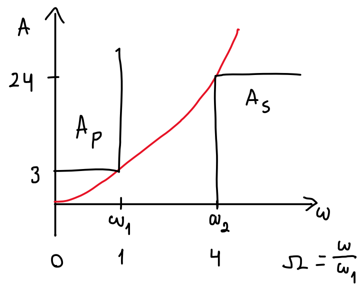
a)

$A_p = 3\,dB \qquad f_1 = 10\,kHz$

$A_s = 24\,dB \qquad f_2 = 40\,kHz$



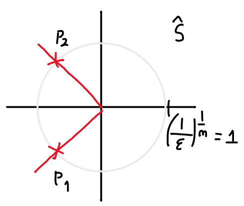
Butterworth

$A(\Omega) = 10 \log\left(1 + \varepsilon^2 \Omega^{2m}\right)$

$A(1) = 10 \log(1+\varepsilon^2) \leqslant A_p \Rightarrow 10\log(1+\varepsilon^2) = 3dB \Rightarrow \varepsilon = \sqrt{10^{\frac{3}{10}} - 1} \approx 1$

$A(4) = 10 \log(1 + 4^{2m}) \leqslant A_s \Rightarrow 10\log(1+4^{2m}) = 24dB \Rightarrow 10^{\frac{24}{10}} - 1 \leqslant 4^{2m} \Rightarrow m = 2$

2ª ordem $\Rightarrow$



$P_2 = -\frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}$

$P_1 = -\frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}$

$\Rightarrow \quad P_1 + P_2 = -\sqrt{2}$

$P_1 \cdot P_2 = 1$

Polinómio : $H(\hat{S}) = (\hat{S} - P_1)(\hat{S} - P_2) = \hat{S}^2 - (P_1 + P_2)\hat{S} + P_1 \cdot P_2 = \hat{S}^2 + \sqrt{2}\,\hat{S} + 1$

$$T(s) = \frac{1}{H(\hat{S})}\Bigg|_{\hat{S} = \sqrt{\varepsilon}\frac{s}{\omega_1}} = \frac{\omega_1^2}{s^2 + \sqrt{2}\omega_1 s + \omega_1^2} = \frac{3.947 \times 10^9}{s^2 + 8.886 \times 10^4 s + 3.947 \times 10^9}$$

b)

$$\frac{V_o}{V_i} = \frac{-Y_1 Y_3}{(Y_1 + Y_2 + Y_3 + Y_4)Y_5 + Y_3 Y_4}$$

Passa - baixo : $Y_1 = \frac{1}{R_1} \quad Y_3 = \frac{1}{R_3} \quad Y_5 = sC_5 \Rightarrow$

$Y_2 = sC_2 \quad Y_4 = \frac{1}{R_4}$

$Y_1 = Y_3 = Y_4 = \frac{1}{R} \qquad C_2 = 3nF$

$Y_2 = sC_2 \quad Y_5 = sC_5 \qquad C_5 = 667pF$

$$\frac{\frac{1}{R^2}}{\left(\frac{3}{R} + sC_2\right)sC_5 + \frac{1}{R^2}} = \frac{\frac{1}{R^2 C_2 C_5}}{s^2 + \frac{3}{RC_2}s + \frac{1}{R^2 C_2 C_5}}$$

$$\frac{\omega_1^2}{s^2 + \sqrt{2}\omega_1 s + \omega_1^2}$$

$\Rightarrow$

$\omega_1^2 = \frac{1}{R^2 C_2 C_5} \Rightarrow R = \frac{1}{\omega_p \sqrt{C_2 C_5}} = 11,25\,k\Omega$

$\sqrt{2}\omega_1 = \frac{3}{RC_2} \Rightarrow R = \frac{3}{\sqrt{2}\,\omega_1 C_2} = 11,25\,k\Omega$