4.1)

$$\begin{cases} u = Ri + L\frac{di}{dt} + \frac{1}{C}\int i\,dt \\ u_R = Ri \\ u_L = L\frac{di}{dt} \\ u_C = \frac{1}{C}\int i\,dt \\ u = u_R + u_L + u_C \end{cases}$$

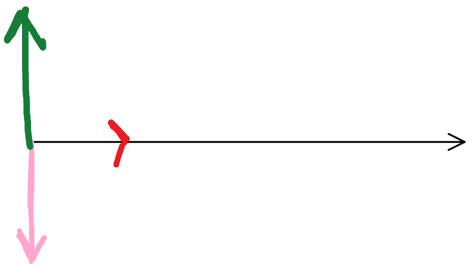
$$\begin{cases} \bar{U} = R\bar{I} + j\omega L\bar{I} + \frac{1}{j\omega C}\bar{I} \\ \bar{U}_R = R\bar{I} \\ \bar{U}_L = j\omega L\bar{I} \\ \bar{U}_C = -\frac{j}{\omega C}\bar{I} \\ \bar{U} = \bar{U}_R + \bar{U}_L + \bar{U}_C \end{cases}$$

2) $f = 10\,kHz \rightarrow \omega = 20\pi \cdot 10^3$ rad/s

$$\omega L = \frac{1}{\omega C} \Leftrightarrow C = \frac{1}{\omega^2 L} = 0,25833 \times 10^{-6}\ F$$

3) $$Q_0 = \frac{\omega L}{R} \Leftrightarrow R = \frac{\omega L}{Q_0} = 20,9\ \Omega$$

4)



$\bar{U} = \sqrt{2}\,V$; $\bar{I} = \frac{\sqrt{2}}{R} = 47,8\,e^{0j}$ mA

$\bar{U}_g = \sqrt{2}\,V$

$\bar{U}_L = \sqrt{2}\,\frac{\omega L}{R}\,e^{\frac{\pi}{2}j} = 3\sqrt{2}\,e^{\frac{\pi}{2}j}\ V$

$\bar{U}_C = \sqrt{2}\,\frac{1}{RC\omega}\,e^{-\frac{\pi}{2}j} = 3\sqrt{2}\,e^{-\frac{\pi}{2}j}\ V$

5) $$\bar{P} = \frac{1}{2}\bar{U}\bar{I}^* = \frac{1}{2}\cdot\sqrt{2}\cdot\frac{\sqrt{2}}{R} = \frac{1}{R}$$

$$P = Re[\bar{P}] = P_g = R\cdot I_{ef}^2 = R\cdot\left(\frac{\sqrt{2}}{\sqrt{2}R}\right)^2 = R\frac{1}{R^2} = \frac{1}{R}$$

$$P = Re[\bar{P}] = P_g \checkmark$$

$$P_q = Im[\bar{P}] = 2\omega\left(W_{m_{av}} - W_{e_{av}}\right) = 0 \checkmark$$

$$W_{m_{av}} = \frac{1}{2}L\left(\frac{\sqrt{2}}{\sqrt{2}R}\right)^2 = \frac{L}{2R^2}$$

$$W_{e_{av}} = \frac{1}{2}C\left(U_{C_{ef}}\right)^2 = \frac{1}{2R^2C\omega^2}$$

Como $C = \frac{1}{\omega^2 L}$

$$\frac{L}{2R^2} = \frac{1}{2R^2C\omega^2} \Rightarrow W_{m_{av}} = W_{e_{av}}$$

7)

$$\begin{cases} u_g = R i_L + L \dfrac{di_L}{dt} \\ u_R = R i_L \\ u_L = L \dfrac{di_L}{dt} \\ u_g = u_R + u_L \end{cases} \qquad \begin{cases} \bar{U}_g = R\bar{I}_L + j\omega L \bar{I}_L = (R + j\omega L) I_L \\ \bar{U}_R = R\bar{I}_L \\ \bar{U}_L = j\omega L \bar{I}_L \\ \bar{U}_g = \bar{U}_R + \bar{U}_L \end{cases}$$

8)

$$\bar{Z}_L = \frac{\bar{U}_g}{\bar{I}_L} = \frac{U_{g\,ef}}{I_{L\,ef}} e^{j\varphi} = 100\, e^{j\frac{\pi}{3}} = 50 + 86.6\, j$$

$\to R = 50\,\Omega$

$\omega L = 86.6 \to L = \frac{86.6}{\omega}\ H = 0{,}276\ H$

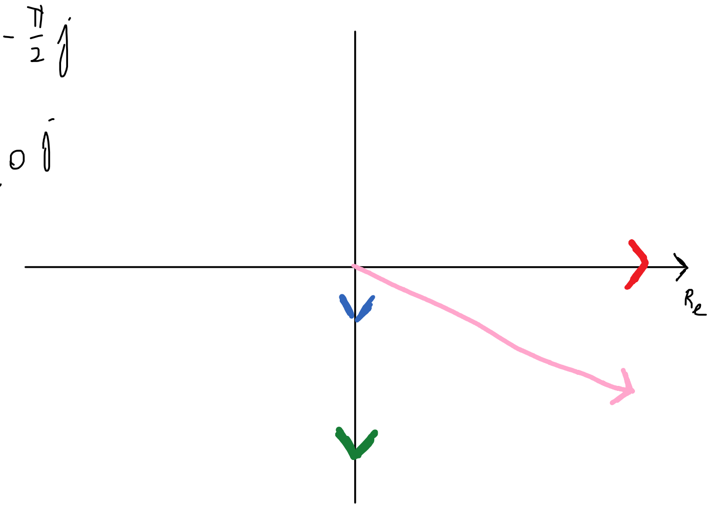
9)

$$\bar{U}_R = R\bar{I}_L = \sqrt{2} I_{ef} R\, e^{-\frac{\pi}{2}} = 115\sqrt{2}\, e^{-\frac{\pi}{2} j}$$

$$\bar{U}_L = j\omega L \bar{I}_L = \sqrt{2} I_{ef}\, \omega L\, e^{0} = \sqrt{2} \cdot 199\, e^{0 j}$$

$$\bar{U}_g = 230\sqrt{2}\, e^{-\frac{\pi}{6} j}$$

$$\bar{I}_L = 2.3\sqrt{2}\, e^{-\frac{\pi}{2} j}$$



10)

$$i_C = C \frac{du_g}{dt} \rightarrow \bar{I}_C = j\omega C \bar{U}_g$$

$$i_g = i_L + i_C \qquad \bar{I}_g = \bar{I}_L + \bar{I}_C$$

$$\underbrace{\arg(\bar{U}_g) - \arg(\bar{I}_L)}_{\varphi}$$

$$I_{Lef} \sin(\varphi) = I_{efC} = \omega C\, U_{gef} \Leftrightarrow C = \frac{I_{Lef}\sin(\varphi)}{\omega\, U_{gef}} = 27{,}6\ mF \qquad \bar{I}_C = 1.99\sqrt{2}\, e^{j\frac{\pi}{3}}$$

Logo $\bar{I}_g = 1{,}15\sqrt{2}\, e^{-j\frac{\pi}{6}}$

ou

$$\cancel{\tfrac{1}{2}} L I_{Lef}^2 = \cancel{\tfrac{1}{2}} C U_{Cef}^2 \Leftrightarrow C = \frac{L I_{Lef}^2}{U_{gef}^2} = 27{,}6\ mF$$

$$\bar{P} = \frac{1}{2} \bar{U}_g \bar{I}_g^{*} = 264.5\ VA \longrightarrow \begin{cases} P = Re\{\bar{P}\} = 264.5\,W = R I_{Lef}^2 \\ P_q = Im\{\bar{P}\} = 0 = 2\omega[W_m - W_e] \end{cases}$$

✓

II) fecha $t=-T=-\frac{1}{f}=-0{,}02$

abre $t=0$

1) A corrente em RL é dada por i_L; As condições iniciais são $i_L(-T^-)=i_L(+T^-)=0$ dado que antes o circuito estava em aberto. $i_L = i_L$ forçada $+ i_L$ livre

forçada:

$i_L(t)=\sqrt{2}\,I_{ef_L}\cos(\omega_g t-\frac{\pi}{2})=0$ Logo $0=0+\boxed{i_L \text{ livre}} \Rightarrow 0$

2)

$$i_L R + L\frac{di_L}{dt}+\frac{1}{C}\int i_L\,dt=0 \qquad s^2+2\beta s+\omega_0^2=0$$

$$\frac{d^2 i_L}{dt^2}\cdot L+\frac{di_L}{dt}R+C i_L=0 \Longrightarrow \frac{d^2 i_L}{dt^2}+\frac{di_L}{dt}\frac{R}{L}+\frac{1}{CL}i_L=0$$

$$2\beta=\frac{R}{L} \qquad \omega_0^2=\frac{1}{CL} \qquad \beta=\frac{R}{2L}=90{,}6\,s^{-1}$$

$$\omega_0=\frac{1}{\sqrt{CL}}=362{,}3\ \text{rad/s}$$

$\beta<\omega_0$ Oscilatório amortecido com $\omega=\sqrt{\omega_0^2-\beta^2}=350{,}8$ rad/s

$\omega_0=362{,}3$ rad/s

$\beta=90{,}6\ s^{-1}$

3) solução do tipo $i_L(t)=I e^{-\beta t}\cos(\omega t+\theta)$ $\quad \delta=\operatorname{arctan}\left(\frac{\omega}{\beta}\right)$

Condições iniciais: $i_L(0^+)=i_L(0^-)=0$ $\Rightarrow \cos(\theta)=0 \Rightarrow \theta=-\frac{\pi}{2}$

$$u_C(0^+)=u_C(0^-)=u_g(0)=U_0=281{,}7\,V$$

$$u_C(t)=-\frac{1}{C}\int \cdot\,dt=\sqrt{\frac{L}{C}}\,I e^{-\beta t}\cos(\omega t+\theta-\delta+\pi)=\sqrt{\frac{L}{C}}\,I\sin(\pi-\delta) \Rightarrow I=2{,}91A$$

4.3) 1)

$$\begin{cases} i_g = i_C + i_L \\ u_R = R i_L \\ u_L = L \dfrac{di}{dt} \\ u_C = \dfrac{1}{C}\int i\,dt \\ u_g = u_C = u_R + u_L \end{cases} \Rightarrow \begin{cases} \bar{I}_g = \bar{I}_C + \bar{I}_L \\ \bar{U}_R = R\bar{I}_L \\ \bar{U}_L = j\omega L \bar{I}_L \\ \bar{U}_C = \dfrac{1}{j\omega C}\bar{I}_C \\ \bar{U}_g = \bar{U}_C = \bar{U}_R + \bar{U}_L \end{cases}$$

2)

$$\bar{U}_g = \bar{U}_R + \bar{U}_L = \bar{I}_L (R + j\omega L) \Leftrightarrow I_{ef} = \frac{U_{ef}}{R + j\omega L}$$

$$P_g = R I_{ef}^2 = R\frac{U_{ef}^2}{R^2 + (\omega L)^2} \qquad \frac{\partial P}{\partial R} = -\frac{U^2(R^2 - (L\omega)^2)}{R^2 + (L\omega)^2} \qquad \frac{U^2(R^2-(L\omega)^2)}{R^2+(L\omega)^2} = 0 \Leftrightarrow R^2 = L\omega^2 \Leftrightarrow$$

$$\Leftrightarrow R = L\omega = 100\,\Omega$$

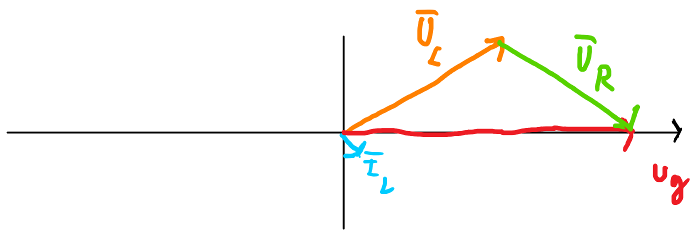
$$P_g\Big|_{R=100\Omega} = 100\frac{(200)^2}{100^2 + 100^2} = 200\ W$$

3)

$$\bar{I}_L = \frac{\bar{U}_g}{R + j\omega L} = 1.41\sqrt{2}\, e^{(\alpha - \frac{\pi}{4})j}$$

$$\bar{U}_R = R\bar{I}_L = 141\sqrt{2}\, e^{(\alpha - \frac{\pi}{4})j}$$

$$\bar{U}_L = j\omega L \bar{I}_L = 141\sqrt{2}\, e^{(\alpha + \frac{\pi}{4})j}$$



4) $W_m = W_e \Leftrightarrow \cancel{\frac{1}{2}} L i_L^2 = \cancel{\frac{1}{2}} C u_c^2 \Leftrightarrow C = \dfrac{L i_L^2}{u_c^2} = 15,9\ \mu F$

5)

$$\bar{I}_C = j\omega C \bar{U}_C \Leftrightarrow \bar{I}_C = \sqrt{2}\, e^{(\alpha + \frac{\pi}{2})j}$$

$$\bar{I} = \bar{I}_C + \bar{I}_L = \sqrt{2}\, e^{\alpha j}$$

$$\bar{P} = \frac{1}{2}\bar{U}\cdot\bar{I}^* = \frac{1}{2}\cdot\sqrt{2}\cdot 200 e^{\alpha j}\cdot\sqrt{2}\, e^{-\alpha j} = 200 e^{0j} = 200\ VA$$

$$P = Re\{\bar{P}\} = 200\ W = R I_{Lef}^2$$

$$P_q = Im\{\bar{P}\} = 0 = W_m - W_e$$

6)

$i_L(0) = 2A$

$\frac{d_{iL}}{dt} = 0$

$$\frac{d^2 i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{CL} i = 0$$

$\beta = \frac{R}{2L} = 157.085\, s^{-1}$

$\omega_0 = \frac{1}{\sqrt{CL}} = 444.5\, rad/s$

$\omega_0 > \beta$ oscilatório

$\omega = \sqrt{\omega_0^2 - \beta^2} = 415.8\, rad/s$

$\delta = \arctan\left(\frac{\omega}{\beta}\right) = 1.21\, rad$

$i_L(t) = I e^{-\beta t} \cos(\omega t + \theta)$

$\frac{di_L}{dt}(t) = \omega_0 I e^{-\beta t} \cos(\omega t + \theta + \pi - \delta)$

$i_L(0) = I \cos(\theta) = 2A$

$\frac{di_L}{dt}(0) = \cos(\theta + \pi - \delta) = 0$

$$\begin{cases} I\cos(\theta) = 2A \\ \cos(\theta + \pi - \delta) = 0 \end{cases}$$

$i_L(t) = 2{,}14\, e^{-\beta t} \cos(\omega t - 0{,}361)$

4) 1)

$$\begin{cases} u_g = u_{C1} + u_R \\ u_{C1} = \frac{1}{C_1}\int i_1\,dt \\ u_R = R\,i_1 \end{cases} \Leftrightarrow \begin{cases} \bar{U}_g = \bar{U}_{C1} + \bar{U}_R \\ \bar{U}_{C1} = \frac{1}{j\omega C_1}\bar{I}_1 \\ \bar{U}_R = R\bar{I}_1 \end{cases}$$

$$\bar{U}_g = \frac{1}{j\omega C_1}\bar{I}_1 + R\bar{I}_1 \Leftrightarrow$$

$$\Leftrightarrow \bar{I}_1 = \frac{\bar{U}_g}{\frac{1}{j\omega C_1} + R}$$

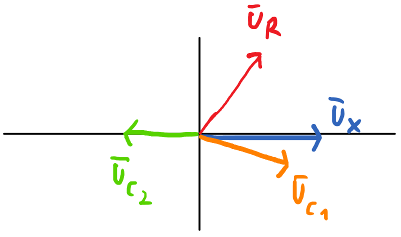
2) $$P = R\,I_{1ef}^2 = R \cdot \frac{U_{ef}^2}{\left(\frac{1}{\omega C_1}\right)^2 + R^2} = R\,\frac{U_{ef}^2(\omega C_1)^2}{1+(\omega C_1 R)^2} \Leftrightarrow P - R\,\frac{(U_{ef}\,\omega C_1)^2}{1+(\omega C_1 R)^2} = 0 \Leftrightarrow$$

$$\Leftrightarrow P(1+(\omega C_1 R)^2) - R(U_{ef}\,\omega C_1)^2 = 0 \Leftrightarrow P + P\omega^2 C_1^2 R^2 - R(U_{ef}\,\omega C_1)^2 = 0$$

$$\Leftrightarrow \begin{array}{l} a = \omega^2 C_1^2 P \\ b = -U_{ef}^2\,\omega^2 C_1^2 \\ c = P \end{array}$$

$$R = \frac{U_{ef}^2\omega^2 C_1^2 \pm \sqrt{(U_{ef}^2\,\omega^2 C_1^2)^2 - 4\,\omega^2 C_1^2 P^2}}{2\,\omega^2 C_1^2 P} = \begin{cases} R = 300\,\Omega \\ \boxed{R = 100\,\Omega} \end{cases}$$

3) $$\bar{U}_g = \left(\frac{1}{j\omega C_1} + R\right)\bar{I}_1 \Leftrightarrow \frac{230\,e^{j0}}{\underbrace{\frac{1}{j\omega C_1} + R}} = \bar{I}_1 \Leftrightarrow \bar{I}_1 = 1{,}15\sqrt{2}\,e^{\frac{\pi}{3}j}$$



$$\bar{U}_R = R\bar{I}_1 = 115\sqrt{2}\,e^{\frac{\pi}{3}j} \qquad \bar{U}_{C_1} = \frac{1}{j\omega C_1}\bar{I}_1 = 200\sqrt{2}\,e^{-\frac{\pi}{6}j}$$

4) $\varphi = -\frac{\pi}{3}$

Em Ressonancia

$$\bar{I}_2 = \sqrt{2}\,I_{1ef}\sin(\varphi)\,e^{j\frac{\pi}{2}} = \sqrt{2}\,e^{-\frac{\pi}{2}j} \qquad \bar{I} = \bar{I}_1 + \bar{I}_2 = 0{,}575\sqrt{2}\,e^{j0}$$

$$\bar{U}_{C_2} = \frac{1}{j\omega C_2}\bar{I}_2 = 84{,}2\sqrt{2}\,e^{j\pi} \qquad \bar{U}_X = \bar{U}_g - \bar{U}_{C_2} = 374{,}16\sqrt{2}\,e^{j0}$$

5) $$W_e = W_{eC_1} + W_{eC_2} = \frac{1}{2}C_1 U_{C_1 ef}^2 + \frac{1}{2}C_2 U_{C_2 ef}^2 = 0{,}5\,J$$

$$W_m = W_e \Leftrightarrow \frac{1}{2}L I_{2ef}^2 = 0{,}5 \Leftrightarrow L = 1\,H$$

elemento X = Bobine (indutivo)

II)

$$\begin{cases} u_{C_2} + L\dfrac{di_2}{dt} + Ri_2 - u_{C_1} = 0 \\ i_2 = C_2 \dfrac{du_{C_2}}{dt} \\ q_0 = q_1 + q_2 = C_1 u_{C_1} + C_2 u_{C_2} \end{cases} \Rightarrow \frac{C_1+C_2}{C_1} u_{C_2} + LC_2 \frac{d^2 u_{C_2}}{dt^2} + RC_2 \frac{du_{C_2}}{dt} = \frac{q_0}{C_1}$$

$$\frac{d^2 u_{C_2}}{dt^2} + 2\beta \frac{du_{C_2}}{dt} + \omega_0^2 u_{C_2} = \frac{\omega_0^2}{C_1+C_2} q_0$$

$\beta = \dfrac{R}{2L}$ $\qquad C = \dfrac{C_1 C_2}{C_1+C_2} = 12{,}37\ \mu F$

2) $i = \dfrac{dq_0}{dt}$ $\qquad \omega_0 = \dfrac{1}{\sqrt{LC}}$

$t>0$; $i=0 \Rightarrow q_0$ é constante!

Para $t<0$: $\bar{I} = 0{,}575\sqrt{2}e^{j0}$ $\qquad i(t) = 0{,}575\sqrt{2}\cos(\omega t)$

$$\int i(t)dt = q_0(t) = 1.83\sqrt{2}\sin(\omega t)\ mC = 1.83\sqrt{2}\cos(\omega t - \tfrac{\pi}{2})\ mC$$

$q_0(0) = 0 \rightarrow$ regime forçado de u_{C2} é nulo

3) $\beta = 50\,s^{-1}$ $\qquad \beta < \omega_0$ regime oscilatório amortecido

$\omega_0 = \dfrac{1}{\sqrt{LC}} = 284.3$ rad/s

$\omega = \sqrt{\omega_0^2 - \beta^2} = 279{,}9$ rad/s

$u_{C_2}(t) = Ue^{-\beta t}\cos(\omega t + \theta)$ $\qquad \delta = \arctan\left(\dfrac{\omega}{\beta}\right)$

4) $i_2 = C_2 \dfrac{du_{C_2}}{dt}$ $\qquad i_2(0) = 0 \Rightarrow \left.\dfrac{du_{C_2}}{dt}\right|_{t=0} = 0$; $\begin{cases} -119.02 = U\cos(\theta) \\ 0 = \cos(\theta+\delta) \end{cases} \Leftrightarrow \begin{cases} U = -120{,}9\,V \\ \theta = -10.18^\circ \end{cases}$

$u_{C2}(0) = -119.02\ V$

$$i_2(t) = U \cdot \frac{1}{\sqrt{\frac{L}{C}}} \cdot e^{-\beta}\cos(\omega t + \theta + \delta) = -0.425\, e^{-\beta}\sin(\omega t)$$