1

.1) $\sum f = F_{total}$

$F_{total} = ma$

Em x:

força aplicada - $F \sin(\theta)$

Em z:

força aplicada - $F\cos(\theta)$

- gravidade: $-mg$

Logo

$m\ddot{x} = F\sin(\theta)$

$m\ddot{z} = F\cos(\theta) - mg$

$\tau = \gamma\ddot{\theta}$

$\sum \text{torques} = \gamma\,\omega$

Sendo $\tau = \gamma\ddot{\theta}$

.2) $x = x_0$, $z = z_0$, $\dot{x}_0 = \dot{z}_0 = 0 \Rightarrow \ddot{x}_0 = \ddot{z}_0 = 0$

$$\begin{cases} m\ddot{x} = F\sin(\theta) \\ m\ddot{z} = F\cos(\theta) - mg \\ \tau = \gamma\ddot{\theta} \end{cases} \Leftrightarrow \begin{cases} 0 = F\sin(\theta) \\ 0 = F\cos(\theta) - mg \\ \tau = \gamma\ddot{\theta} \end{cases} \begin{matrix} \Rightarrow \theta_0 = 0 \\ \Rightarrow F_0 = mg \\ \Rightarrow \tau_0 = 0 \end{matrix}$$

.3) $$m\,\delta\ddot{z} \approx 0 + \left.\frac{\partial F\cos(\theta) - mg}{\partial F}\right|_{eq} \delta F + \left.\frac{\partial F\cos(\theta) - mg}{\partial \theta}\right|_{eq} \delta\theta \Leftrightarrow m\,\delta\ddot{z} = \delta F \Leftrightarrow \frac{\delta\ddot{z}}{\delta F} = 1 \Rightarrow \frac{\Delta Z(s)}{\Delta F(s)} = \frac{1}{s^2}$$

.2) $d_2(t) = u(t)$

$E(s) = -Y(s)$

$E(s)\cdot K(s)\cdot G(s) + D_2(s) = Y(s)$

$-Y(s)K(s)G(s) + D_2(s) = Y(s)$

$$\frac{D(s)}{1 + K(s)G(s)} = Y(s)$$

$$\lim_{s\to 0} s\cdot Y(s) = \frac{s\cdot\frac{1}{s}}{1 + k\cdot\frac{s+1}{s(s-1)}} = \frac{1}{\infty} = 0$$

$d_1(t) = u(t)$

$\left(E(s)K(s) + D_1(s)\right)\cdot G(s) = Y(s)$

$E(s) = -Y(s)$

$-Y(s)k(s)G(s) + D_1(s)G(s) = Y(s)$

$$\frac{D_1(s)\,G(s)}{1 + K(s)G(s)} = Y(s)$$

$$\lim_{s\to 0} s\cdot Y(s) = \frac{\frac{1}{s}\cdot s\cdot\frac{1}{s(s-1)}}{1 + k\frac{s+1}{s(s-1)}} = \frac{\frac{1}{s(s-1)}}{1 + k\frac{s+1}{s(s-1)}} = \frac{1}{k}$$

2

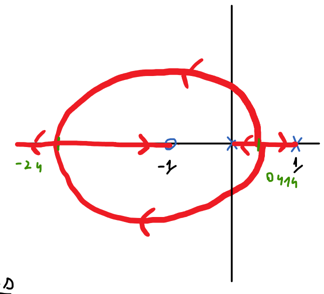
.1) $G(s) = \dfrac{1}{s(s-1)}$

$K(s) = K(s+1)$

$m = 1$

$n = 2$

1 assimptota



$$K = -\frac{s(s-1)}{s+1} = -\frac{s^2 - s}{s+1}$$

$$\frac{dk}{ds} = 0 \Leftrightarrow (2s-1)(s+1) - (s^2 - s) = 0 \Leftrightarrow 2s^2 - s + 2s - 1 - s^2 + s = 0 \Leftrightarrow$$

$\Leftrightarrow s^2 + 2s - 1 = 0 \Leftrightarrow$ $s = 0{,}414$, $s = -2{,}41$

$1 + G(s)K(s) = 0$ $\quad 1 + k\dfrac{j\omega + 1}{j\omega(j\omega - 1)} = 0 \Leftrightarrow$

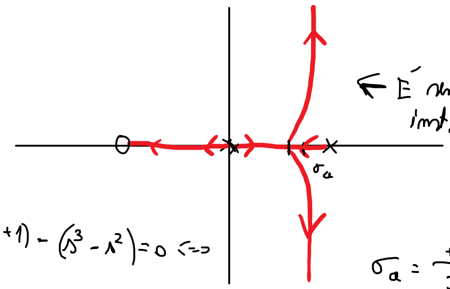
$j\omega = s$

$\Leftrightarrow j\omega K + K = -j\omega(j\omega - 1) \Leftrightarrow j\omega K + K = -j\omega + \omega^2$

O sistema é estável para $k > 1$

$$\begin{cases} \omega K = \omega \\ K = \omega^2 \end{cases} \Rightarrow \begin{matrix} \omega = 1 \\ K = 1 \end{matrix}$$

3) $K\dfrac{s+1}{s^2(s-1)}$



← É sempre instável!

$\sigma_a = \dfrac{+1}{3} = 0.66$

$$\frac{dK}{ds} = 0 \Leftrightarrow (3s^2 - 2s)(s+1) - (s^3 - s^2) = 0 \Leftrightarrow$$

$\Leftrightarrow 2s^3 + 2s^2 - 2s = 0$

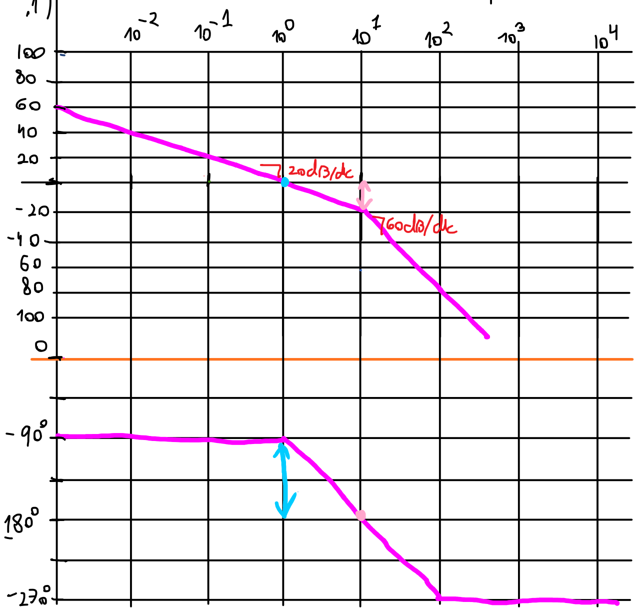
$s = 0$

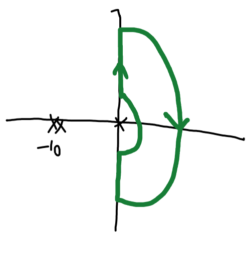
$s = 0.618$

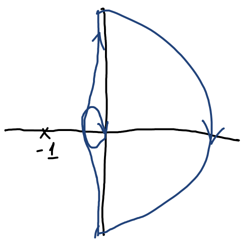
$s = -1.618 \Rightarrow$ Não pertence

3 .4) Colocar outro zero no semi-plano esquerdo!

.1)







.2) $K = 1$

$GM = 20\,dB$

$\phi_M = 90^0$

$K = 10$

$GM = 0\,dB$

$\phi_M = 0^0$

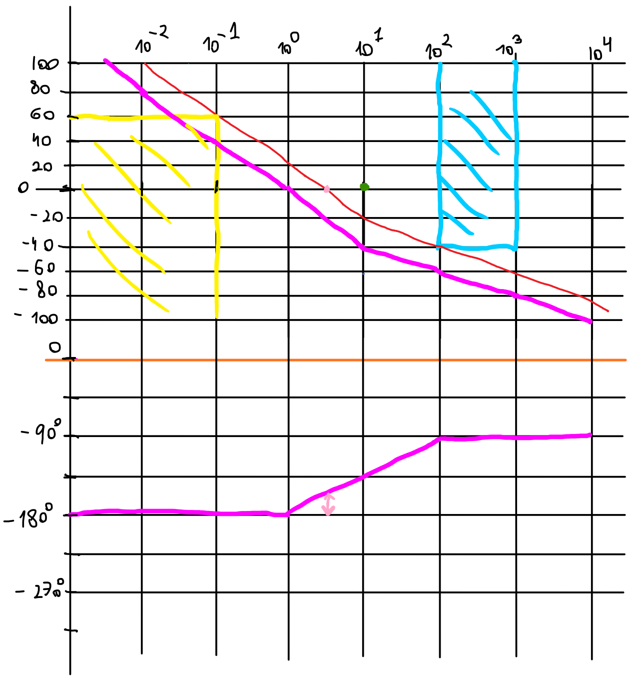
.3) $20^0 < 90^0 - \zeta \Rightarrow 70^0, \zeta$

$\zeta \in [0; \frac{7\pi}{18}]$

$$-2\arctan\left(\frac{\omega}{10}\right) - 90^0 + 180^0 = 0 \Leftrightarrow 2\arctan\left(\frac{\omega}{10}\right) = 90 \Leftrightarrow \frac{\frac{\omega}{10} + \frac{\omega}{10}}{1 - \frac{\omega^2}{100}} = \infty \Rightarrow 1 - \frac{\omega^2}{100} = 0 \Rightarrow \omega = 10$$

$$k = \left|\frac{10\,(10 + 10)^2}{100}\right| = 20 \Rightarrow k \in [0, 20[$$

4



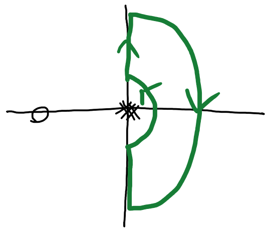
i) $m \geq 0$

ii) $|P(s)K(s)|_{dB} \geq 60\,dB$

iii) $|P(s)K(s)|_{dB} \leq -40\,dB$

iv) $G_M \geq 40\,dB$

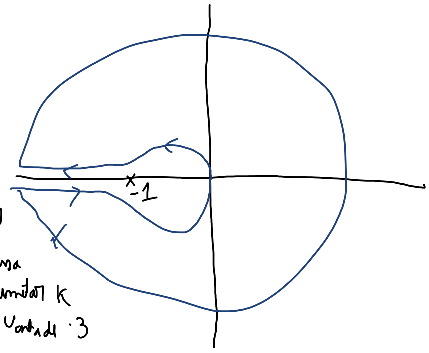
v) $P_M \geq 20°$



$m = 0 \quad K = 1$

$z = 10$

$K \cdot \frac{1}{s} \cdot \frac{1}{s} \cdot \frac{s+z}{z}$



Para aumentar K à vontade ·3

Fase $m=0$, $z=10$, $K=10$

Respeita i, ii, iii

$G_M = \infty$

$P_M = 22.5$

Respeita tudo mas P_M podia ser melhor

Em diagrama real (não no assimptótico) pode ser diferente

1
8
6
4
2
-
-4
1
-
18
-