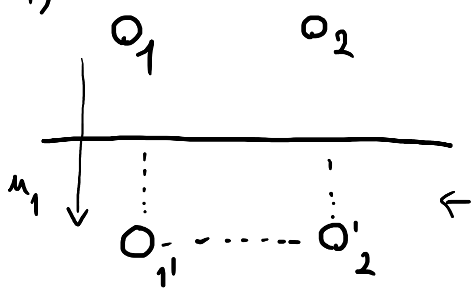
1.1) 1)



Aplicar uma carga em 1 e 1'

← Método das imagens (ver doc auxiliar)

Distribuição filiforme de carga Q e comprimento ℓ

$$V_i = \frac{Q_1}{2\pi\varepsilon_0 \ell}\ln\left(\frac{r_2}{r_1}\right)$$

r_2 = dist. de 1' ao centro de i

r_1 = dist. de 1 ao centro de i

$$u_1 = \frac{Q_1}{2\pi\varepsilon L}\ln\left(\frac{2h}{r_0}\right)$$

$$S_{11} = \frac{u_1}{Q_1} = \frac{1}{2\pi\varepsilon L}\ln\left(\frac{2h}{r_0}\right) = \frac{4}{3}\times 10^8 = S_{22}$$

$$u_2 = \frac{Q_1}{2\pi\varepsilon L}\ln\left(\frac{\sqrt{d^2+(2h)^2}}{d}\right)$$

$$S_{21} = \frac{u_2}{Q_1} = \frac{1}{2\pi\varepsilon L}\ln\left(\frac{\sqrt{d^2+(2h)^2}}{d}\right) = \frac{1}{3}\times 10^8 = S_{12}$$

$$\text{Logo } [S] = \begin{bmatrix} S_{11} & S_{21} \\ S_{12} & S_{22} \end{bmatrix} = \begin{bmatrix} 4/3 & 1/3 \\ 1/3 & 4/3 \end{bmatrix} 10^8\ F^{-1}$$

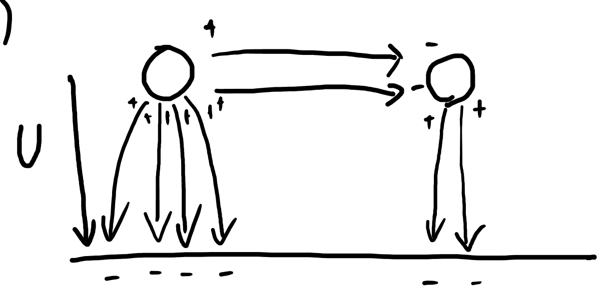
2) $$[C] = [S]^{-1} = \frac{1}{\det[S]}\begin{bmatrix} 4/3 & -1/3 \\ -1/3 & 4/3 \end{bmatrix} 10^8 = \begin{bmatrix} 4/5 & -1/5 \\ -1/5 & 4/5 \end{bmatrix} 10^{-8} = \begin{bmatrix} 8 & -2 \\ -2 & 8 \end{bmatrix} nF$$

3) $$\hat{C}_{10} = \hat{C}_{20} = \sum_{j=1}^{2} C_{1j} = 6\ nF$$

$$\hat{C}_{11} = -C_{11} = -8\ nF\ ;\ \hat{C}_{22} = -C_{22} = -8\ nF$$

$$\hat{C}_{12} = \hat{C}_{21} = -C_{21} = -C_{12} = 2\ nF$$

4)



$$\begin{cases} Q_1 = ? \\ Q_2 = 0 \end{cases} \Leftrightarrow \begin{cases} Q_1 = U_1 C_{11} + U_2 C_{12} \\ Q_2 = U_1 C_{21} + U_2 C_{22} \end{cases}$$

$$\Downarrow$$

$U_1 = 100\ \text{kV}$ $\quad U_2 = 25\ \text{kV}$

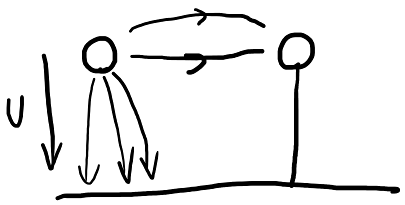
$Q_2 = 0$ $\quad Q_1 = 750\ \mu C$

$$W_e = \frac{1}{2} \begin{bmatrix} Q_1 & Q_2 \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \end{bmatrix} = 37{,}5\ \text{J}$$

$$Q_2 = \hat{C}_{21}(U_2 - U_1) + \hat{C}_{20} U_2 = 0$$

$$\Leftrightarrow \bar{Q}_2 = \hat{C}_{21}(U_2 - U_1) = -150\ \mu C$$

5)



$U_1 = 100\ \text{kV}$

$U_2 = 0\ \text{V}$

$$\begin{cases} Q_1 = U_1 C_{11} = 800\ \mu F \\ Q_2 = U_1 C_{12} = -200\ \mu F \end{cases}$$

$$W_e = \frac{1}{2} \begin{bmatrix} Q_1 & Q_2 \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \end{bmatrix} = 40\ \text{J}$$

$40 - 37{,}5 > 0 \Rightarrow$ Existe dissipação de energia

6)

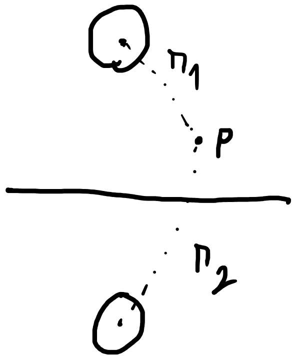
$$\boxed{E = \frac{\lambda}{2\pi\varepsilon} \frac{1}{r}} = \frac{Q}{2\pi\varepsilon l} \frac{1}{r} = 960000 = 9{,}6\ \text{kV/cm}$$

dado

a aproximação filiforme,,

1.2)

1)



$$V_p = \frac{Q}{2\pi\varepsilon_0 l}\ln\left(\frac{r_2}{r_1}\right)$$

vezi 1.1) - 1)

2)

$$C = \frac{Q}{V_p} = \frac{2\pi\varepsilon_0 l}{\ln\left(\frac{r_2}{r_1}\right)} = \frac{2\pi\varepsilon_0 l}{\ln\left(\frac{2h}{r_0}\right)} = 15,1\,nF$$

$r_2 = 2h$

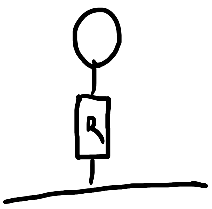
$r_1 = r_0$

3) $U_1 = 10\,kV$

$$C \cdot U_1 = Q \Leftrightarrow Q = 0.151\,mC$$

$$E = \frac{\lambda}{2\pi\varepsilon}\frac{1}{r} = \frac{Q}{2\pi\varepsilon l}\frac{1}{r} = 5,436\,kV/cm$$

4)



$$W_j = W_e = \frac{1}{2}U_1 q_1 = 0,755\,J$$

5)

$$F_e = -\nabla W_e = -\frac{\partial}{\partial y}\frac{1}{C(y)}\frac{Q^2}{2} = -\frac{Q^2}{2}\frac{\partial}{\partial y}\left(\frac{U(y)}{Q}\right) = -\frac{Q}{2}E_y =$$

$$= -\frac{Q^2}{2\varepsilon_0 2\pi y l} = -705\vec{e}_y\,[N]$$

1.3 1) O o cabra 1 todo → Logo $\hat{c}_{12} = 0$

$$Q_1 = \hat{c}_{10} V_1$$

$$Q_2 = \hat{c}_{20} V_2$$

2) $[C] = \begin{bmatrix} c_{11} & 0 \\ 0 & c_{22} \end{bmatrix}$ $\quad c_{11} = \hat{c}_{10}$ $\quad c_{22} = \hat{c}_{20}$

$$\begin{bmatrix} 0{,}2 & 0 \\ 0 & 0{,}2 \end{bmatrix} \mu F$$

3) $V_1 + V_2 = 100 \, kV$

$Q_0 = 0$

$Q_1 = ?$

$Q_2 = ?$

$$\left\{ \begin{array}{l} V_1 + V_2 = 100 \, kV \\ Q_1 = V_1 C_1 \\ Q_2 = V_2 C_2 \\ \underbrace{Q_0}_{0} = Q_1 + Q_2 \end{array} \right. \Leftrightarrow \left\{ \begin{array}{l} V_1 = 50 \, kV \\ V_2 = -50 \, kV \\ Q_1 = 10 \, mC \\ Q_2 = -10 \, mC \end{array} \right.$$

IST

$$W_e = \frac{1}{2} \begin{bmatrix} Q_1 & Q_2 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = 500 \, J$$

4) $V_2 = 0 \quad V_1 = 50 \, kV$

$Q_2 = 0 \quad Q_1 = 10 \, mC$ $\rightarrow Q_0 = 10 \, mC$

5)

$$W_e' = \frac{1}{2} \begin{bmatrix} Q_1 & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ 0 \end{bmatrix} = \frac{1}{2} Q_1 V_1 =$$

$$= 250$$

$$W_J = 500 - 250 = 250$$

$$c = \frac{q}{V}$$

6)

$$E = 0.10 \, E_d$$

$$\varepsilon_{r1} = 2{,}5 \, \varepsilon_0$$

$$E = \frac{Q}{2\pi\varepsilon_1 l} \frac{1}{r_1} \Leftrightarrow 0.1 \, E_d = \frac{Q}{2\pi\varepsilon_1 l} \frac{1}{r_1} \Leftrightarrow r_1 = \frac{Q}{2\pi \cdot 2{,}5 \varepsilon_1 \, l (0.1) E_d} =$$

$$= 0{,}036 \, m$$

Para o r_2 ⇒ fazer igual logo $0{,}036 + 0{,}036 = 0{,}072 \, m$

1.4) 1) $U = U_1 = U_2 = 1V$

$Q = Q_1 + Q_2 = 6mC$

$C = \frac{Q}{U} = 6mF$

$C = \hat{C}_{10} // \hat{C}_{20}$

$\hat{C}_{10} = \frac{Q_1}{U_1} = 3mF$

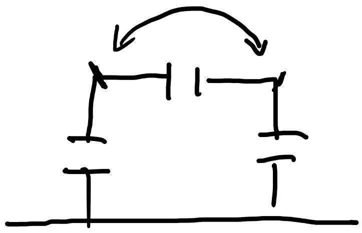
$\hat{C}_{20} = \frac{Q_2}{U_2} = 3mF$

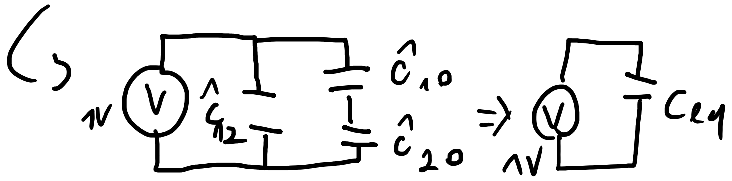
$\hat{C}_{10} // \hat{C}_{20} = \hat{C}_{20} + \hat{C}_{10} = 6mF$ //

2) $U = U_1 - U_2 = 1V$

$Q = Q_1 = 3,5 mC$

$C = \frac{Q}{V} = 3,5 mF$





$C_{eq} = \hat{C}_{12} // (\hat{C}_{10} S \hat{C}_{20}) =$

$= \hat{C}_{12} + \left(\frac{1}{\hat{C}_{10}} + \frac{1}{\hat{C}_{20}}\right)^{-1} = \hat{C}_{12} + 1.5 mF$

$3,5 = \hat{C}_{12} + 1.5 mF$

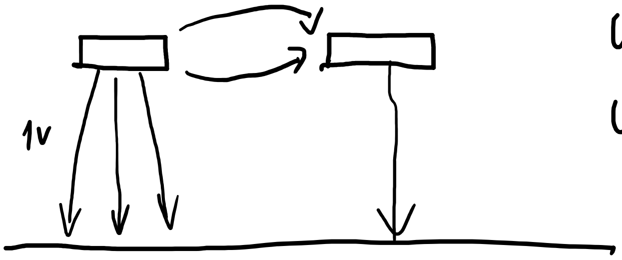
$\Rightarrow \hat{C}_{12} = 2 mF$

3) $C_{11} = C_{22} = \hat{C}_{10} + \hat{C}_{12} = 5mF$

$C_{12} = C_{21} = -\hat{C}_{12} = -2mF$

$[C] = \begin{bmatrix} 5 & -2 \\ -2 & 5 \end{bmatrix} mF$

4)



$U_1 = 1V$ $\quad Q_1 = ?$

$U_2 = ?$ $\quad Q_2 = 0$

$W_e = \frac{1}{2} Q_1 U_1 = 2.1 mJ$

$(Q) = [C](U) = \begin{cases} Q_1 = C_{11}U_1 + C_{12}U_2 \\ Q_2 = C_{21}U_1 + C_{22}U_2 \end{cases} \Leftrightarrow \begin{cases} Q_1 = 4.2 mC \\ U_2 = 0,4V \end{cases}$

5)

$Q_1 = 4.2\,mC$

$V_2 = 0$

$V_1 = ?$

$Q_2 = ?$

$$\begin{cases} Q_1 = C_{11} V_1 + C_{12} V_2 \\ Q_2 = C_{21} V_1 + C_{22} V_2 \end{cases} \Leftrightarrow \begin{cases} V_1 = 0,84\,V \\ Q_2 = -1,68\,mC \end{cases}$$

$$W_e' = \frac{1}{2} Q_1 V_1 = 1,764\,mJ$$

$W_J = W_e - W_e' > 0 \Rightarrow$ dissipação energia no fio

6) Ligar 1 a 0

$$W_J = W_e' = 1,764\,mJ$$

1,5)

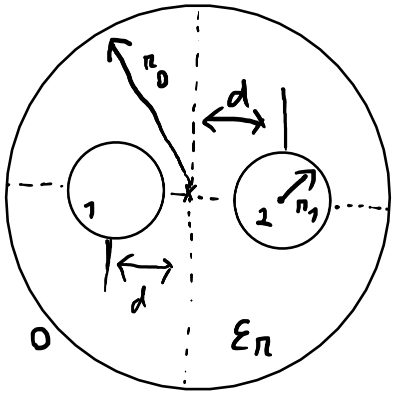
$l = 1\,Km$

$r_1 = 2\,mm$

$d = 1\,cm$

$r_0 = 2\,cm$

$\varepsilon_r = 2,3$



$S = \frac{U}{q}$ $(U) = [S]\,(Q)$

1) $U_1 = 1\,KV$

$|Q_1| = 63,65\,\mu C$

$U_2 = 107,2\,V$

$Q_2 = 0$

$$\begin{cases} U_1 = S_{11}\, Q_1 \\ U_2 = S_{21}\, Q_1 \end{cases}$$

$S_{11} = S_{22}$

$S_{21} = S_{12}$

$S_{11} = S_{22} = 15,71\,MF^{-1}$

$\Leftrightarrow S_{21} = S_{12} = 1,684\,MF^{-1}$

$$[S] = \begin{bmatrix} 15,71 & 1,684 \\ 1,684 & 15,71 \end{bmatrix} \quad W_e = \frac{1}{2} QV = 31,825\,mJ$$

2. $$[C] = [S]^{-1} = \begin{bmatrix} 64.4 & -6,93 \\ -6,93 & 64.4 \end{bmatrix} mF$$

$\hat{C}_{10} = \hat{C}_{20} = C_{11} + C_{12} = 57.47\,mF$

$\hat{C}_{12} = -C_{12} = 6,93\,mF$

3) Ver resolução prof

$U_1 = ?$

$Q_2 = ?$

4) $$\begin{cases} U_1 = S_{11}\, Q_1 + S_{12}\, Q_2 \\ U_2 = S_{21}\, Q_1 + S_{22}\, Q_2 \end{cases} \Leftrightarrow \begin{cases} - \\ \dfrac{U_2 - S_{21} Q_1}{S_{22}} = Q_2 \end{cases} \Leftrightarrow \begin{cases} U_1 = 988,35\,V \\ Q_2 = -6.82\,\mu C \end{cases}$$

$Q_1 = 63,65\,\mu C$

$U_2 = 107,2\,V$

$W_e' = \frac{1}{2} Q_1 U_1 = 31,454\,mJ$

$W_g = W_e - W_e' = 0,371\,mJ$ //