Q1.1) $V = 1\ m/s$, $\theta_0 = \frac{\pi}{2}$, $d_0 = 0$

$\dot{d} = V\cos(\theta) = f(\theta)$

$f(\theta_0) = 0$ Ponto de equilíbrio

$$\dot{d} \approx f(\theta_0) + \left.\frac{\partial f(\theta)}{\partial \theta}\right|_{eq} (\theta - \theta_0)$$

$$\delta\dot{d} \approx -V\sin(\theta)\Big|_{eq} \delta\theta \iff \delta\dot{d} \approx -\delta\theta$$

$$-\frac{\delta\dot{d}}{\delta\theta} = 1 \xrightarrow{TL} -\frac{s\,\Delta D(s)}{s\,\Delta\Theta(s)} = 1 \iff P_1(s) = \frac{\Delta D(s)}{\Delta\Theta(s)} = -\frac{1}{s}$$

Q1.2) $\omega_0 = 0$, $m_{d_0} = 0$, $\theta_0 = \frac{\pi}{2}$, $I = 1$

$I\dot{\omega} = T_a - 10\,m_d \iff$

$\iff \dot{\omega} = T_a - 10\,m_d = -10\omega - \omega^2 - 10 m_d = f(\omega, m_d)$

$f(\omega_0, m_{d_0}) = 0$

$$\delta\dot{\omega} \approx f(\omega_0, m_{d_0}) + \frac{\partial f}{\partial \omega}\delta\omega + \frac{\partial f}{\partial m_d}\delta m_d =$$

$$\approx 0 + (-10)\delta\omega + (-10)\delta m_d$$

$$\delta\dot{\omega} \approx -10\delta\omega - 10\delta m_d \rightarrow \delta m_d = \frac{\delta\dot{\omega} + 10\delta\omega}{-10} \xrightarrow{TL} \Delta N_d(s) = \frac{\Delta W(s)\,(s+10)}{-10}$$

$$\delta\dot{\theta} \approx \delta\omega \xrightarrow{TL} \Delta\Theta(s) = \frac{\Delta W(s)}{s}$$

$$P_2(s) = \frac{\Delta\Theta(s)}{\Delta N_d(s)} = \frac{\frac{\Delta W(s)}{s}}{\frac{\Delta W(s)(s+10)}{-10}} = -\frac{10}{s(s+10)}$$

Q1.3) $V = 3$ $\quad P_1(s) = -\frac{3}{s}$ $\quad$ Logo $P(s) = V \cdot \frac{10}{s^2(s+10)}$

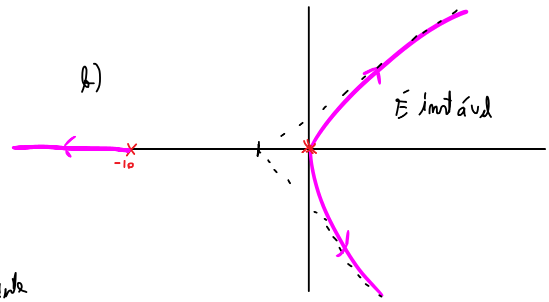
$P_2(s) =$ Não se altera $\quad$ V é um ganho!

Q2.1) a) $G(s) = K \cdot \frac{10}{s^2(s+10)}$

$$H(s) = \frac{G(s)}{1+G(s)} = \frac{K \cdot \frac{10}{s^2(s+10)}}{\frac{s^2(s+10) + K10}{s^2(s+10)}} = \frac{K10}{s^2(s+10) + K10} =$$

$$= \frac{K10}{s^3 + 10s^2 + K10}$$ → Pelo critério de Hurwitz como falta um coeficiente não é estável!

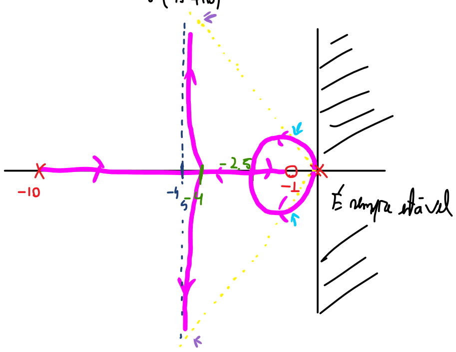
b)



Q2.2) Em cadeia aberta

$$G(s) = \frac{K(s+1)10}{s^2(s+10)}$$

Entre -10 e -1 temos traçado pois temos um número ímpar de polos e zeros à direita



$$1 + \frac{K(s+1)10}{s^2(s+10)} = 0 \iff K = -\frac{s^2(s+10)}{(s+1)10}$$

$$\frac{dK}{ds} = 0 \iff \frac{(3s^2+20s)(s+1)10 - s^2(s+10)10}{((s+1)10)^2} = 0$$

$$(3s^2+20s)(s+1) - s^2(s+10) = 0$$

$$3s^3 + 3s^2 + 20s^2 + 20s - s^3 - s^2 10 = 0$$

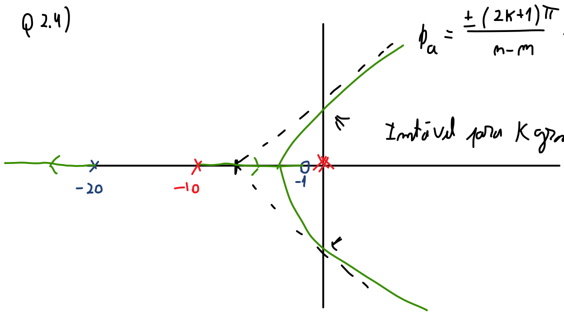
$$2s^3 + 13s^2 + 20s = 0 \iff 2s^2 + 13s + 20 = 0 \iff \begin{cases} s = -4 \checkmark \\ s = -\frac{5}{2} \checkmark \end{cases}$$

Q2.3) $\theta = \arcsin(0.5) = 30^\circ$

Aí existem duas soluções possíveis (a e b)

$$\sigma = \frac{-10+1}{2} = -4.5 \checkmark$$

Q2.4)



$$\phi_a = \frac{\pm(2K+1)\pi}{n-m} = \begin{cases} \frac{\pi}{3} \\ -\frac{\pi}{3} \\ \pi \end{cases}$$

Instável para K grande o suficiente

Q 3 1) $G(s) = \dfrac{10}{s^2(s+10)}$ $\quad C(s) = \dfrac{10k(s+1)}{s^2(s+10)}$

$k(s) = k(s+1)$

Intuitivamente é trivial que os erros estáticos de posição e velocidade são 0 dado que $C(s)$ tem 2 integradores $\left(\frac{1}{s^2}\right)$

$$\begin{cases} E(s) = R(s) - Y(s) \\ Y(s) = E(s)\cdot C(s) \end{cases} \Leftrightarrow \begin{cases} E(s) = R(s) - E(s)\cdot C(s) \\ - \end{cases} \Rightarrow \begin{cases} E(s) = \dfrac{R(s)}{1 + C(s)} \end{cases}$$

$R(s) = \frac{1}{s}$ (escalão)

$$\lim_{t\to\infty} e(t) = \lim_{s\to 0} s\,E(s) = \lim_{s\to 0} \frac{s\cdot\frac{1}{s}}{1 + \frac{10k(s+1)}{s^2(s+10)}} = \frac{1}{1+\infty} = 0$$

$R(s) = \frac{1}{s^2}$ (rampa)

$$\lim_{t\to\infty} e(t) = \lim_{s\to 0} s\,E(s) = \lim_{s\to 0} \frac{s\cdot\frac{1}{s^2}}{1 + \frac{10k(s+1)}{s^2(s+10)}} = \lim_{s\to 0} \frac{\frac{1}{s}(1)}{\frac{1}{s}\left(s + \frac{10k(s+1)}{s(s+10)}\right)} = \frac{1}{0+\infty} = 0$$

Q 3.2) $R(s) = Z(s) = 0$

$$\begin{cases} Y(s) = G(s)\cdot U(s) \\ U(s) = W(s) + E(s)K(s) \\ E(s) = -Y(s) \end{cases} \Rightarrow \begin{cases} Y(s) = G(s)\cdot\left(W(s) - Y(s)k(s)\right) \\ U(s) = W(s) - Y(s)k(s) \\ - \end{cases}$$

$$Y(s) = G(s)\cdot W(s) - G(s)Y(s)k(s)$$

$$Y(s) = \frac{G(s)\cdot W(s)}{1 + G(s)K(s)}$$

$$e = \lim_{s\to 0} s\,Y(s) = \lim_{s\to 0} \frac{s\cdot\frac{\alpha}{s}\cdot\frac{10}{s^2(s+10)}}{1 + \frac{10k(s+1)}{s^2(s+10)}} = \lim_{s\to 0} \frac{\alpha\frac{10}{s^2(s+10)}}{\frac{s^2(s+10)+10k(s+1)}{s^2(s+10)}} = \lim_{s\to 0} \alpha\frac{10}{s^2(s+10)+10k(s+1)} = \frac{\alpha}{K}$$

Vai ter um erro $\frac{\alpha}{K}$ isto acontece no que é equivalente a ter uma referencia $= \frac{1}{k(s+1)\,s}$ logo o erro não é nulo

Q 3.3) $Z(s) = \frac{\beta}{s}$

$$\begin{cases} Y(s) = E(s)\cdot K(s)\,G(s) \\ E(s) = -\left(Z(s) + Y(s)\right) \end{cases} \Rightarrow \begin{cases} Y(s) = -\left(Z(s) + Y(s)\right)K(s)G(s) \\ - \end{cases}$$

$$Y(s) = -\frac{Z(s)K(s)\,G(s)}{1 + K(s)\,G(s)}$$

$$e = \lim_{s\to 0} s\,Y(s) = -\frac{\cancel{s}\,\frac{\beta}{\cancel{s}}\cdot\frac{10k(s+1)}{\cancel{s^2(s+10)}}}{\frac{s^2(s+10)+10k(s+1)}{\cancel{s^2(s+10)}}} = \lim_{s\to 0} -\frac{\beta\cdot 10k(s+1)}{s^2(s+10)+10k(s+1)} = -\frac{\beta\cdot\cancel{10k}}{\cancel{10k}} = -\beta$$