$$P(A) = 1 - P(\bar{A})$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(B|A)\,P(A)}{P(B)}$$

$$P(A \cap B) = P(A) \times P(A|B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

2.1) $P(A) + P(B) = x$

$P(A \cap B) = y$

a) $1 - P(A \cup B) = 1 - (x - y) = 1 - x + y$

b) $P(A \cup B) - P(A \cap B) = P(A \cup B) + P(A \cup B) - P(A) - P(B) =$

$= 2(x - y) - x = x - 2y$

c) $P(A \cup B) = x - y$

d) $1 - P(A \cap B) = 1 - y$

2.2) a) $\Omega = \{\underbrace{B}_{A}, \underbrace{PB}_{B}, \underbrace{PPB}_{C}, \underbrace{PPPB}_{D}, \underbrace{PPPPB}_{E}, \underbrace{PPPPPB}_{F}\}$

$P(A) = \frac{1}{2}$

$P(B) = \frac{1}{2} \times \frac{5}{9}$

$P(C) = \frac{1}{2} \times \frac{4}{9} \times \frac{5}{8}$

$P(D) = \frac{1}{2} \times \frac{4}{9} \times \frac{3}{8} \times \frac{5}{7}$

$P(E) = \frac{1}{2} \times \frac{4}{9} \times \frac{3}{8} \times \frac{2}{7} \times \frac{5}{6}$

$P(F) = \frac{1}{2} \times \frac{4}{9} \times \frac{3}{8} \times \frac{2}{7} \times \frac{1}{6} \times \frac{5}{5}$

$P(\text{A Ganhar}) = P(A) + P(C) + P(E) = \frac{83}{126}$

$P(\text{B Ganhar}) = P(B) + P(D) + P(F) = \frac{43}{126}$

c) $\Omega = \{$ 1- B

2- P B

3- P P B

4- P P P B

⋮

n - $(P)^n$ B

$\boxed{P = \frac{1}{2}}$ A

$\boxed{P = \frac{1}{2}^2}$ B

$\boxed{P = \frac{1}{2}^3}$ A

$\boxed{P = \frac{1}{2}^4}$ B

$P(A) = \sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^{2n-1} = \frac{2}{3}$

$P(B) = \sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^{2n} = \frac{1}{3}$

2.3) A: Existir petróleo

B: Sair petróleo à primeira

$P(B|\bar{A}) \rightarrow$ Probabilidade de sair petróleo à primeira sendo que não existe petróleo $\Rightarrow 0$ //

a) $P(A) = 0,8$

$P(B|A) = 0,5$

TOTAL PROBABILITY LAW

$$P(B) = P(B|A) \times P(A) + \overbrace{P(B|\bar{A})}^{0} \times P(\bar{A}) = 0,8 \times 0,5 = 0,4$$

b) $$P(A|\bar{B}) = \frac{P(A \cap \bar{B})}{P(\bar{B})} = \frac{P(\bar{B}|A) \times P(A)}{P(\bar{B})} = \frac{(1-0,5) \times 0,8}{1-0,4} = \frac{2}{3} \approx 0,66\ldots$$

2.4) a) A: Ter Cancro $\qquad P(B|A) = 0.99$

B: Testar Positivo $\qquad P(\bar{B}|\bar{A}) = 0.95$

BAYE'S RULE

$$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B|A)P(A) + P(B|\bar{A})P(\bar{A})} = 0.09$$

b) $$P(A|B_1 \cap B_2) = \frac{P(B_1 \cap B_2|C)P(C)}{P(B_1 \cap B_2)} =$$

$$= \frac{P(B_1|C)P(B_2|C)P(C)}{P(B_1 \cap B_2|C) \cdot P(C) + P(B_1 \cap B_2|\bar{C}) \cdot P(\bar{C})} =$$

$$= \frac{P(B_1|C)P(B_2|C)P(C)}{P(B_1|C)P(B_2|C)P(C) + P(B_1|\bar{C})P(B_2|\bar{C}) \cdot P(\bar{C})} \approx 66.33\%$$