

7.1) a) 99%

$$T = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} \sim t_{n-1}$$

$$S = 0.3906$$

$$\bar{x} = 2.2833$$

$$\gamma = 0.99 \quad P(-\alpha \leq T \leq \alpha) = \gamma \Leftrightarrow \alpha = F_{t, \alpha}^{-1}\left(\frac{1+\gamma}{2}\right) =$$

$$\alpha = 0.01$$

$$\left[\bar{x} + \alpha \cdot \frac{s}{\sqrt{n}} ; \bar{x} - \alpha \cdot \frac{s}{\sqrt{n}} \right] = \text{IAC}_{99\%}(\mu)$$

$$\left[2.2833 + 0.3445 ; 2.2833 - 0.3445 \right] = \text{IAC}_{99\%}(\mu)$$

b) $\gamma = 0.99$

$$Z = \frac{(n-1)S^2}{\sigma^2} \sim \chi^2_{(n-1)}$$

$$P(a \leq Z \leq b) = \gamma \Leftrightarrow a = F_x^{-1}\left(\frac{1-\gamma}{2}\right) = 4.575$$

$$b = F_x^{-1}\left(\frac{1+\gamma}{2}\right) = 26.76$$

$$a \leq \frac{(n-1)S^2}{\sigma^2} \leq b$$

$$\frac{a}{(n-1)S^2} \leq \frac{1}{\sigma^2} \leq \frac{b}{(n-1)S^2} \Leftrightarrow \sqrt{\frac{(n-1)a}{a}} \leq \sigma \leq \sqrt{\frac{(n-1)b}{b}} \Rightarrow \text{IAC}_{99\%}(\sigma) = [0.2504 ; 0.6057]$$

7.2)

a) $X \sim \text{Bern}(p)$

$$V = \bar{x}(1-\bar{x})$$

$$S = \sqrt{V}$$

$$Z = \frac{\bar{x} - p}{\frac{s}{\sqrt{n}}} \sim N(0,1)$$

$$\text{and } \alpha = \Phi^{-1}\left(\frac{1+\gamma}{2}\right)$$

$$P(-\alpha \leq Z \leq \alpha) = \gamma \Leftrightarrow P\left(\bar{x} - \alpha \sqrt{\frac{\bar{x}(1-\bar{x})}{n}} \leq p \leq \bar{x} + \alpha \sqrt{\frac{\bar{x}(1-\bar{x})}{n}}\right) = \gamma$$

$$\gamma = 0.95$$

$$\text{IAC}(p)_{95\%} = \left[0.17 - 1.96 \sqrt{\frac{0.1411}{100}} ; 0.17 + 1.96 \sqrt{\frac{0.1411}{100}} \right] = [0.09638 ; 0.2436]$$

$$b) \text{IAC}(p)_{98\%} = \left[\bar{x} - \alpha \sqrt{\frac{\bar{x}(1-\bar{x})}{n}} ; \bar{x} + \alpha \sqrt{\frac{\bar{x}(1-\bar{x})}{n}} \right] \quad \alpha \sqrt{\frac{p(1-p)}{n}} = 0.02 \quad \Phi^{-1}\left(\frac{1+0.98}{2}\right) = 2.3263$$

$$\rightarrow p_{\max} = \frac{1}{2}$$

$$\frac{p(1-p)}{\left(\frac{0.02}{2.0537}\right)^2} = n \Rightarrow n_{\max} = 3383$$

$N_{\max}$ 172 cascos de mais 3283 peças