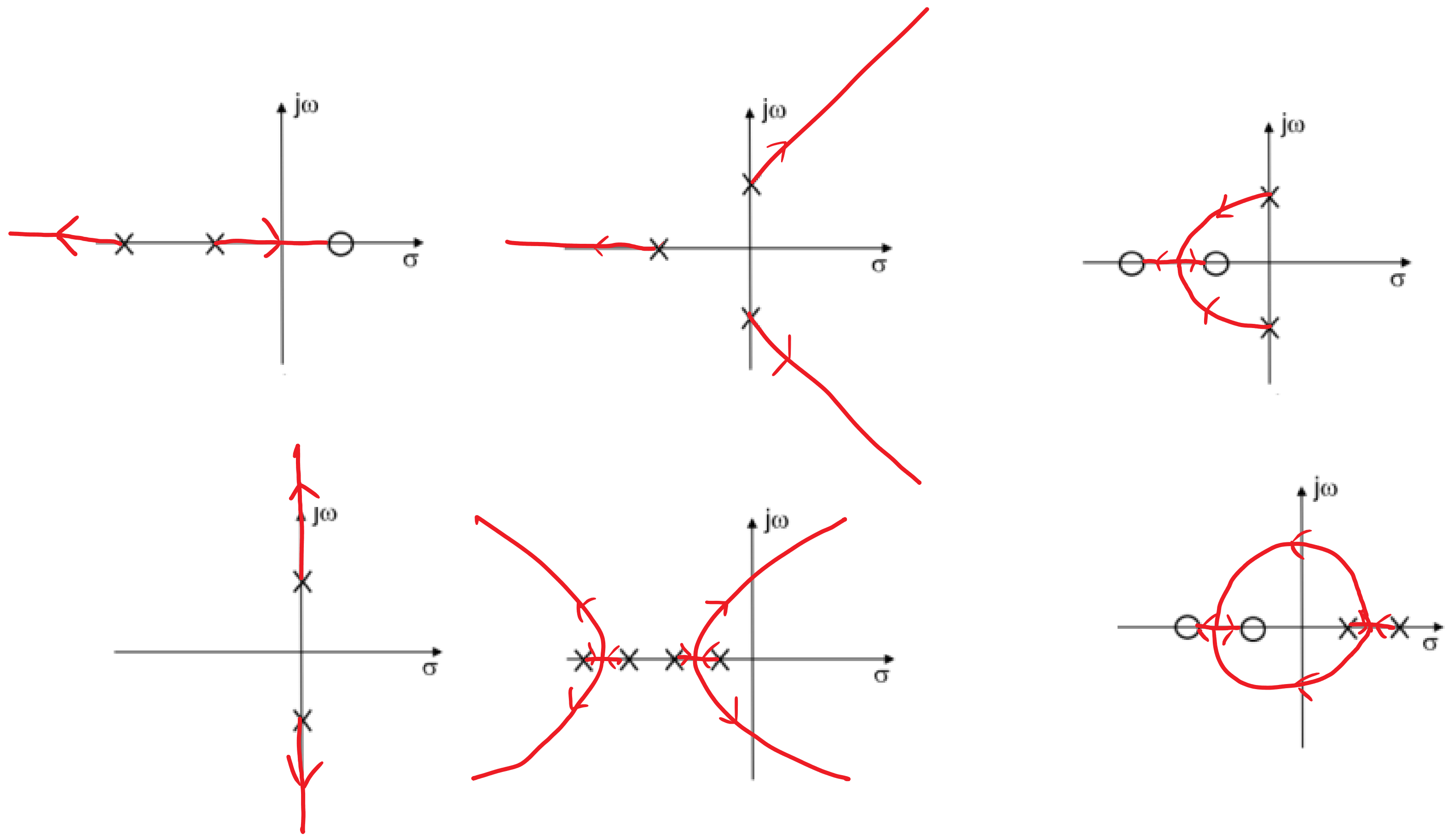
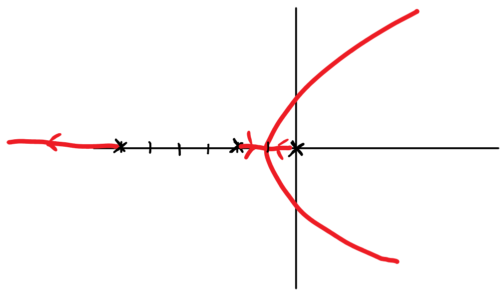
1)



2) a) $k \cdot G(s) \cdot H(s) = k \cdot \frac{1/L}{(s+\frac{R}{L})} \cdot \frac{K_m/J}{(s+\frac{B}{J})} \cdot \frac{1}{s} \cdot K_p = k \frac{4}{(s+6)(s+2)s}$



$\sigma = \frac{-6-2+0}{3} = -\frac{8}{3}$

$k = -\frac{s(s+6)(s+2)}{4} \Rightarrow \frac{dk}{ds} = \frac{-3s^2-16s-12}{4} \Rightarrow \frac{dk}{ds} = 0 \Big\} s = -0{,}9028$

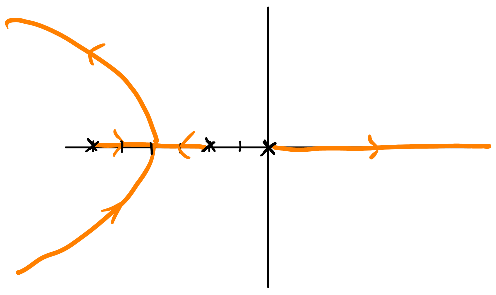
$\phi = \frac{\pi}{3}; \pi, \frac{5\pi}{3}$

$1 + kG(s)H(s) = 0 \qquad s = j\omega$

$4k = -(j\omega)(j\omega+6)(j\omega+2) \Leftrightarrow 4k = (+\omega^2 - j\omega 6)(j\omega+2) \Leftrightarrow 4k = \omega^3 j + \omega^2 6 + 2\omega^2 - j\omega 12$

$\Leftrightarrow -\omega^3 j - \omega^2 8 + \omega 12 j + 4k = 0 \Leftrightarrow \begin{cases} -\omega^3 + \omega 12 = 0 \\ -\omega^2 8 + 4k = 0 \end{cases} \Leftrightarrow \begin{cases} \omega = 0 \vee \omega^2 - 12 = 0 \\ k = 2\omega^2 \end{cases} \Leftrightarrow \begin{cases} \omega = 0 \\ k = 0 \end{cases} \vee \begin{cases} \omega = \sqrt{12} \\ k = 24 \end{cases}$

b)



$\sigma = \frac{-6-2+0}{3} = -\frac{8}{3}$

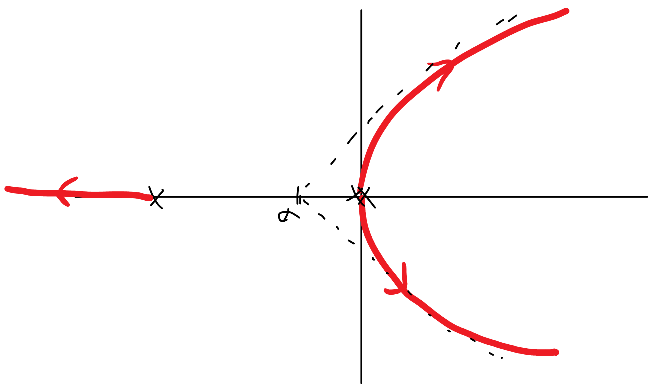
$k = -\frac{s(s+6)(s+2)}{4} \Rightarrow \frac{dk}{ds} = \frac{-3s^2-16s-12}{4} \Rightarrow \frac{dk}{ds} = 0 \Big\} s = -4{,}43$

$\phi = \frac{\pi}{3}; \pi, \frac{5\pi}{3}$

c) O sistema é estável entre [0,24]

d) para $k > 0$ é igual a b)!

3) a) $\alpha = 0 \qquad K G(s) H(s) = K \cdot \frac{1}{s(s+12)} \cdot \frac{1}{s}$



$\phi = \frac{\pi}{3}; \pi; \frac{5\pi}{3}$

$\sigma = \frac{-12+0+0}{3} = -4$

$1 + K G(s)H(s) = 0 \Leftrightarrow k = -s^2(s+12) = -s^3 - 12s^2$

$\frac{dk}{ds} = -3s^2 - 24s \qquad -3s^2 - 24s = 0 \Leftrightarrow \begin{cases} s = 0 \\ -3s - 24 = 0 \end{cases} \Leftrightarrow \begin{cases} \boxed{s = 0} \\ s = -8 \end{cases}$

$k = -s^3 - 12s^2 \qquad s = j\omega$

$s^3 + 12s^2 + k = 0 \Leftrightarrow -\omega^3 j - 12\omega^2 + k = 0$

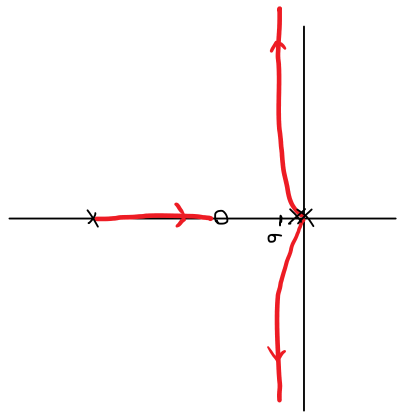
$\begin{cases} -\omega^3 = 0 \\ -12\omega^2 + k = 0 \end{cases} \Leftrightarrow k = 0$

$s = j0$

b)

$\alpha = 0.2 \quad K\,G(s)H(s) = K \cdot \frac{1}{s(s+12)} \cdot \left(\frac{1}{s} + \alpha\right) =$

$= K \cdot \frac{1}{s(s+12)} \cdot \left(\frac{1+0.2s}{s}\right) = K \cdot \frac{0.2(5+s)}{s^2(s+12)}$



$\sigma = \frac{-12-0-0+5}{2} = -3.5$

$\phi = \frac{\pi}{2} ; \frac{3\pi}{2}$

$1 + K\frac{0.2(s+5)}{s^2(s+12)} = 0 \Leftrightarrow K = \frac{-s^2(s+12)}{0.2(s+5)}$

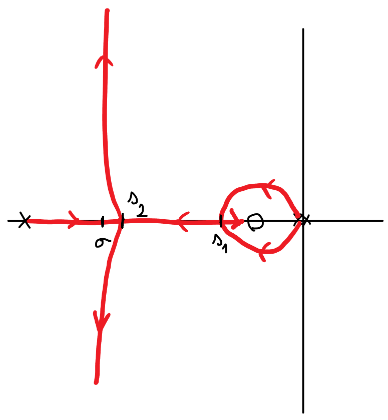
$\frac{dK}{ds} = 0 \Rightarrow s = 0$

O sistema passa a ser estável para $K>0$

c)

$\alpha = 1 \quad K\,G(s)H(s) = K \cdot \frac{1}{s(s+12)} \cdot \left(\frac{1}{s} + \alpha\right) =$

$= K \cdot \frac{1}{s(s+12)} \cdot \left(\frac{1+s}{s}\right) = K \cdot \frac{s+1}{s^2(s+12)}$



$\sigma = \frac{-12+1}{2} = -5.5$

$\phi = \frac{\pi}{2} ; \frac{3\pi}{2}$

$K = \frac{-s^2(s+12)}{(s+1)}$

$\frac{dK}{ds} = 0 \Rightarrow s_1 = -2.314$

$s_2 = -5.186$

Com $n-m \geq 2$

$\sum \text{polos} = -12$

$2 \times -5.186 + x = -12$

$x = -1.628$

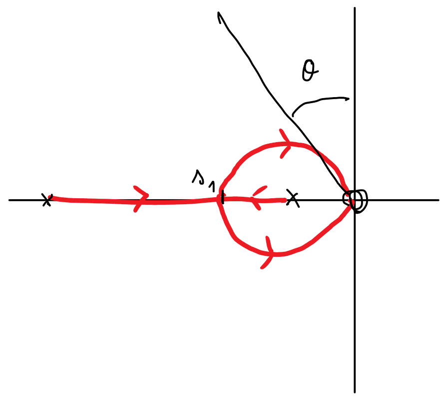
d) Para $s_2 = -5.186$ há um polo duplo com $K = -\left(\frac{s+1}{s^2(s+12)}\right)^{-1} = 43.78$ com o outro polo em $s = -1.628$

Para $s_1 = -2.314$ há um polo duplo com $K = 39.5$ com o outro polo em $s = -7.38$

4) a)

$\frac{\ddot{Y}_m(s)}{Y_g(s)} = \frac{s^2(2s+2)}{(C_a+1)s^2 + 4s + 2} \rightarrow C_a s^2 + s^2 + 4s + 2 = 0 \Leftrightarrow$

$\Leftrightarrow C_a s^2 = -(s^2+4s+2) \Leftrightarrow 1 + C_a \frac{s^2}{s^2+4s+2} = 0$



$K\,G(s)H(s) = C_a \frac{s^2}{(s+2+\sqrt{2})(s-\sqrt{2}+2)}$

$\phi = \pi ;$

$C_a = -\frac{s^2+4s+2}{s^2}$

$\frac{dC_a}{ds} = \frac{4s+4}{s^3}$

$\frac{dC_a}{ds} = 0 \Rightarrow s_1 = -1$

f) $\varphi = 0.69$

$(C_a+1)s^2 + 4s + 2 \Rightarrow s^2 + \frac{4}{(C_a+1)}s + \frac{2}{(C_a+1)}$

$$\begin{cases} \frac{4}{C_a+1} = 2\xi\omega \\ \frac{2}{C_a+1} = \omega^2 \end{cases} \Rightarrow \frac{2}{C_a+1} = \xi\omega = \omega^2 \Rightarrow \xi = \omega = 0.69$$

$C_a = 3.21$

$\Rightarrow$ Polos em $-0.48 + j0.5$

$-0.48 - j0.5$

5) a) $0 < k_1 < \infty$
$\lambda = 2$

$$K\,G(s)\,H(s) = K_1 \frac{10(1+0{,}2s)}{s(s+\lambda)(s+11)}$$

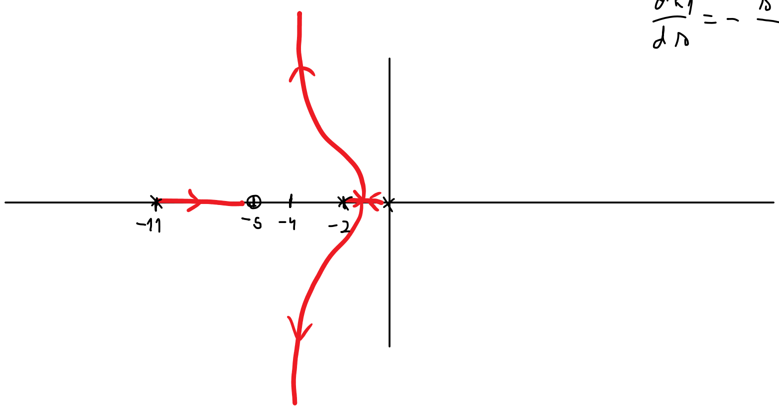
$n = 3$
$m = 1$

$$K_1 = -\frac{s(s+\lambda)(s+11)}{10(1+0{,}2s)}$$

$$\frac{dK_1}{ds} = -\frac{s^3 + 14s^2 + 65s + 55}{(s+5)^2}$$

$s^3 + 14s^2 + 65s + 55 = 0 \iff s \approx -1{,}08$

$$\sigma_a = \frac{-2-11+5}{2} = \frac{-8}{2} = -4$$

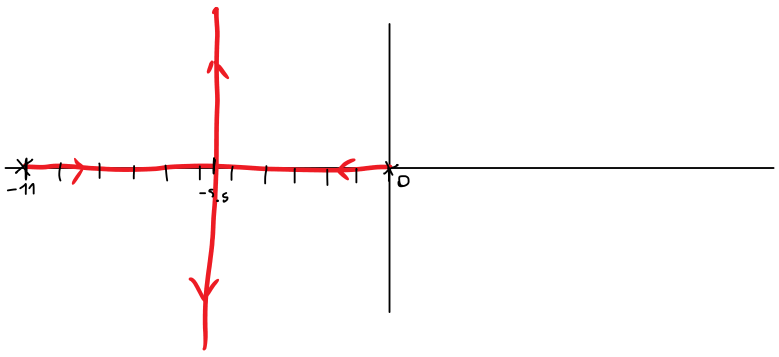


b) $\lambda = 5$

$$K\,G(s)\,H(s) = K_1 \frac{10(1+0{,}2s)}{s(s+\lambda)(s+11)}$$

$$K_1 = -\frac{s(s+\lambda)(s+11)}{10(1+0{,}2s)}$$

$$\frac{dK_1}{ds} = -\frac{2s+11}{2} \Rightarrow 2s + 11 = 0 \iff s = -5{,}5$$

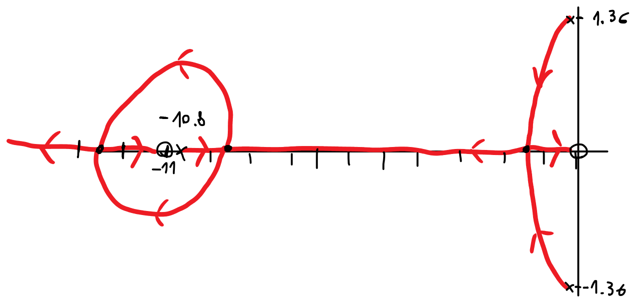


c) $$K\,G(s)\,H(s) = K_1 \frac{10(1+0{,}2s)}{s(s+\lambda)(s+11)} \Rightarrow$$
$K_1 = 2$
$(s+\lambda)(s^2+11s) + 4s + 20 = 0$

$\lambda(s^2+11s) + s^3 + 11s^2 + 4s + 20 = 0$

$$1 + \lambda \frac{s^2+11s}{s^3+11s^2+4s+20} = 0 \qquad \lambda = -\frac{s^3+11s^2+4s+20}{s^2+11s} \qquad \frac{d\lambda}{ds} = -\frac{s^4+22s^3+117s^2-40s-220}{s^2(s+11)^2} \Rightarrow \frac{d\lambda}{ds} \Rightarrow$$

$s = 1{,}358$
$s = -1{,}365$
$s = -9{,}608$
$s = -12{,}486$



6)

$S\% = 20.5\% \Rightarrow \xi = 0.45$

$t_{5\%} = 0.75 \Rightarrow t_s = \frac{3}{\xi w_n} \Rightarrow w_n = 8.89$ rad/s

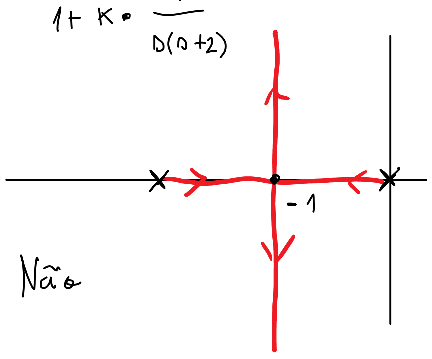
$\theta = \arcsin(\xi) = 26.7^\circ$

$-\xi w_n = -4$

Polos em $s = -4 \pm j8$

a) Root locus

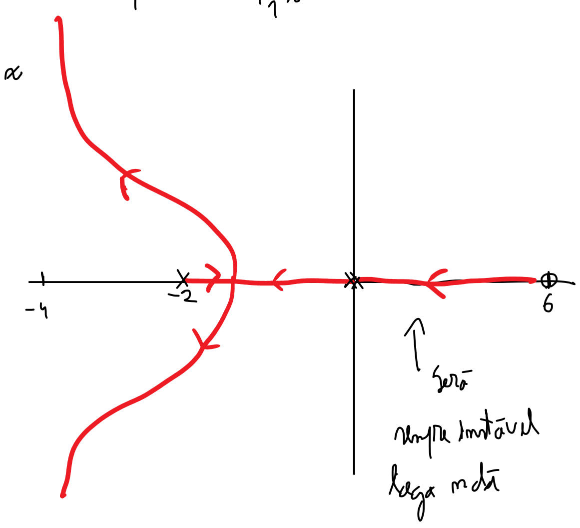
$$1 + K \cdot \frac{1}{s(s+2)}$$



Não

b) $$C(s) = K\left(1 + \frac{1}{T_1 s}\right) = K\left(\frac{T_1 s + 1}{T_1 s}\right)$$

$\frac{1}{T_1} = \alpha$

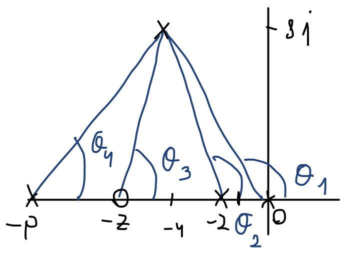


$$\sigma = \frac{-2-0-0+\alpha}{3-1} \leq -4 \Rightarrow -\alpha \geq 6$$

c) $$C(s) = K \frac{s+z}{s+p}$$

$$\sigma_a = \frac{-2-p-0+z}{2} \leq 4 \Rightarrow p \geq 6 + z$$

Pela condição de argumento $\theta_3 - \theta_4 \approx 41^\circ$



i) $z = 4 \Rightarrow \theta_3 = 90^\circ \Rightarrow \theta_4 = 49^\circ$

$$\frac{8}{p'} = tg(\theta_4) \Rightarrow p = p' + 4 \approx 11$$

Condição de modulo $s = -4 + 8j$

$$|K| = \frac{|s+11|\,|s+2|\,|s|}{|s+4|} = 98$$

Erro? $$K_v = \lim_{s\to 0} s\,G(s) = s \cdot \frac{s+4}{s+11} \cdot K \cdot \frac{1}{s(s+2)} = \frac{4 \cdot K}{11 \cdot 2} = \frac{196}{11} \qquad e_v(\infty) = \frac{11}{196} = 0.056 \text{ FALHA!}$$

ii) $p = +\infty \Rightarrow \theta_4 = 0 \Rightarrow \theta_3 \approx 41^\circ$

$$\frac{8}{z'} = tg(\theta_3) \Rightarrow z = z' + 4 \approx 13$$

$|K| \approx 6$

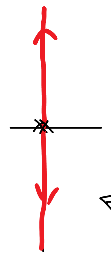
$$K_v = \lim_{s\to 0} s\,G(s) = s \cdot K \cdot (s+13) \cdot \frac{1}{s(s+2)} = \frac{13K}{2} \qquad e_v(\infty) = \frac{2}{13K} = \frac{1}{39} = 0.026 \,!$$

Tem o necessário

7) a) $KG(s)H(s) = K\frac{1}{s^2}$

$1+K\frac{1}{s^2}=0$

$n=2$ $\quad m=0$ $\quad \sigma_a = 0$ $\quad \phi_a = \left\{\frac{\pi}{2}, -\frac{\pi}{2}\right\}$



$K=-s^2$

$\frac{dK}{ds} = -2s \qquad \frac{dK}{ds}=0 \Rightarrow s=0$

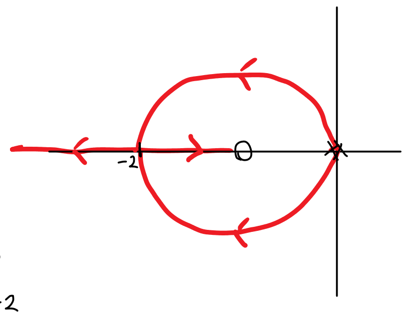
Os polos estarão sempre sobre o eixo Imaginário logo o sistema é marginalmente estável

b) $KG(s)H(s) = K\frac{s+1}{s^2}$

$1+K\frac{s+1}{s^2}=0$

$K=-\frac{s^2}{s+1} \qquad \frac{dK}{ds} = -\frac{2s(s+1)-s^2}{\dots}$

$2s(s+1)-s^2=0 \Leftrightarrow s^2+2s=0 \begin{cases} s=0 \\ s=-2 \end{cases}$



$n=2$ $\quad \phi_a = \pi$

$m=1$ $\quad \sigma_a = 1$

Logo é sempre estável

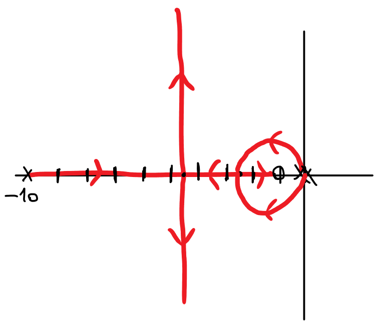
c) $KG(s)H(s) = K_p \cdot \frac{s+1}{s+p} \cdot \frac{1}{s^2}$ $\qquad \sigma_a = -\frac{10+1-0-0}{2} = -4.5$

$p=10$ $\qquad \phi_a = \frac{\pm(2k+1)\pi}{2} = \left\{\frac{\pi}{2}, -\frac{\pi}{2}\right\}$

$1+K\frac{s+1}{s^2(s+10)}$ $\qquad K = -\frac{s^2(s+10)}{s+1} = -\frac{s^3+10s^2}{s+1}$

$n=3$

$m=1$ $\qquad \frac{dK}{ds} = -\frac{(3s^2+20s)(s+1)-s^3-10s^2}{\dots}$



$(3s^2+20s)(s+1)-s^3-10s^2=0 \Leftrightarrow$

$\Leftrightarrow 3s^3+20s^2+3s^2+20s-s^3-10s^2=0 \Leftrightarrow$

$\Leftrightarrow 2s^3+13s^2+20s=0 \Leftrightarrow \begin{array}{l} s=0 \\ s=-4 \\ s=-\frac{5}{2} \end{array}$