

a) $f_s = 40 \text{ kHz}$

Transformação bilinear: $D = \frac{2}{T} \cdot \frac{1-z^{-1}}{1+z^{-1}}$

$$D^2 = \left(\frac{2}{T} \cdot \frac{1-z^{-1}}{1+z^{-1}} \right)^2 = \frac{4}{T^2} \cdot \frac{1-2z^{-1}+z^{-2}}{1+2z^{-1}+z^{-2}}$$

$$T = \frac{1}{f_s}$$

$$T(z) = T(s) \Big|_{s = \frac{2}{T} \frac{1-z^{-1}}{1+z^{-1}}} = \frac{\frac{4}{T^2} \frac{1-2z^{-1}+z^{-2}}{1+2z^{-1}+z^{-2}}}{\frac{4}{T^2} \frac{1-2z^{-1}+z^{-2}}{1+2z^{-1}+z^{-2}} + 7,4 \times 10^4 \frac{2}{T} \frac{1-z^{-1}}{1+z^{-1}} + 2,53 \times 10^9} = \frac{\frac{4}{T^2} (1-2z^{-1}+z^{-2})}{2,53 \times 10^9 (1+2z^{-1}+z^{-2}) + 7,4 \times 10^4 \frac{2}{T} \cdot (1-z^{-1})(1+z^{-1}) + \frac{4}{T^2} (1-2z^{-1}+z^{-2})}$$

$$= \frac{\frac{4}{T^2} (1-2z^{-1}+z^{-2})}{\underbrace{\left(\frac{4}{T^2} + 7,4 \times 10^4 \frac{2}{T} + 2,53 \times 10^9 \right)}_A + \underbrace{\left(2 \cdot 2,53 \times 10^9 - \frac{8}{T^2} \right)}_B z^{-1} + \underbrace{\left(2,53 \times 10^9 - 7,4 \times 10^4 \frac{2}{T} + \frac{4}{T^2} \right)}_C z^{-2}} = \frac{4}{AT^2} \cdot \frac{1-2z^{-1}+z^{-2}}{1 + \frac{B}{A} z^{-1} + \frac{C}{A} z^{-2}}$$

$T = 25 \mu\text{s}$

$$A = \frac{4}{T^2} + 7,4 \times 10^4 \frac{2}{T} + 2,53 \times 10^9 = 14,85 \times 10^9$$

$$\frac{4}{AT^2} = 0,431$$

$$B = 2 \cdot 2,53 \times 10^9 - \frac{8}{T^2} = -7,74 \times 10^9$$

$$\frac{B}{A} = -0,521$$

$$T(z) = 0,431 \frac{1-2z^{-1}+z^{-2}}{1-0,521z^{-1}+0,203z^{-2}}$$

$$C = 2,53 \times 10^9 - 7,4 \times 10^4 \frac{2}{T} + \frac{4}{T^2} = 3,01 \times 10^9$$

$$\frac{C}{A} = 0,203$$

b) $z = e^{j\omega T} = -1 \quad T(-1) = 1 \Rightarrow 20 \log_{10}(1) = 0 \text{ dB}$
 $\omega = 2\pi \cdot 20 \text{K}$

ou $D = \frac{2}{T} \frac{1-z^{-1}}{1+z^{-1}}$

$s = j\tilde{\omega}$

$z = e^{j\omega T}$

$\tilde{\omega} \rightarrow \text{freq. analoga}$

$\omega \rightarrow \text{freq. digital}$

$\tilde{\omega} = \frac{\omega}{T} \tan\left(\frac{\omega T}{2}\right) \Rightarrow \tilde{\omega} = \infty$

c) $T(z) = 0,431 \frac{1-2z^{-1}+z^{-2}}{1-0,521z^{-1}+0,203z^{-2}}$

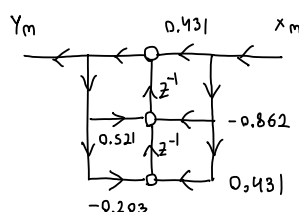
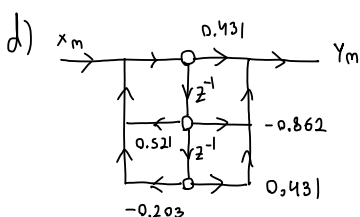
$T(s) \Big|_{s=j\infty} = 1 \Rightarrow 20 \log_{10}(1) = 0 \text{ dB}$

$$T(z) = \frac{Y(z)}{X(z)} \Rightarrow Y_m - 0,521 Y_{m-1} + 0,203 Y_{m-2} = 0,431 X_m - 0,862 X_{m-1} + 0,431 X_{m-2}$$

Polos: $1 - 0,521z^{-1} + 0,203z^{-2} = 0 \Rightarrow z^2 - 0,521z + 0,203 = 0 \Leftrightarrow$

$$\Leftrightarrow z = \frac{0,521 \pm \sqrt{0,521^2 - 4 \cdot 0,203}}{2} = 0,26 \pm 0,37j \quad |z| = 0,45 < 1$$

Está dentro do círculo unitário, logo é estável.



(Transposto)