4.1) a)

$Y \backslash X$	1	2	3	$P(Y=y)$
1	1/9	0	1/18	3/18
2	0	1/3	1/9	8/18
3	1/9	1/6	1/9	7/18
$P(X=x)$	4/18	9/18	5/18	1

i) $f_X(x) = \begin{cases} \frac{4}{18} & ; x=1 \\ \frac{9}{18} & ; x=2 \\ \frac{5}{18} & ; x=3 \end{cases}$

ii) $F_Y(y) = \begin{cases} \frac{3}{18} & ; y=1 \\ \frac{11}{18} & ; y=2 \\ 1 & ; y=3 \end{cases}$

iii) $P(X+Y \leq 4) = \frac{1}{9} + 0 + \frac{1}{18} + 0 + \frac{1}{3} + \frac{1}{9} = \frac{11}{18}$

iv) $f_{X|Y=1}(x) = \begin{cases} 2/3 & x=1 \\ 0 & x=2 \\ 1/3 & x=3 \end{cases}$ $\qquad f_{X|Y=3}(x) = \begin{cases} 2/7 & x=1 \\ 3/7 & x=2 \\ 2/7 & x=3 \end{cases}$

v) $E(X|Y=1) = \sum_{x=1}^{3} x f_{X|Y=1}(x) = 1 \cdot \frac{2}{3} + 2 \cdot 0 + 3 \cdot \frac{1}{3} = \frac{2}{3} + 1 = \frac{5}{3}$

b) $E(X|Y) = \sum_{x=1}^{3} x \, f_{X|Y=y}(x)$

c) $P(X=1 \cap Y=1) = P(X=1) \cdot P(Y=1) \Rightarrow \frac{1}{9} \neq \frac{3}{18} \cdot \frac{4}{18} \Leftrightarrow \frac{1}{9} \neq \frac{1}{27}$ Logo X e Y não são independentes

d) $V(X+Y) = E((X+Y)^2) - E(X+Y)^2 = \frac{349}{18} - \left(\frac{77}{18}\right)^2 = \frac{353}{324} \approx 1.0895$

$E(X+Y) = \sum_{x=1}^{3} \sum_{y=1}^{3} (x+y) f_{X,Y}(x,y) = 2 \cdot \frac{1}{9} + 4 \cdot \frac{1}{18} + 4 \cdot \frac{1}{3} + 5 \cdot \frac{1}{9} + 4 \cdot \frac{1}{9} + 5 \cdot \frac{1}{6} + 6 \cdot \frac{1}{9}$
$= \frac{77}{18}$

$E((X+Y)^2) = \sum_{x=1}^{3} \sum_{y=1}^{3} (x+y)^2 f_{X,Y}(x,y) = \frac{349}{18}$

4.2)

$$f_{X,Y}(x,y) = \begin{cases} 2 & , 0 < x < y < 1 \\ 0 & c.c. \end{cases}$$

$$Cov(X,Y) = E\left[(X - E(X))(Y - E(Y))\right]$$

$$Cov(X,Y) = E(XY) - E(X)E(Y)$$

$$E(X) = \sum_x x f_X(x) \lor \int_{-\infty}^{+\infty} x f_X(x)$$

$$V(X) = \sum_x (x - E(X))^2 f_X(x) \lor \int_{-\infty}^{+\infty} (x - E(X))^2 f_X(x)$$

$$V(X) = E(X^2) - E(X)^2$$

$$\sigma_X = \sqrt{V(X)}$$

a)

$$\rho_{X,Y} = Corr(X,Y) = \frac{\sigma_{XY}}{\sigma_X \sigma_Y} = \frac{Cov(X,Y)}{\sqrt{V(X)V(Y)}}$$

$$E(XY) = \int_0^1 \int_0^y xy \cdot 2 \, dx \, dy = \int_0^1 y^3 \, dy = \frac{1}{4}$$

$$f_X = \int_{-\infty}^{+\infty} f_{X,Y}(x,y) dy = \int_x^1 2 \, dy = \begin{cases} 2(1-x) & , 0 < x < 1 \\ 0 & , c.c. \end{cases}$$

$$f_Y = \int_{-\infty}^{+\infty} f_{X,Y}(x,y) dx = \int_0^y 2 \, dx = \begin{cases} 2y & , 0 < y < 1 \\ 0 & , c.c. \end{cases}$$

$$E(X) = \int_{-\infty}^{+\infty} x f_X \, dx = \int_0^1 2x - 2x^2 \, dx = \left[x^2 - \frac{2x^3}{3}\right]_0^1 = 1 - \frac{2}{3} = \frac{1}{3}$$

$$E(X^2) = \int_{-\infty}^{+\infty} x^2 f_X \, dx = \int_0^1 2x^2 - 2x^3 \, dx = \frac{2}{3} - \frac{2}{4} = \frac{1}{6}$$

$$\rightarrow V(X) = \frac{1}{6} - \frac{1}{9} = \frac{1}{18}$$

$$E(Y) = \int_{-\infty}^{+\infty} y f_Y \, dy = \int_0^1 2y^2 \, dy = \left[\frac{2y^3}{3}\right]_0^1 = \frac{2}{3}$$

$$E(Y^2) = \int_{-\infty}^{+\infty} y^2 f_Y \, dy = \int_0^1 2y^3 \, dy = \left[\frac{2y^4}{4}\right]_0^1 = \frac{1}{2}$$

$$\rightarrow V(Y) = \frac{1}{2} - \frac{4}{9} = \frac{1}{18}$$

$$Cov(X,Y) = E(XY) - E(X)E(Y) = \frac{1}{4} - \frac{1}{3} \cdot \frac{2}{3} = \frac{1}{4} - \frac{2}{9} = \frac{1}{36}$$

$$Corr(X,Y) = \frac{Cov(X,Y)}{\sqrt{V(X)V(Y)}} = \frac{\frac{1}{36}}{\frac{1}{18}} = \frac{1}{2} \neq 0 \Rightarrow$$ Não são independentes

b) $V(X \mid Y = y)$

$$f_{X|Y=y} = \frac{f_{X,Y}(x,y)}{f_Y(y)} = \frac{2}{2y} = \frac{1}{y}$$

$$E(X \mid Y = y) = \int_0^y x \cdot \frac{1}{y} \, dx = \frac{y}{2}$$

$$E(X^2 \mid Y = y) = \int_0^y x^2 \cdot \frac{1}{y} \, dx = \frac{y^2}{3}$$

$$\rightarrow V(X \mid Y = y) = \frac{y^2}{3} - \frac{y^2}{4} = \frac{y^2}{12}$$

c)

$$E(X) = \int_0^1 x f_X(x)\,dx = \frac{1}{3}$$

$$f_X(x) = \int_{-\infty}^{+\infty} f_{XY}\,dy = \int_x^1 2\,dy = 2(1-x)$$

$$E(X) = E\left[E(X|Y)\right]$$

$$E\left[E(X|Y)\right] = \int_0^1 \frac{y}{\cancel{2}} \cdot \cancel{2}y\,dy = \left[\frac{y}{3}\right]_0^1 = \frac{1}{3}$$

$E(X|Y) = \frac{y}{2}$

f.d.p : $f_Y(y) = 2y$ $\quad 0 < y < 1$

4.3)

$X \sim N(3; 0{,}0004)$

$Y \sim N(0{,}3; 0{,}000025)$

a) $E(X+2Y) = E(X) + 2E(Y) = 3{,}6$ cm

Como são independentes

$V(X+2Y) = V(X) + 4V(Y) = 0.0005$ cm

$\sigma = \sqrt{V(X+2Y)} \approx 0.02$

b) $2Y + X \sim N(3{,}6; 0.0005)$

Reduzindo a normal $\rightarrow Z = \frac{2Y+X-\mu}{\sigma} \sim N(0,1)$

$P(X+2Y > 3.62) = 1 - P(X-2Y \leq 3.62)$

$1 - P(Z \leq 0.8944) = 1 - 0.8133 = 0.1867$