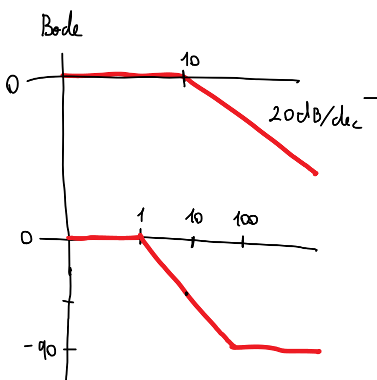
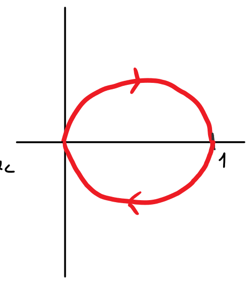
1) i) $G(s) = \frac{10}{s+10}$ $K=1$

Bode

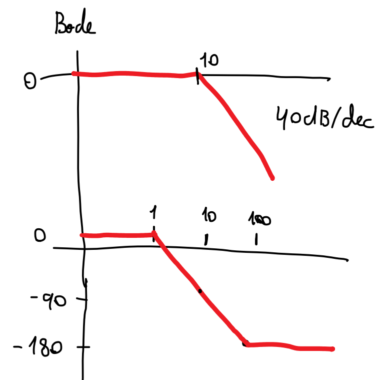


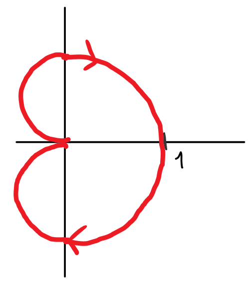


P = 0
N = 0
Z = 0

ii) $G(s) = \frac{100}{(s+10)^2}$ $K=1$

Bode

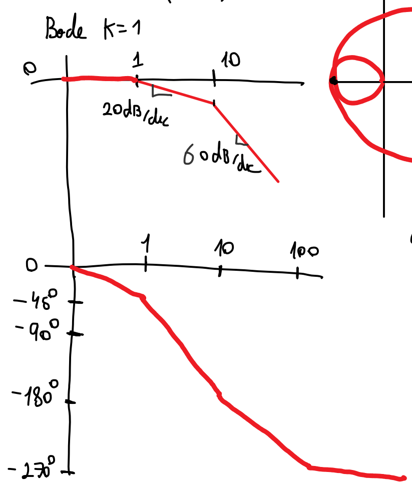


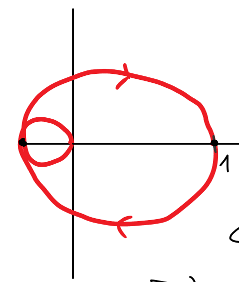


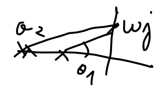
P = 0
N = 0
Z = P + N = 0

iii) $G(s) = \frac{100}{(s+1)(s+10)^2}$

Bode K = 1







$\theta_1 = \arctan\left(\frac{\omega}{1}\right)$
$\theta_2 = \arctan\left(\frac{\omega}{10}\right)$

$-\theta_1 - \theta_2 - \theta_2 = -180$
$\hookrightarrow \omega = \pm\sqrt{120} \approx 11$

$G(j\sqrt{120}) = \frac{1}{24.2}$ $\hookrightarrow K = 24.2$

Pelo Root locus: $s = j\sqrt{120}$

$1 + K \frac{100}{(s+1)(s+10)^2} = 0 \Rightarrow K = 24.2$

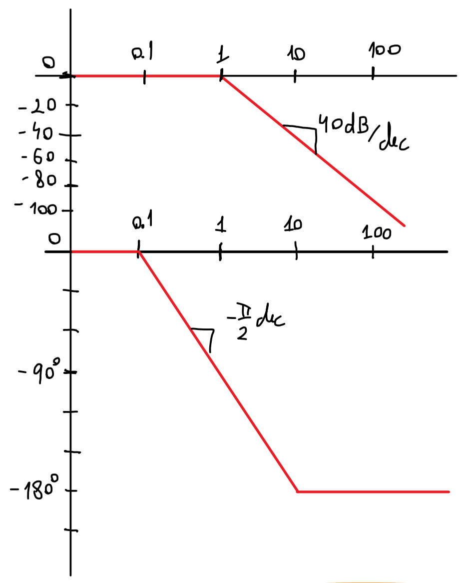
Ao aumentar K podemos afastar -1!

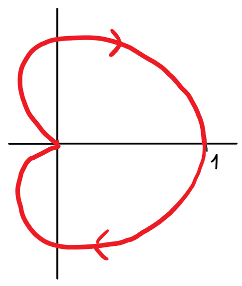
{P = 0, N = 0} {P = 0, N = 2}

Z = 0 ↑ Estável Z = 2 ↓ Instável

2) i) $KG(s)H(s) = K \frac{1}{(s+1)^2}$

K = 1



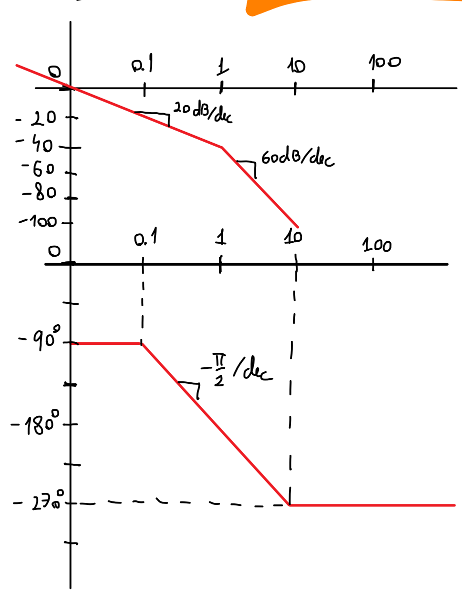


P = 0
N = 0 É estável
Z = 0 para K > -1

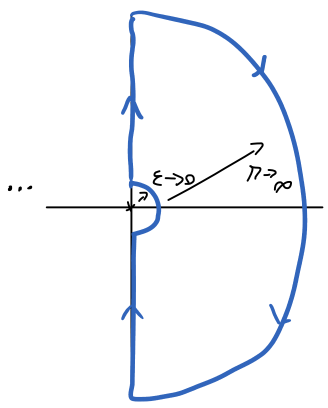
Margem de Ganho = ∞

ii) $KG(s)H(s) = K \frac{1}{s(s+1)^2}$

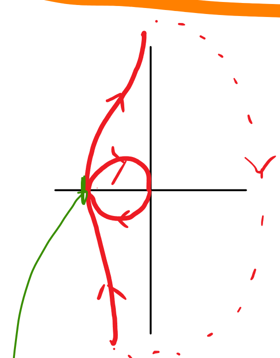
K = 1



! Pole na origem!



P = 0



$\arg(KG(s)H(s)) = 180° = -\arg(s) - \arg(s+1)^2 \Leftrightarrow \arctg(\omega) = 45°$

$\omega = tg(45°) = 1 \Rightarrow |KG(s)H(s)|_{\omega=1} = \frac{|K|}{2}$ Ponto de intersecção!

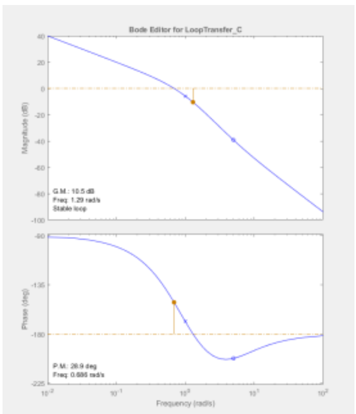
Logo $-1 < -\frac{K}{2} \Leftrightarrow 2 > K \Rightarrow N = 0 \Rightarrow Z = 0 \Rightarrow$ Estável

Logo $K \in [0, 2]$ para ser estável!

$\omega_\pi = 1$

$MG = \frac{1}{|KG(s)H(s)|_{\omega = \omega_\pi}} = 2$ Margem de ganho = 2

iii) $KG(s)H(s) = K \frac{5+s}{5s} \cdot \frac{1}{(s+1)^2}$



! Polo na origem!

A mesma coisa!

$\arg(KG(s)H(s)) = 180° \Leftrightarrow$

$\Leftrightarrow \underbrace{-\arg(5s)}_{-90°} - \arg(s+1)^2 + \arg(5+s) = 180° \Leftrightarrow$

$-90° - 2\arctg(\omega) + \arctg\left(\frac{\omega}{5}\right) = 180° \Leftrightarrow$

$\Leftrightarrow -\arctg(\omega) - \arctg(\omega) = -\arctg\left(\frac{\omega}{5}\right) + 270°$

$\Leftrightarrow \frac{-\omega - \omega}{1 - \omega^2} = \frac{\frac{\dots}{\tan(270)} + 1}{\frac{1}{\tan(270)} + \frac{\omega}{5}} \Leftrightarrow \frac{-2\omega}{1-\omega^2} = \frac{1}{\frac{\omega}{5}} \Leftrightarrow \omega = \pm\sqrt{\frac{5}{3}}$

$|KG(s)H(s)|_{\omega = \pm\sqrt{\frac{5}{3}}} = 0.3|K|$

$-1 < -0.3K$
$3.33 > K$ → N = 0, P = 0, Z = 0 | Estável

Para K = 1 Margem de ganho 3.33

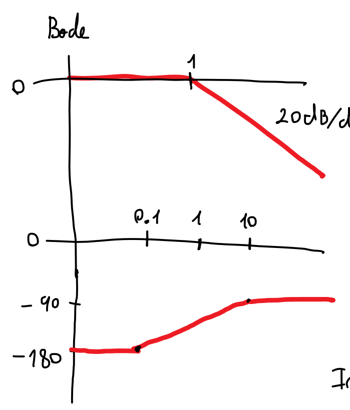
$0 < K < 3.33$

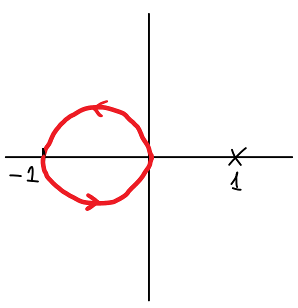
3)

a) $G(s) = \frac{1}{s-1}$

K = 1

Bode





K < 1	K = 1	K > 1
P = 1	P = 1	P = 1
N = 0	N = 0	N = -1
Z = 1		Z = 0

Instável ← Z = 1; K = 1 → Estabilidade Marginal; Z = 0 → Estável

b) $G(s) = \frac{1}{s - p}$ $p \in [\frac{1}{2}, 10]$

Por inferência da alínea anterior

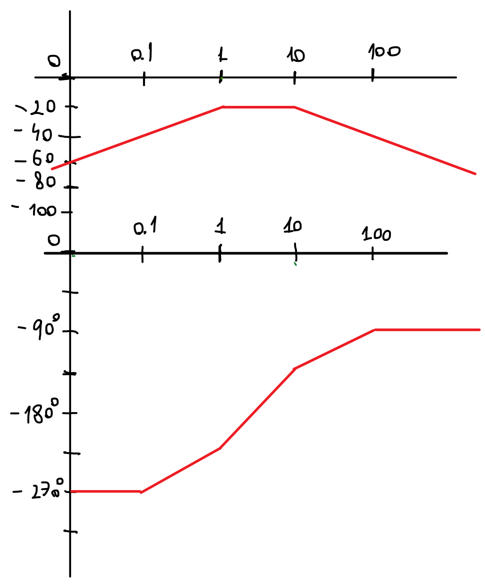
Para $p = \frac{1}{2}$ é sempre estável

Para $p = 10$

K < p	K > p
P = 1	P = 1
N = 0	N = -1
Z = 1	Z = 0
Instável	Estável

Logo K > 10 para ser estável

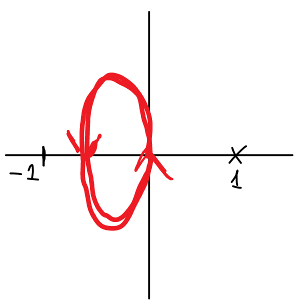
c) $G(s) = \frac{s}{(s-1)(s-10)} = \frac{1}{10} \cdot s \cdot \frac{10}{s-10} \cdot \frac{1}{s-1}$



$90° - \left(90° + \arctg\left(\frac{1}{\omega}\right)\right) - \left(90° + \arctg\left(\frac{10}{\omega}\right)\right) = 180°$

$\hookrightarrow \arctg\left(\frac{1}{\omega}\right) + \arctg\left(\frac{10}{\omega}\right) = 90° \Rightarrow K = 11$

Para ser estável



K = 1

P = 2
N = 0
Z = 2

K = 11

P = 2
N = 2
Z = 0

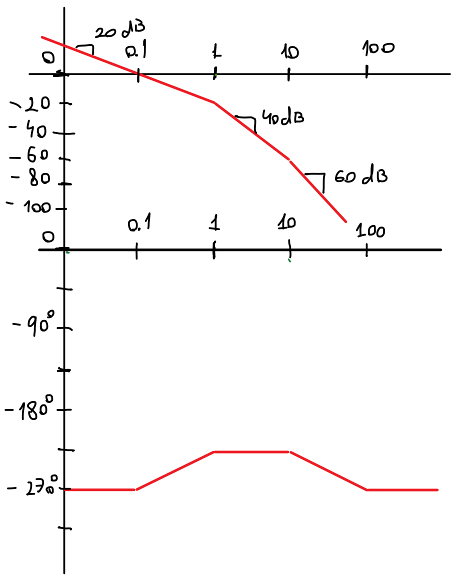
4)

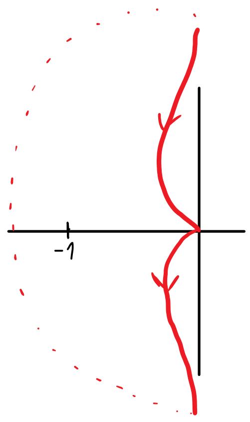
$$\frac{1}{s}\cdot\frac{1}{(s-1)}\cdot\frac{10}{(s+10)}\cdot\frac{1}{10}$$

$$G(s)=\frac{m}{(s-1)(s+10)} \quad , m>0$$

a) m = 1

$$C(s)=\frac{K}{s}$$

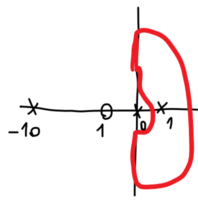




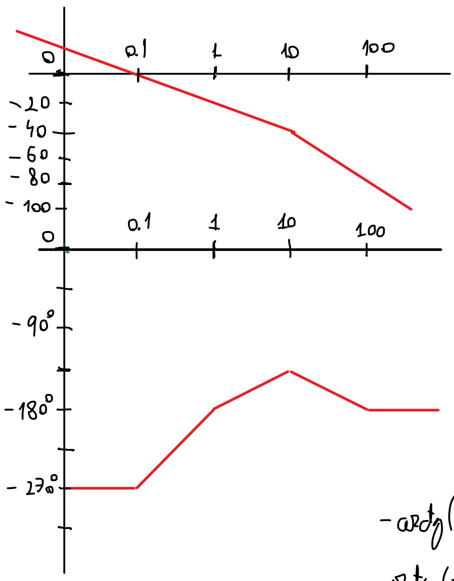
P = 1
N = 1
Z = 2
$\forall K>0$

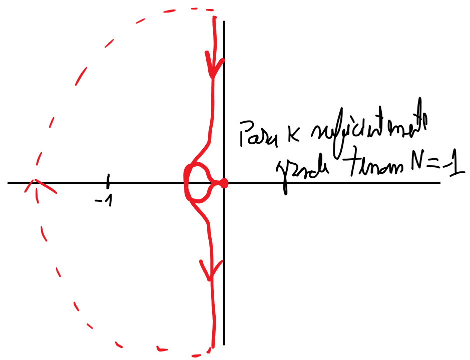
b) $C(s)=K\,\frac{(s+1)}{s}$

$$\frac{1}{10}\cdot\frac{1}{s-1}\cdot\frac{10}{s+10}\cdot\frac{1}{s}\cdot(s+1)$$



P = 1





Para K suficientemente grande temos N = 1

$K^* = ?$

$$-\text{arctg}\left(\frac{\omega}{10}\right)+\text{arctg}(\omega)-90-\left(90+\text{arctg}\left(\frac{1}{\omega}\right)\right)=180^\circ$$

$$-\text{arctg}\left(\frac{\omega}{10}\right)+\text{arctg}(\omega)-\text{arctg}\left(\frac{1}{\omega}\right)=360^\circ$$

$$\text{arctg}\left(\frac{\omega}{10}\right)+\text{arctg}\left(\frac{1}{\omega}\right)=-\cancel{360^\circ}+\text{arctg}(\omega)$$

$$\frac{\frac{\omega}{10}+\frac{1}{\omega}}{1-\frac{1}{10}}=\frac{0+\omega}{1} \iff \frac{\omega}{10}+\frac{1}{\omega}=\frac{9\omega}{10} \Rightarrow \omega=\sqrt{\frac{10}{8}}$$

$s=j\omega$

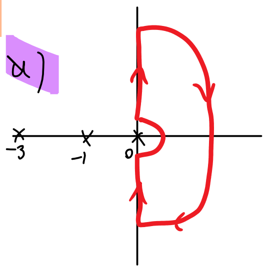
$$1+K^*\cdot\frac{1}{10}\cdot\frac{1}{s-1}\cdot\frac{10}{s+10}\cdot\frac{1}{s}\cdot(s+1)=0 \qquad K^*=11{,}2430 \text{ Para ser estável}$$

c) $MG^- = \dfrac{1}{K\,G(j\omega)H(j\omega)}\Big|_{\omega=\omega_\pi}$ $\omega_\pi=1$ $\left(1-\text{ocorrência}\right)$ $\Rightarrow m>0{,}1005$

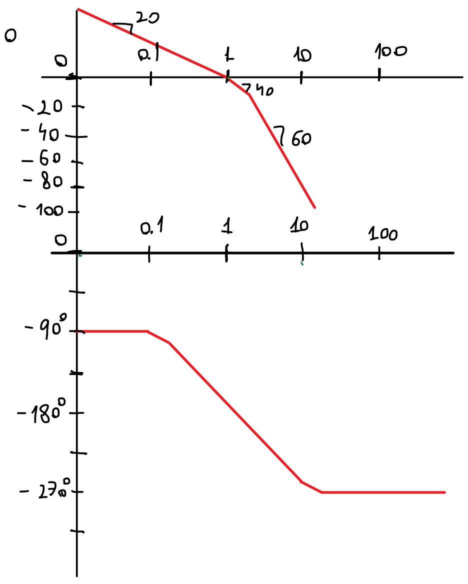
d) $MG^+=\infty$ (aproxima-se de 180° mas nunca chega)

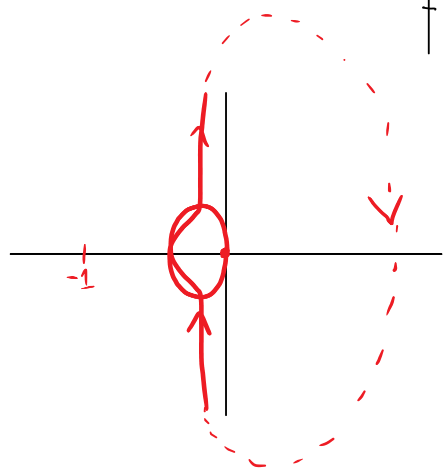
5)

a)



P = 0





$s(s+3)(s+1)$

$(s^2+3s)(s+1)$

$s^3+3s^2+s^2+3s$

Para K < 0 é sempre instável

$$-\text{arctg}\left(\frac{\omega}{3}\right)-\text{arctg}(\omega)-90=180$$

$$\frac{\frac{\omega}{3}+\omega}{1-\frac{\omega^2}{3}}=\infty \qquad \frac{\omega^2}{3}=1 \Rightarrow \omega=\sqrt{3}$$

$\omega=\sqrt{3}$

$1+K^*\cdot H(\omega_j) \Rightarrow K^*=4$

Para K > 4 é instável

É estável para $K\in[0;4]$

b)

K < 0	K > 4
P = 0	P = 0
N = -1	N = 2
Z = 1	Z = -2
1 Polo	2 Polos

6)

$$\arg(K\,H(s)\,G(s))=180^\circ$$

$$-\omega T-90^\circ-\text{arctg}(\omega)=180^\circ$$

$$-\omega T-\text{arctg}(\omega)=270^\circ$$

$$\frac{\tan(\omega T)+\omega}{1+\tan(\omega T)\,\omega}=\infty \Rightarrow 1+\tan(\omega T)\,\omega=0$$

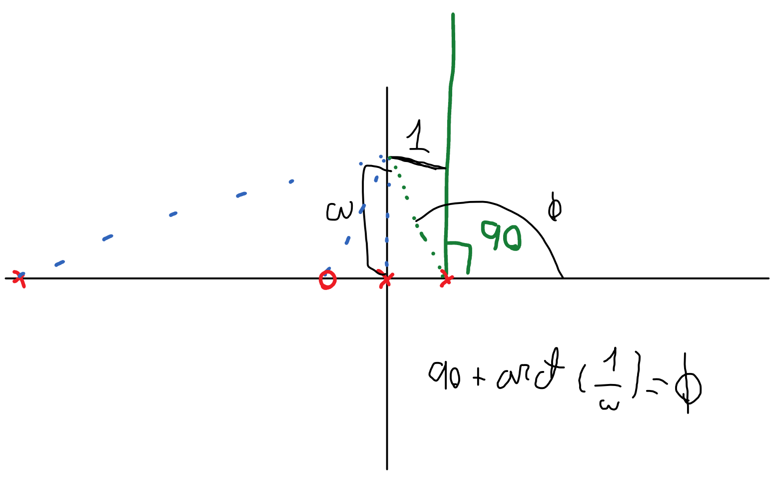
$$\iff \tan(\omega T)=-\frac{1}{\omega} \iff$$

$$\iff 1+K\,G(s)H(s)=0 \iff 1+K\cdot\frac{e^{-Ts}}{s(s+1)}=0$$

$$\iff K=-\frac{s(s+1)}{e^{-Ts}} \qquad s=j\omega$$

...

7) a)



$$90+\text{arctg}\left(\frac{1}{\omega}\right)=\phi$$