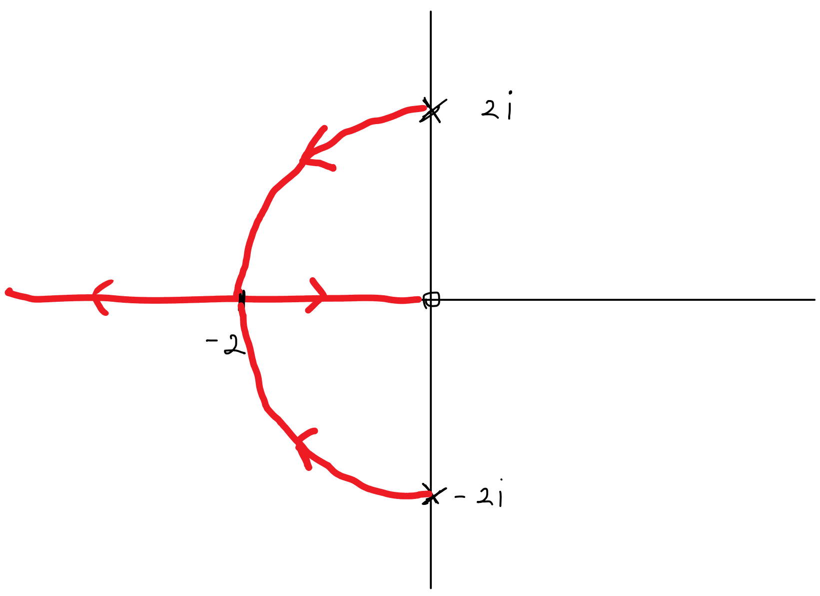
g) $$\frac{\frac{\alpha(K_p+K_d s)}{s^2+\beta}}{1+\frac{\alpha(K_p+K_d s)}{s^2+\beta}} = \frac{\alpha(K_p+K_d s)}{s^2+\beta+\alpha(K_p+K_d s)}$$

$(s^2+\beta+\alpha K_p) + K_d\alpha s = 0$

$\Downarrow$

$1 + K_d\frac{\alpha s}{s^2+\beta+\alpha K_p} = 0 \iff 1 + K_d\frac{4s}{s^2+4} = 0$

$s^2+\beta+\alpha K_p = 0 \iff$
$s = \sqrt{-(\beta+\alpha K_p)} = \sqrt{-2}$



Entrada e saída

$K_d = \frac{-(s^2+4)}{0.4s}$

$\frac{dK_d}{ds} = -\frac{5s^2-20}{2s^2} \Rightarrow -5s^2+20 = 0 \iff s^2 = \frac{20}{5} \iff s = \pm 2$

+2 não pode ser, mas −2 é um break in/away point

$\phi_a = \frac{\pm(2K+1)\pi}{n-m} \therefore K=0,\ \phi_0 = 180$

$\alpha K_d = 2\zeta\omega_n$

$t_s(2\%) = \frac{4}{\zeta\omega_n} = \frac{8}{\alpha K_d} = \frac{2}{K_d}$

Logo K_d tal que t_s é mínima é $\boxed{K_d \to \infty}$