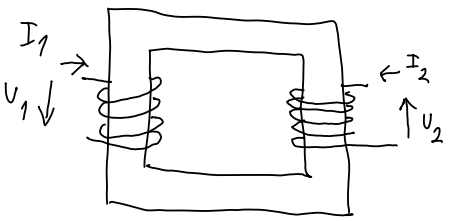
5.1) $f = 50\ Hz$



$U_{1ef} = 230V \quad I_{1ef} = 0,23A \quad P_1 = 16mW$

$U_{2ef} = 115V$

$R_2 = \frac{R_1}{v^2}$

$\bar{U}_1 = R_1 \bar{I}_1 + j\omega L_{11} \bar{I}_1 + j\omega L_M \bar{I}_2$

$-\bar{U}_2 = R_2 \bar{I}_2 + j\omega L_{22} \bar{I}_2 + j\omega L_M \bar{I}_1$

Em vazio $\bar{I}_2 = 0$

$\bar{U}_1 = R_1 \bar{I}_1 + j\omega L_{11} \bar{I}_1$

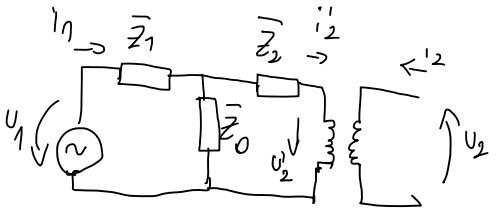
$\bar{U}_2 = j\omega L_M \bar{I}_1$

2) $v = \frac{U_{1f}}{U_{2f}} = 2$

$R_1 = \frac{P_1}{I_{1f}^2} = 0.3\Omega \qquad R_2 = \frac{R_1}{v^2} = 0.075\Omega$

$L_M = \frac{U_1}{\omega I_1} = 1,59\ H \qquad L_{11} = vL_M = 3,18H \qquad L_{22} = \frac{L_M}{v} = 0,796H$

3)



$\bar{U}'_2 = -v\bar{U}_2$

$\bar{I}'_2 = -v\bar{I}_2$

$$\begin{cases} \bar{U}_1 = \bar{Z}_1 \bar{i}_1 + \bar{Z}_0 (\bar{i}_1 - \bar{i}'_2) \\ \bar{U}'_2 = \bar{Z}_0 (\bar{i}_1 - \bar{i}'_2) - \bar{Z}_2 \bar{i}'_2 \end{cases}$$

$$\begin{cases} \bar{U}_1 = (\bar{Z}_1 + \bar{Z}_0) i_1 + \frac{\bar{Z}_0}{v} i_2 \\ \bar{U}_2 = -\frac{\bar{Z}_0}{v} i_1 - \frac{\bar{Z}_1 + \bar{Z}_0}{v^2} i_2 \end{cases}$$

$\Downarrow$

$\frac{\bar{Z}_0}{v} = j\omega L_M \Leftrightarrow \bar{Z}_0 = j\omega L_M v = j\omega L_{11} \Rightarrow vL_M = L_{11} = 3,18H$

$\bar{Z}_1 + \bar{Z}_0 = R_1 + j\omega L_{11} \Leftrightarrow \bar{Z}_1 = R_1 + j\omega L_{11} - j\omega L_{11} = R_1$ $\begin{cases} r_1 = R_1 = 0,3\Omega \\ X_{11} = 0 \end{cases}$ $\qquad L_{22} - \frac{L_{11}}{v} = 0$

$\frac{\bar{Z}_2 + \bar{Z}_0}{v^2} = R_2 + j\omega L_{22} \Leftrightarrow \bar{Z}_2 + \bar{Z}_0 = R_2 v^2 + j\omega L_{22} v^2 \Leftrightarrow \bar{Z}_2 = R_2 v^2 + j\omega L_{22} v^2 - j\omega L_{11} = R_1$ $\begin{cases} r'_2 = R_1 = 0,3\Omega \\ X'_{22} = 0 \end{cases}$

5.2) 1)

$$R_m = \frac{\ell}{\mu S} = \frac{\ell}{\mu_0 \mu_r S} = 240162\ H^{-1}$$

$$R_{m ar} = \frac{\delta}{\mu_0 S} \Leftrightarrow R_m \mu_0 S = \delta \Leftrightarrow \delta \simeq 0.504\ mm$$

2)

$$\begin{cases} \phi_0 + \phi_1 + \phi_2 = 0 \\ R_m(\phi_1 - \phi_0) = N_1 i_1 \\ R_m(\phi_0 - \phi_2) = N_2 i_2 \end{cases} \Leftrightarrow \begin{cases} -\phi_0 = \phi_1 + \phi_2 \\ \phi_1 - \phi_0 = \frac{N_1 i_1}{R_m} \\ \phi_0 - \phi_2 = \frac{N_2 i_2}{R_m} \end{cases} \Leftrightarrow \begin{cases} 2\phi_1 + \phi_2 = \frac{N_1 i_1}{R_m} \\ -2\phi_2 - \phi_1 = \frac{N_2 i_2}{R_m} \end{cases}$$

$$\begin{cases} \phi_2 = \frac{N_1 i_1}{R_m} - 2\phi_1 \\ -\frac{2N_1 i_1}{R_m} + 4\phi_1 - \phi_1 = \frac{N_2 i_2}{R_m} \end{cases} \Leftrightarrow \begin{cases} \phi_0 = \frac{N_1}{3R_m}\left(\frac{i_2}{2} - i_1\right) \\ \phi_1 = \frac{1}{3}\left(\frac{N_2 i_2}{R_m} + \frac{2N_1 i_1}{R_m}\right) = \frac{N_1}{3R_m}\left(\frac{i_2}{2} + 2i_1\right) \\ \phi_2 = -\frac{N_1}{3R_m}\left(i_2 + i_1\right) \end{cases}$$

Como $i_2 = 0$ $\quad \phi = BS$ $\quad \frac{B}{\mu} = H$

$$\phi_0 = \frac{-N_1 i_1}{3R_m} = -0.83\ mWb \qquad B_0 = \frac{\phi_0}{S} = -0.499\ T \qquad H_0 = \frac{B_0}{\mu_0} = -398\ kA/m$$

$$\phi_1 = \frac{2N_1 i_1}{3R_m} = 1.67\ mWb \Rightarrow B_1 = \frac{\phi_1}{S} = 0.997\ T \Rightarrow H_1 = \frac{B_1}{\mu_r \mu_0} = 1.59\ kA/m$$

$$\phi_2 = -\frac{N_1 i_1}{3R_m} = -0.83\ mWb \qquad B_2 = \frac{\phi_2}{S} = -0.499\ T \qquad H_2 = \frac{B_2}{\mu_r \mu_0} = -0.796\ kA/m$$

3) $\psi_1 = N_1 \phi_1 = 1002\ Wb$ $\quad \psi_2 = -N_2 \phi_2 = 249\ Wb$

$$\begin{cases} \psi_1 = L_{11} i_1 + L_{12} i_2 \\ \psi_2 = L_{21} i_1 + L_{22} i_2 \end{cases} \rightarrow \begin{cases} \psi_1 = L_{11} i_1 \\ \psi_2 = L_M i_1 \end{cases} \quad \begin{cases} L_{11} = 1\ H \\ L_M = 0.25\ H \end{cases}$$

$i_2 = 0$

4) $W_{m1} = \frac{1}{2} B_1 H_1 \ell S = 0.335\ J$ $\quad W_{mT} = W_{m1} + W_{m2} + W_{m0} = 0.5\ J$

$W_{m2} = \frac{1}{2} B_2 H_2 \ell S = 0.084\ J$ $\rightarrow L_{11} = \frac{2W_m}{i_1^2} = 1\ H$

$W_{m0} = \frac{1}{2} B_0 H_0 \delta S = 0.084\ J$

$$\frac{L_{11}}{L_{22}} = \left(\frac{N_1}{N_2}\right)^2 \rightarrow L_{22} = 0.25\ H$$

$$K = \frac{0.25}{\sqrt{1 \cdot 0.25}} = 0.5$$

1)

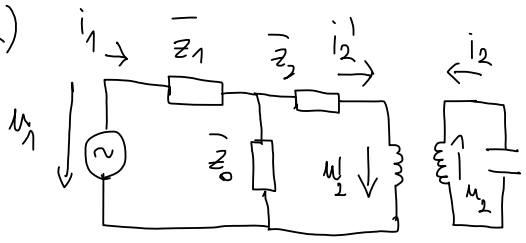
$$u_1 = R_1 i_1 + L_{11}\frac{di_1}{dt} + L_M\frac{di_2}{dt}$$

$$-u_2 = R_2 i_2 + L_M\frac{di_1}{dt} + L_{22}\frac{di_2}{dt}$$

$$\bar{U}_1 = R_1\bar{I}_1 + j\omega L_{11}\bar{I}_1 + j\omega L_M\bar{I}_2$$

$$-\bar{U}_2 = R_2\bar{I}_2 + j\omega L_{22}\bar{I}_2 + j\omega L_M\bar{I}_1$$

2)



$$u_2' = -\nu u_2 \qquad i_2' = -\frac{1}{\nu} i_2$$

$$u_1 = \bar{z}_1 i_1 + \bar{z}_0(i_1 - i_2')$$

$$-u_2' = \bar{z}_2 i_2' + \bar{z}_0(i_1 - i_2')$$

$$\bar{z}_0 = j\omega L_M \nu \rightarrow \lambda_0$$

$$\bar{z}_1 = R_1 + j\omega(L_{11} - \nu L_M) \rightarrow \lambda_1$$

$$\bar{z}_2 = R_2\nu^2 + j\omega\nu^2\left(L_{22} - \frac{L_M}{\nu}\right) \rightarrow \lambda_2$$

$$\frac{L_M}{L_{22}} \leq \nu \leq \frac{L_{11}}{L_M}$$

$$\Downarrow$$

$$1 \leq \nu \leq 4$$

$R_1 = R_2 = 0$

Com $\nu = 1$

$$\lambda_0 = L_M = 0.250\,H$$

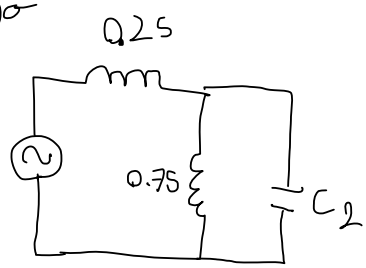
$$\lambda_1 = L_{11} - L_M = 0{,}750\,H$$

$$\lambda_2 = L_{22} - L_M = 0\,H$$

$$\bar{z}_2' = \nu^2\bar{z}_2$$

$$C_2' = \frac{1}{\nu^2}C_2$$

Logo



3)

$$u_2(t) = \sqrt{2}\,U_{2ef}\cos(\omega t)$$

$U_{2ef} = 230\,V$

$\omega = 2\pi \cdot 50 = 100\pi$

$$u_2 = 230\sqrt{2}\,e^{j0}$$

$$u_2' = -\nu u_2 = -u_2 = 230\sqrt{2}\,e^{j\pi}$$

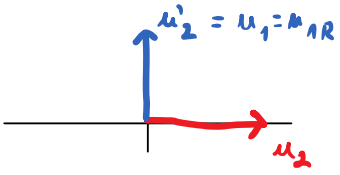
$$i_2' = j\omega C_2' u_2' = 2{,}9\sqrt{2}\,e^{j(-\frac{\pi}{2})}$$

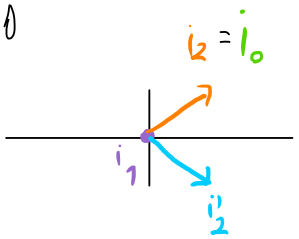
$$i_2 = -\frac{1}{\nu}i_2' = -2{,}9\sqrt{2}\,e^{-\frac{\pi}{2}j} = 2{,}9\sqrt{2}\,e^{\frac{\pi}{2}j}$$

$$i_0 = \frac{u_2'}{j\omega L} = 2{,}9\sqrt{2}\,e^{\frac{\pi}{2}j}$$

$$i_1 = i_2' + i_0 = 0$$

$$u_{1F} + u_{1R} = u_2' = 230\sqrt{2}\,e^{j\pi}$$





4)

$$\bar{P} = \frac{1}{2}\bar{U}_1\bar{I}_1^* = 0 + j0$$

$$P = P_y = 0$$

$$P_q = 2\omega[W_M - W_e] = 0$$

$$W_M = \frac{1}{2}L I_0^2 = 1.073\,J$$

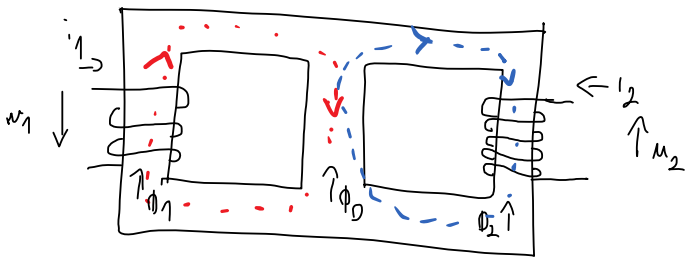
$$W_e = \frac{1}{2}C_2' U_2'^2 = 1.073\,J$$

$\rightarrow 0$

3)

1) $R_{m1} = R_{m2} = 3R_m$

$R_{m_0} = 6{,}67 \times 10^5\ H^{-1}$



$L_M = \frac{N_2}{N_1} L_{11}$

$L_{22} = \left(\frac{N_2}{N_1}\right)^2 L_{11}$

$$\begin{cases} R_{m_1}\phi_1 - R_m\phi_0 = n_1 i_1 \\ R_m\phi_0 - R_{m_2}\phi_2 = n_2 i_2 \\ \phi_1 + \phi_2 + \phi_0 = 0 \end{cases} \Leftrightarrow \begin{cases} 3R_m\phi_1 - R_m\phi_0 = N i_1 \\ R_m\phi_0 - 3R_m\phi_2 = N i_2 \\ \phi_1 + \phi_2 + \phi_0 = 0 \end{cases}$$

2) $$\begin{cases} 3R_m\phi_1 - R_m\phi_0 = N \\ R_m\phi_0 - 3R_m\phi_2 = 0 \\ \phi_1 + \phi_2 + \phi_0 = 0 \end{cases} \Leftrightarrow \begin{cases} -12R_m\phi_2 - 3R_m\phi_2 = N \\ \phi_0 = 3\phi_2 \\ \phi_1 = -4\phi_2 \end{cases} \Leftrightarrow \begin{cases} \phi_2 = -\frac{N}{15R_m} = -0{,}05\ mWb \\ \phi_0 = 3\phi_2 = -0{,}15\ mWb \\ \phi_1 = -4\phi_2 = 0{,}2\ mWb \end{cases}$$

3) $\Psi_1 = N\phi_1 = 100\ mWb$

$\Psi_2 = N\phi_2 = 25\ mWb$

$$\begin{cases} \Psi_1 = L_{11} i_1 + L_M i_2 \\ \Psi_2 = L_M i_1 + L_{22} i_2 \end{cases} \Leftrightarrow \begin{cases} L_{11} = 100\ mH \\ L_M = 25\ mH \end{cases}$$

$L_{22} = \left(\frac{N_2}{N_1}\right)^2 L_{11} \Rightarrow L_{22} = L_{11}$ //

$k = \frac{|L_M|}{\sqrt{L_{22} L_{11}}} = \frac{1}{4}$

4)

$W_m = \frac{1}{3} 3R_m\phi_1^2 + \frac{1}{2} R_m\phi_0^2 + \frac{1}{2} \cdot 3R_m\phi_2^2 = 40 + 7{.}5 + 2{.}5 = 50\ mJ$

$L_{11} = \frac{2W_m}{i_1^2} = 100\ mH$

II 1)

$$\frac{L_M}{L_{22}} \leq v \leq \frac{L_{11}}{L_M} \Leftrightarrow \frac{1}{4} \leq v \leq 4$$

Dado que $u'_2 = -v u_2$

$i'_2 = -\frac{1}{v} i_2$

$$\begin{cases} u'_2 = Z_a \cdot i'_2 \\ u_2 = Z i_2 \end{cases} \Leftrightarrow \begin{cases} -v u_2 = Z_a \cdot \left(+\frac{1}{v} i_2\right) \\ - \end{cases} \Leftrightarrow \begin{cases} v^2 u_2 = Z_a i_2 \\ u_2 = Z i_2 \end{cases} \Leftrightarrow \begin{cases} v^2 Z = Z_a \\ - \end{cases}$$

Logo $Z = \frac{1}{v^2} Z_a \Rightarrow C = \frac{1}{v^2} C_a$ Como $C = C_a$ temos que $v = 1$ (é valido já que está entre $[\frac{1}{4}; 4]$)

$R = r = 10\,\Omega$

$l = L_{11} - v L_M = 75\,mH$

$L = v L_M = 25\,mH$

$\lambda = v^2 L_{22} - v L_M = 75\,mH$

2)

$$\bar{U}_0 = \left(j\omega\lambda + \frac{1}{j\omega C}\right) \bar{I}'_2 \qquad \bar{I}_2 = -\bar{I}'_2 \ (v=1) \qquad \bar{U}'_2 = \frac{1}{j\omega C} \bar{I}'_2 \qquad \bar{U}_2 = -\frac{1}{v} \bar{U}'_2$$

$$\bar{U}_\lambda = j\omega\lambda \bar{I}'_2 \qquad \bar{U}_0 = \bar{U}_\lambda + \bar{U}'_2 \qquad \bar{U}_0 = j\omega L \bar{I}_0 \qquad \bar{I}_1 = \bar{I}_0 + \bar{I}_2 \qquad \bar{U}_1 = (R + j\omega R)\bar{I}_1 \qquad \bar{U}_g = \bar{U}_1 + \bar{U}_0$$

3) $\bar{I}_1 = 0 \Rightarrow \bar{I}_0 + \bar{I}'_2 = 0$

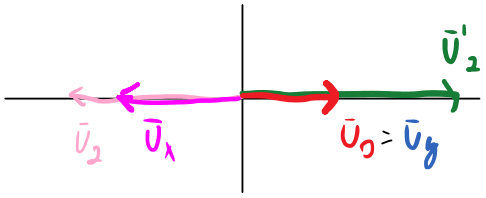
$$\begin{cases} \bar{I}_0 = \frac{\bar{U}_0}{Z_0} \\ \bar{I}'_2 = \frac{\bar{U}_\lambda + \bar{U}'_2}{Z_{\lambda 2}} = \frac{\bar{U}_0}{\bar{Z}_{\lambda 2}} \end{cases} \Rightarrow \frac{1}{j\omega L} + \frac{1}{j\omega\lambda + \frac{1}{j\omega C}} = 0 \Leftrightarrow C = \frac{1}{\omega^2(L+\lambda)} = 1{,}0135 \times 10^{-4} =$$

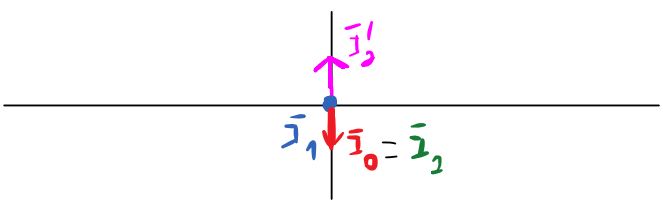
4) $\bar{I}_1 = 0 \Rightarrow \bar{U}_0 = \bar{U}_g = 40\sqrt{2}\, e^{j0}\ V$

$\bar{I}_0 = -\bar{I}'_2 = \bar{I}_2 = 5\sqrt{2}\, e^{-j\frac{\pi}{2}}\ A$

$\bar{U}'_2 = -\bar{U}_2 = 160\sqrt{2}\, e^{j0}\ V$

$\bar{U}_\lambda = 120\sqrt{2}\, e^{j\pi}\ V$





5)

$$\bar{P} = \frac{1}{2}\,\bar{U}_1 \cdot \bar{I}_1^{*} = \frac{1}{2} \cdot 40 \cdot \sqrt{2} \cdot e^{j0} \cdot 0 = 0 \qquad \begin{cases} Re\{\bar{P}\} = P_g = 0 \\ Im\{\bar{P}\} = P_q = 0 \end{cases}$$

$$P_g = I_1^2 R = 0$$

$$P_q = 2\omega \left[W_m - W_e \right]$$

$$W_m = \frac{1}{2}\left(L_1 I_{1ef}^2 + L I_{0ef}^2 + L_2 I_{2ef}^2 \right) \simeq 1{,}3\ J$$

$$W_e = \frac{1}{2} C \bar{U}_{2ef}^2 = 1{,}3\ J$$

Logo $P_q = 0$ Teorema Confirmado

6)

$$u'_2 = \frac{1}{C}\int i'_2\,dt \iff C\frac{du'_2}{dt} = i'_2$$

$$\lambda \frac{di'_2}{dt} + L\frac{di'_2}{dt} + \frac{1}{C}\int i'_2\,dt = 0 \iff (\lambda + L)\cdot C \frac{d^2 u'_2}{dt^2} + u'_2 = 0 \iff$$

$$\iff \frac{d^2 u'_2}{dt^2} + \frac{1}{(\lambda + L)C}\,u'_2 = 0$$

$$\downarrow$$

$$s^2 + 2\beta_0 + \omega_0^2 = 0 \qquad \begin{cases} \beta = 0 \\ \omega_0^2 = \dfrac{1}{(\lambda + L)C} \end{cases} \iff \begin{cases} \beta = 0 \\ \omega_0 = \omega = 314{,}19\ rad/s \end{cases}$$

$$\omega = \sqrt{\omega_0^2 + \beta^2}$$

$$u'_2(0^-) = u'_2(0^+) = 226{,}3V$$

$$i'_2(0^-) = i'_2(0^+) = 0$$

5.4 / 1) a) $R_m(\phi_1 - \phi_0) = N_1 i_1$

$R_m(\phi_0 - \phi_2) = N_2 i_2$

$\phi_0 + \phi_1 + \phi_2 = 0$

b) $\begin{cases} R_m(\phi_1 - \phi_0) = N_1 \\ \phi_0 = \phi_2 \\ 2\phi_0 = -\phi_1 \end{cases} \Leftrightarrow \begin{cases} \phi_0 = -\frac{N_1}{3R_m} = -1,423\ mWb \\ \phi_2 = -1,423\ mWb \\ \phi_1 = 2,846\ mWb \end{cases}$

c) $\Psi_1 = \phi_1 N_1 = 1\ Wb$

$\Psi_2 = -\phi_2 N_2 = 1\ Wb$

$L_{11} = \frac{\Psi_1}{i_1} = 1H$

$L_M = \frac{\Psi_2}{i_1} = 1H$

$L_{22} = \left(\frac{N_2}{N_1}\right)^2 L_{11} = 4L_{11} = 4H$

) $\Psi_1 = L_{11} i_1 + L_M i_2$

$\Psi_2 = L_M i_1 + L_{22} i_2$

$K = \frac{1}{\sqrt{4}} = \frac{1}{2} = 0.5$

d) $W_m = \frac{1}{2} \sum R_m \phi^2 =$

$= \frac{1}{2} R_m (\phi_1^2 + \phi_2^2 + \phi_0^2) = 0.5J$

$W_m = \frac{1}{2} L_{11} i_1^2 \Leftrightarrow L_{11} = \frac{W_{m2}}{i_1^2} = 1H$ //

2) a)

$u_1 = R_1 i_1 + L_{11}\frac{di_1}{dt} + L_M \frac{di_2}{dt}$

$-u_2 = R_2 i_2 + L_M \frac{di_1}{dt} + L_{22}\frac{di_2}{dt}$

b) $u_1 = L_{11}\frac{di_1}{dt} + L_M \frac{di_2}{dt}$

$-u_2 = L_M \frac{di_1}{dt} + L_{22}\frac{di_2}{dt}$

$L = \nu L_M \Leftrightarrow \frac{L}{L_M} = \nu \Leftrightarrow \nu = 1$

$\lambda_{11} = L_{11} - \nu L_M = 0$

$\lambda = \lambda'_2 = \nu^2 (L_{22} - \frac{L_M}{\nu}) = 3H$

$R = R'_a = \nu^2 R_a = R_a = 554,2\ \Omega$

3) a) $\bar{U}_1 = \bar{U}_\lambda + \bar{U}'_2$

$\bar{U}_\lambda = j\omega\lambda \bar{I}'_2$

$\bar{U}'_2 = R\bar{I}'_2$

$\bar{U}_1 = j\omega L \bar{I}_L$

$\bar{I} = \bar{I}_C + \bar{I}_1$

$\bar{I}_1 = \bar{I}_L + \bar{I}'_2$

$\bar{I}'_2 = -\bar{I}_2$

$\bar{U}'_2 = -\bar{U}_2$

$\bar{I}_C = j\omega C \bar{U}_1$

b) $W_e = W_m$

$\frac{1}{2} C u_1^2 = \frac{1}{2}(L i_L^2 + \lambda i_2'^2)$

$C = \frac{L i_L^2 + \lambda i_2'^2}{u_1^2} = \frac{1}{\omega^2 L} + \frac{\lambda}{R^2 + (\omega\lambda)^2} = 12,66mF$

c) $\bar{U}_1 = 544,2\sqrt{2}\, e^{j0}$

$\bar{I}_L = 1,73\sqrt{2}\, e^{-j\frac{\pi}{2}}$

$\bar{I}_C = 2,17\sqrt{2}\, e^{j\frac{\pi}{2}}$

$\bar{I}'_2 = \frac{\bar{U}_1}{j\omega\lambda + R} = 0,5\sqrt{2}\, e^{-\frac{\pi}{8}j}$

$\bar{I}_2 = 0,5\sqrt{2}\, e^{\frac{2\pi}{3}j}$

$\bar{U}'_2 = 272,1\sqrt{2}\, e^{-\frac{\pi}{8}j}$

$\bar{U}_2 = 272,1\sqrt{2}\, e^{\frac{2\pi}{3}j}$

$\bar{U}_1 = 471,3\sqrt{2}\, e^{\frac{\pi}{6}j}$

$\bar{I}_1 = 2,2\sqrt{2}\, e^{-j(1,4586)}$

$\bar{I} = 0,25\sqrt{2}\, e^{j0}$

3. d) $\bar{P} = \frac{1}{2} \cdot \bar{U} \cdot \bar{I}^* = \frac{1}{2} \cdot (5442\sqrt{2}) \cdot (0.25\sqrt{2}) = 136.05 \quad VA$

$P_y = R I_{2ef}^2 = 136.05\ W$

$P_y = Re\{\bar{P}\}$ //

5.5)

F_{mm}

$H \cdot 2\pi \cdot r = N_1 i_1 + \frac{N_2}{2} i_2$

$B(r) = \frac{\mu_r \mu_0}{2\pi r} \left(N_1 i_1 + \frac{N_2}{2} i_2\right)$

$R_m = \frac{F_{mm}}{\phi} = \frac{\int H\, ds}{\int_S \vec{B}_m \vec{ds}} = \frac{N_1 i_1 + \frac{N_2}{2} i_2}{\int_0^a \int_{r_1}^{r_2} \frac{\mu_r \mu_0}{2\pi r} \left(N_1 i_1 + \frac{N_2}{2} i_2\right) dr\, dl} =$

$= \frac{1}{a \left(\frac{\mu_r \mu_0}{2\pi} \ln(r_2/r_1)\right)} = \frac{2\pi}{\mu_r \mu_0\, a \ln(r_2/r_1)} = 5 \times 10^5\ H^{-1}$

2)

$R_m \phi_1 = N_1 i_1 + \frac{N_2}{2} i_2$

$R_m \phi_2 = N_1 i_1 + \frac{N_2}{2} i_2$

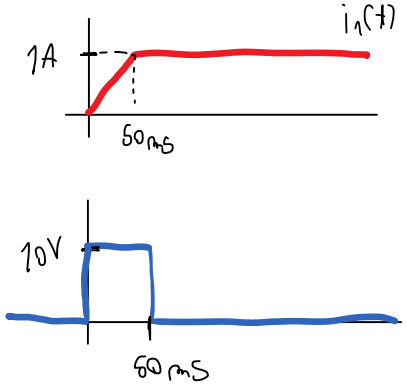
$\rightarrow \phi_1 = \phi_2$

3) $\psi_1 = 2\phi N_1 \quad L_{11} = \frac{\psi_{11}}{i_1} = \frac{2N_1^2}{R_m} = 1H$

$\psi_2 = \phi N_2$

$L_{22} = \frac{\psi_{22}}{i_2} = \frac{N_2^2}{2R_m} = 0{,}25H$

4)



$-u_2(t) = R i_1 + L_M \frac{di_1(t)}{dt}$ (R = 0)

$u_2(t) = -L_M \frac{di_1}{dt}$

$L_M = \frac{\psi_{22}}{i_1} = \frac{\psi_{11}}{i_2} = \frac{N_1 N_2}{R_m} = 0.5H$

$k = \frac{|L_M|}{\sqrt{L_{11} L_{22}}} = 1$

II

1)

$\frac{L_{11}}{L_M} < v < \frac{L_M}{L_{22}} \Leftrightarrow 2 < v < 2$

$v = 2$

$\bar{Z} = v^2 \bar{Z}_a$ Logo $R = v^2 R_a$

$X = -v^2 \frac{1}{C_a \omega}$

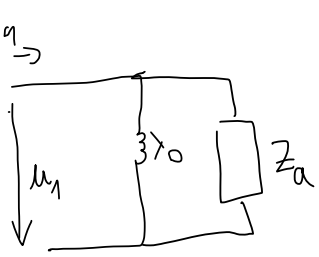
$\lambda_1 = L_{11} - v L_M = 0$

$\lambda_0 = v L_M = 1\,H$

$\lambda_2 = v^2 \left(L_{22} - \frac{L_M}{v}\right) = 0$

$n_2 = 0$

$n_1 = \infty$



2) $P_a = R I'^2_{2ef}$ $\quad I'_{2ef} = \frac{U_{1ef}}{\sqrt{R^2 + X^2}}$ $\quad P_a = \frac{R}{R^2+X^2} U^2_{1ef}$

$\frac{dP_a}{dR} = -\frac{R^2 - X^2}{(R^2+X^2)^2} U^2_{1ef} = 0 \Leftrightarrow R^2 - X^2 = 0 \Leftrightarrow R^2 = X^2 \Rightarrow R = |X|$

$Im\{\lambda_0 // \bar{Z}\} = 0$

$\lambda_0 // \bar{Z} = \left(\frac{1}{j\omega\lambda_0} + \frac{1}{R+Xj}\right)^{-1} = \frac{j\lambda_0\omega(R+jX)}{R + j(\lambda_0\omega + X)} = \frac{j\lambda_0\omega(R+jX)(R - j(\lambda_0\omega + X))}{R^2 + (\lambda_0\omega + X)^2} =$

$Im\{(j\lambda_0\omega R - \lambda_0\omega X)(R - j(\lambda_0\omega + X))\} = \lambda_0\omega R^2 + (\lambda_0\omega + X)\lambda_0\omega X$

$\lambda_0\omega R^2 + \lambda_0^2\omega^2 X + \lambda_0\omega X^2 = 0$

$R^2 + \lambda_0\omega X + X^2 = 0$

$2X^2 + \lambda_0\omega X = 0 \Leftrightarrow X(2X + \lambda_0\omega) = 0 \Leftrightarrow X = -\frac{\lambda_0\omega}{2}$

$R = -X = 3.14\,K\Omega$

$R_a = \frac{R}{v^2} = 785\,\Omega$

$C_a = -\frac{v^2}{\omega X} = 202{,}8\,mF$

II

3) $\bar{U}_2' = \bar{U}_1 = 6{,}28\sqrt{2}\, e^{j0}$

$\bar{I}_0 = \frac{\bar{U}_1}{j\omega L_0} = 0{,}001\sqrt{2}\, e^{-j\frac{\pi}{2}} = \sqrt{2}\, e^{-j\frac{\pi}{2}}$ mA

$\bar{I}_2' = \frac{\bar{U}_1}{Z} = 1{,}41\sqrt{2}\, e^{j\frac{\pi}{4}}$ mA

$\bar{I}_2 = -\nu \bar{I}_2' = 2{,}283\sqrt{2}\, e^{j\frac{3\pi}{4}}$ mA

$\bar{I}_1 = \bar{I}_0 + \bar{I}_2' = \sqrt{2}\, e^{j0}$ mA

$\bar{U}_2 = -\frac{1}{\nu}\bar{U}_2' = 3{,}14\sqrt{2}\, e^{j\pi}$ V

4)

$P = \frac{1}{2}\bar{U}_1 \bar{I}^* = 6{,}28$ mVA

$P_a = R I_{2ef}'^2 = R_a I_{2ef}'^2 = 6{,}28$ mW

$R_4 =$ | $W_m = \frac{1}{2} L_{11} I_{0ef}^2 = 0{,}5\,\mu J$

$W_e = \frac{1}{2} \times U_{Cef}^2 = 0{,}5\,\mu J$ → 50 ✓