1) $\oint_S H_m \, ds = \int_{S_S} j \cdot m_S \, ds = I$

$2\pi r H = N_1 i_1 - N_2 i_2 \;\vdash\!> \; \alpha = \frac{\mu}{2\pi}(N_1 i_1 - N_2 i_2)$

$B = \mu H$

$B = \frac{\alpha}{r}$

2) $\phi = \int_{r_1}^{r_2} \frac{\alpha}{r} a \, dr = a \alpha \ln\left(\frac{r_2}{r_1}\right)$

$\alpha = 2 \times 10^{-2} \; T m$

$\phi = 0{,}2 \; mWb$

3) $R_m = \frac{U_m}{\phi} = \frac{F_{mm}}{\phi} = \frac{N_1 i_1 - N_2 i_2}{a \frac{\mu}{2\pi}(N_1 i_1 - N_2 i_2) \cdot \ln\left(\frac{r_2}{r_1}\right)} = \frac{2\pi}{a \mu \ln\left(\frac{r_2}{r_1}\right)} = 5 \times 10^6 \; H^{-1}$

Απλούστερα

$R_m = \frac{l}{\mu S} = \frac{2\pi\left(\frac{r_2 + r_1}{2}\right)}{\mu a (r_2 - r_1)} = 5{,}1 \times 10^6 \; H^{-1}$

Εγγύτερα

Ίσημο

$\phi = a \frac{\mu}{2\pi}(m_1 i_1 - m_2 i_2) \ln\left(\frac{r_2}{r_1}\right)$

4) $\begin{cases} \Psi_1 = n_1 \phi \\ \Psi_2 = -n_2 \phi \end{cases} \Leftrightarrow \begin{cases} \Psi_1 = n_1 a \frac{\mu}{2\pi} (n_1 i_1 - n_2 i_2) \ln\left(\frac{r_2}{r_1}\right) \\ \Psi_2 = n_2 a \frac{\mu}{2\pi} (n_1 i_1 - n_2 i_2) \ln\left(\frac{r_2}{r_1}\right) \end{cases}$

L_{11} $L_{12} = L_m$

$\begin{cases} \Psi_1 = n_1^2 a \frac{\mu}{2\pi} \ln\left(\frac{r_2}{r_1}\right) i_1 - n_1 n_2 a \frac{\mu}{2\pi} \ln\left(\frac{r_2}{r_1}\right) i_2 \\ \Psi_2 = -n_2 n_1 a \frac{\mu}{2\pi} \ln\left(\frac{r_2}{r_1}\right) i_1 + n_2^2 a \frac{\mu}{2\pi} \ln\left(\frac{r_2}{r_1}\right) i_2 \end{cases}$

$L_{21} = L_m$ L_{22}

$\Psi_1 = 0,2 \text{ wb}$

$\Psi_2 = -0,1 \text{ wb}$

$L_{11} = 0,2 \text{ H}$

$L_{12} = L_m = L_{21} = -0,1 \text{ H}$

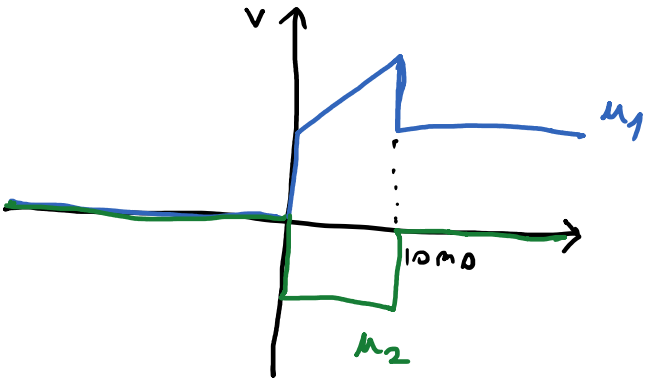
$L_{22} = 0,05 \text{ H}$

$K = \frac{L_M}{\sqrt{L_{11} L_{22}}} = 1$ ← ausência de dispersão

5) $W_m = \frac{1}{2} \frac{B_1^2}{\mu_0} V + \frac{1}{2} \frac{B_2^2}{\mu_0} V = \frac{1}{2} L_{11} i_1^2 + \frac{1}{2} L_{22} i_2^2 = 0,1 \text{ J}$

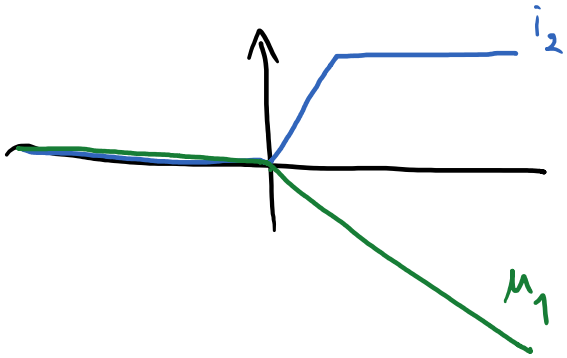
II 1) $\begin{cases} u_1 = r_1 i_1 + \frac{d\Psi_1}{dt} = r_1 i_1 + L_{11} \frac{di_1}{dt} + L_M \frac{di_2}{dt} \\ -u_2 = \frac{d\Psi_2}{dt} = L_M \frac{di_1}{dt} + L_{22} \frac{di_2}{dt} \end{cases}$

$$\begin{cases} u_1 = r_1 i_1 + L_{11} \dfrac{di_1}{dt} \\ -u_2 = L_M \dfrac{di_1}{dt} \end{cases}$$



3) $i_2 = -\dfrac{L_M}{L_{22}} i_1 = 2 i_1$

$u_1 = r_1 i_1$

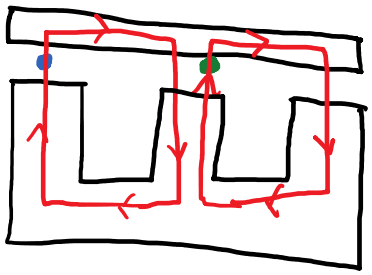


3.2)

1- $R_{m_{EF}} = \frac{l_{EF}}{\mu_{EF} S_{EF}} = \frac{\delta}{\mu_0 S} = \frac{1 \times 10^{-3}}{4\pi \times 10^{-7} \cdot 25 \times 10^{-4}} =$

$= 318310\ H^{-1}$

2-



$\oint \vec{H} d\vec{l} = \int \vec{j} \cdot \vec{n}\, ds$

- $Hl = n_1 i_1 - n_2 i_2 = U_{m1} - U_{m2} = R_m(\phi_1 - \phi_2)$
- $Hl = n_2 i_2 - n_3 i_3 = U_{m2} - U_{m3} = R_m(\phi_2 - \phi_3)$

$$\begin{cases} \phi_1 + \phi_2 + \phi_3 = 0 \\ \phi_1 - \phi_2 = \frac{N(i_1 - i_2)}{R_m} \\ \phi_2 - \phi_3 = \frac{N(i_2 - i_3)}{R_m} \end{cases} \Rightarrow \begin{cases} \phi_1 = \frac{N}{R_m}\left(\frac{2}{3} i_1 - \frac{1}{3}(i_2 + i_3)\right) \\ \phi_2 = \frac{N}{R_m}\left(\frac{2}{3} i_2 - \frac{1}{3}(i_1 + i_3)\right) \\ \phi_3 = \frac{N}{R_m}\left(\frac{2}{3} i_3 - \frac{1}{3}(i_1 + i_2)\right) \end{cases}$$

3) $\phi_1 = \frac{N}{R_m} \frac{2}{3} = 1{,}45\ mWb$

$\phi_2 = \phi_3 = -\frac{1}{3} \frac{N}{R_m} = -0{,}72\ mWb$

4) $B_2 = B_3 = \frac{\phi_2}{S} = -0.29\ T$ $\quad B_2 = \mu_0 H_2 \Leftrightarrow H_2 = \frac{B_2}{\mu_0} = -230\ kA/m$

$B_1 = \frac{\phi_1}{S} = 0{,}58\ T$ $\quad \Rightarrow B_1 = \mu_0 H_1 \Leftrightarrow H_1 = \frac{B_1}{\mu_0} = 461\ kA/m$

5) $\Psi_1 = N\,\phi_1$

$\Psi_2 = N\,\phi_2$

$\Psi_3 = N\,\phi_3$

$\Leftrightarrow$

$\Psi_1 = \frac{V^2}{R_m}\frac{2}{3} = 1\,wb = L_{11}\,i_1$

$\Psi_2 = -\frac{1}{3}\frac{V^2}{R_m} = \Psi_3 = -0{,}5\,wb = L_{21}\,i_1 = L_{31}\,i_1$

$L_{11} = 1\,H$

$L_{21} = 0{,}5\,H$

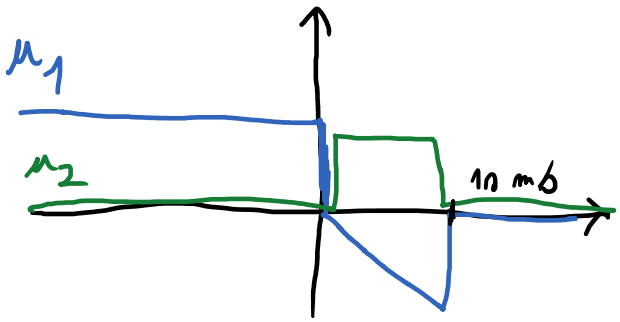
$L_{31} = -0{,}5\,H$

6) $W_m = \frac{1}{2}\frac{B_1^2}{\mu_0}\delta S + \frac{1}{2}\frac{B_2^2}{\mu_0}\delta S + \frac{1}{2}\frac{B_3^2}{\mu_0}\delta S = \frac{\delta S}{2\mu_0}(B_1^2 + B_2^2 + B_3^2) = 0.5\,J$

7) $u_1 = R i_1 + \frac{d\Psi_1}{dt}$ $\quad \Psi_i = L_p i_i + L_M i_j + L_M i_k$

$u_2 = \frac{d\Psi_2}{dt}$

$u_3 = \frac{d\Psi_3}{dt}$



8) $i_2(0) = 0$

$u_3 = 0 \Rightarrow \Psi_3 = 0 \Rightarrow i_3 = -\frac{L_M}{L_p} i_1 = \frac{1}{2} i_1$

$u_1 = R_1 + L_p\frac{di_1}{dt} + L_M\frac{di_3}{dt} = R i_1 + \frac{3}{4} L_p\frac{di_1}{dt}$

$u_2 = L_M\frac{di_1}{dt} + L_M\frac{di_3}{dt} = \frac{3}{2} L_M\frac{di_1}{dt} = -\frac{3}{4} L_p\frac{di_1}{dt}$

3.3)

1) Vamos assumir:

- Ausência de dispersão
- Meio linear
- Campo uniforme em cada troço k

$$R_{m_{ar}} = \frac{\delta}{\mu_0 S} = 5 \times 10^5 \, H^{-1}$$

2) Com base nisto

$$R_{m_t} = R_m + R_{m_{ar}} = 6 \times 10^5 \, H^{-1}$$

$$Hl = U_m = R_m \phi$$

$$\oint H dl = \int \vec{j} \cdot \vec{n} \, ds$$

$\Rightarrow$ Pela Lei de Ampère

$$Hl = NI \rightarrow \phi = \frac{NI}{R_m} = 1 \, mWb$$

3) E - Enrolamento

e - espira

$$\Psi_E = N\phi = L_E I$$

$$\Psi_e = \phi = L_e I$$

$$\Psi_E = 0,6 \, Wb$$

$$\Psi_e = 1 \, mWb$$

$$L_{11} = L_E = 0,6 \, H$$

$$L_{21} = L_e = 1 \, mH$$

$K \neq 1$ Tudo a ausência de dispersão

4)

$$B = \frac{\phi}{S} = 0,63 \, T$$

$$B = \mu_0 H_{ar} \Leftrightarrow H_{ar} = \frac{B}{\mu_0} \Leftrightarrow H_{ar} \simeq 500 \, kA/m$$

5) $W_m = \frac{1}{2} L_{11} i^2 = \frac{1}{2} \cdot 0.6 \cdot 1^2 = 0,3 \, J$

$W_{m_{ar}} = \frac{1}{2} \frac{B}{\mu_0} \delta S = 0.25 \, J$

$W_{m_m} = 0.05 \, J$ //

6) $\vec{E} = \vec{E}' + \underbrace{\vec{v} \times \vec{B}'}_{\hookrightarrow \vec{E}_v}$

$E_v = -Bv$

$\vec{\nabla} \times \vec{E} = -\frac{d}{dt}\vec{B} \Leftrightarrow \oint_{\partial S} \vec{E} \cdot d\vec{l} = -\frac{d}{dt}\int_S \vec{B} \cdot \vec{n}\, dS \Leftrightarrow -Bv_a = \frac{d}{dt}\Psi_a$

$\mu_d = \frac{d}{dt}\Psi_a = -Bv_a$

3.4)

1) $\oint_{dS} H\,dl = \int_S \vec{j} \cdot \vec{n}\, dS \Leftrightarrow Hl = Ni_1 - 2Ni_2$

$\Phi_m = \frac{Ni_1 - 2Ni_2}{R_m}$

$R_m = R_{mm} + R_{m0}$

$R_{m_m} = \frac{\left(l + \frac{w}{2}\right) + \left(\frac{w}{2} + l + \frac{w}{2}\right) + \left(\frac{w}{2} + l\right)}{\mu_R \mu_0 \cdot S_m} = \frac{3l + 2w}{560 \mu_0 S_m} = 0,5 \times 10^6\ H^{-1}$

$R_0 = 2 \cdot \frac{\delta}{\mu_0 S_m} = 10^6\ H^{-1}$

$R_m = 1,5 \times 10^6\ H^{-1}$

$i_2 = 0$

$\Phi_m = \frac{Ni_1}{R_m} = 1\ mWb$

2)

$B_a = \frac{\Phi}{S_a} = 0,945T$

$B_0 = B_m = \frac{\Phi}{S_m} = 0,63T$

$H_a = 0$

$H_m = \frac{B_m}{\mu_m \mu_0} = 893\ A/m$

$H_0 = \frac{B_0}{\mu_0} = 5 \times 10^2\ kA/m$

3) $\psi_1 = N\phi = 1.5\ Wb$

$\psi_2 = 2N\phi = -0.75\ Wb$

$L_{11} = \frac{\psi_1}{i_1} = 1.5\ H$

$L_M = \frac{\psi_2}{i_1} = -0.75 H$

Como $k=1$ (assumimos ausência de dispersão)

$k = \frac{L_M}{\sqrt{L_{11} L_{22}}} \Leftrightarrow L_{22} = \frac{L_M^2}{L_{11}} = 0.375\ H$

4) $W_m = \frac{1}{2} B_0 H_0 2\delta S_m + \frac{1}{2} B_m H_m (3l + 2w) S_m = 0,75\ y$

$0.75 y = \frac{1}{2} L_{11} i_1^2 \Leftrightarrow L_{11} = 1.5 H$ c.q.d.

5) $\vec{F}_m = -\nabla W_m|_\psi = -\frac{\partial f}{\partial x}\Big|_\psi \vec{e}_x = -\frac{\partial}{\partial x}(B_0 H_0 S_m x)\vec{e}_x =$

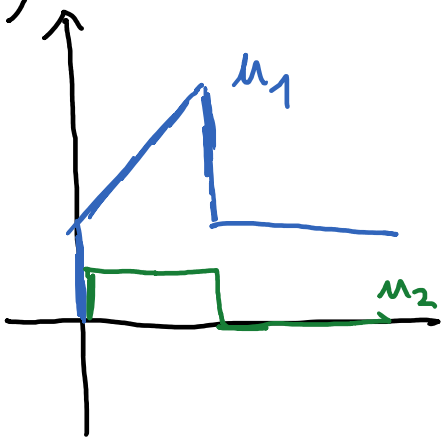
$= -B_0 H_0 S_m \vec{e}_x \Rightarrow \vec{F}_m = -501,48 \vec{e}_x$

LINEAR

6) $u_1 = r_1 i_1 + \frac{d\psi_1}{dt} = r_1 i_1 + L_{11}\frac{di_1}{dt} + L_M \frac{di_2}{dt}$

$-u_2 = r_2 i_2 + \frac{d\psi_2}{dt} = r_2 i_2 + L_M \frac{di_1}{dt} + L_{22}\frac{di_2}{dt}$

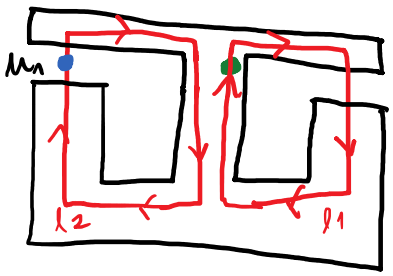
7)



3.4)

1- $R_m = \frac{\delta}{\mu_0 S} = 4,97 \times 10^5 \, H^{-1}$

2- $\phi_1 + \phi_2 + \phi_3 = 0$



$$\begin{cases} \oint_{l_1} \vec{H} \cdot d\vec{l} = \int_S \vec{j} \cdot \vec{n} \, dS = \mu_{1m} \\ \oint_{l_2} \vec{H} \cdot d\vec{l} = \int_S \vec{j} \cdot \vec{n} \, dS = \mu_{3m} \end{cases}$$

$$\begin{cases} \mu_{1m} = R_m \phi_1 = N_1 i_1 - N_2 i_2 \\ \mu_{3m} = R_m \phi_3 = N_3 i_3 - N_2 i_2 \end{cases} \Leftrightarrow \begin{cases} \phi_1 = \frac{N_1 i_1 - N_2 i_2}{R_m} \\ \phi_3 = \frac{N_3 i_3 - N_2 i_2}{R_m} \end{cases}$$

$$\phi_2 = \frac{2N_2 i_2 - N_1 i_1 - N_3 i_1}{R_m}$$

3) $\psi_1 = N_1 \phi_1$

$\psi_2 = N_2 \phi_2$

$\psi_3 = N_3 \phi_3$

$$L = \frac{1}{R_m} \begin{bmatrix} N_1^2 & -N_1 N_2 & 0 \\ -N_2 N_1 & 2N_2^2 & -N_2 N_3 \\ 0 & -N_3 N_2 & N_3^2 \end{bmatrix} = \begin{bmatrix} 0,5 & -0,5 & 0 \\ -0,5 & 1 & -0,5 \\ 0 & -0,5 & 0,5 \end{bmatrix}$$

$$k_{12} = \frac{L_{12}}{\sqrt{L_{11} L_{22}}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} = k_{23}$$

$k_{13} = 0 \rightarrow$ Não há ligamento

3.4)

4- $\phi_1 = 1\,mWb$ $B_1 = 0.625\,T \longrightarrow H_1 = 4,97 \cdot 10^5\,T$

$\phi_2 = -1\,mWb$ $B_2 = -0.625\,T \longrightarrow H_2 = 0$ (dentro de um ferro $H = 0$)

$\phi_3 = 0$ $B_3 = 0 \longrightarrow H_3 = 0$

$$W_m = \frac{1}{2} B H \delta S = 0.2485\,J$$

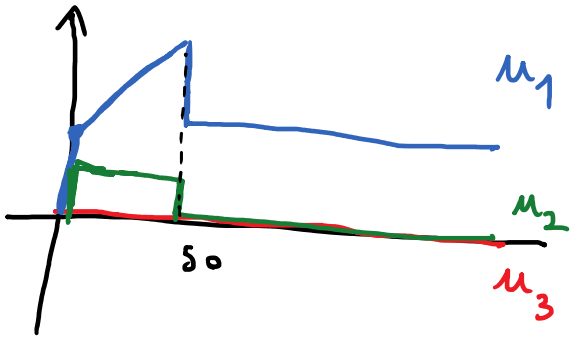
$$W_m = \frac{1}{2} L_{11} i_1^2 \iff L_{11} = \frac{2W_m}{i_1^2} \simeq 0.5\,H \quad C.q.d$$

5)

$$u_1 = r_1 i_1 + \frac{d\psi_1}{dt} \qquad u_1 = r_1 i_1 + L_{11}\frac{di_1}{dt}$$

$$u_2 = r_2 i_2 + \frac{d\psi_2}{dt} \Rightarrow u_2 = L_{21}\frac{di_1}{dt}$$

$$-u_3 = r_3 i_3 + \frac{d\psi_3}{dt} \qquad -u_3 = \underbrace{L_{31}}_{\downarrow 0}\frac{di_1}{dt} \Rightarrow u_3 = 0$$



6) $i_2 = 1\,A$ $\quad i_1 = i_3 = 0$ $\quad \phi_3 = 10^{-3}\,Wb$

$\Big($ $B_3 = -0.625\,T$

$$\oint \underbrace{\vec{E}_0 + \vec{E}_v}_{\vec{E}} d\vec{l} = -\frac{d}{dt}\psi_0 \Rightarrow u_0 = B\,\frac{d}{dt}S(t) = B\,\frac{d}{dt}(ax) = BVa$$

$$u_0 = 1V$$

em 4) $B_3 = 0 \rightarrow u_0 = 0$