1.5.0)

1.5.1) $T_1 = \frac{V_1}{V_i}$

Todos os filtros tem os mesmos polos!

$$V_1 = KV_i + \frac{1}{Q}V_2 - V_3 = KV_i - \frac{1}{Q}\cdot\frac{1}{sT}V_1 - \frac{1}{T^2s^2}V_1 \Leftrightarrow$$

$$\Leftrightarrow V_1 + \frac{1}{QTs}V_1 + \frac{1}{T^2s^2}V_1 = kV_i \Leftrightarrow$$

$$\Leftrightarrow T_1 = \frac{V_1}{V_i} = \frac{K}{\frac{1}{T^2s^2}+\frac{1}{QTs}+1} = \frac{Ks^2}{s^2+\frac{\omega_p}{Q}s+\omega_p^2} \Rightarrow \text{Passa Alto}$$

$T_2 = \frac{V_2}{V_i}$

$$V_2 = -\frac{1}{sT}V_1 = -\frac{1}{sT}\left(KV_i + \frac{1}{Q}V_2 - V_3\right) = -\frac{1}{sT}\left(KV_i + \frac{1}{Q}V_2 + \frac{1}{sT}V_2\right) \Leftrightarrow$$

$$\Leftrightarrow -sTV_2 = KV_i + \frac{1}{Q}V_2 + \frac{1}{sT}V_2 \Leftrightarrow -V_2\left(sT + \frac{1}{Q} + \frac{1}{sT}\right) = kV_i \Leftrightarrow$$

$$\Leftrightarrow T_2 = \frac{V_2}{V_i} = -\frac{K}{sT+\frac{1}{Q}+\frac{1}{sT}} = -\frac{\omega_p K s}{s^2+\frac{\omega_p}{Q}s+\omega_p^2} \Rightarrow \text{Passa Banda}$$

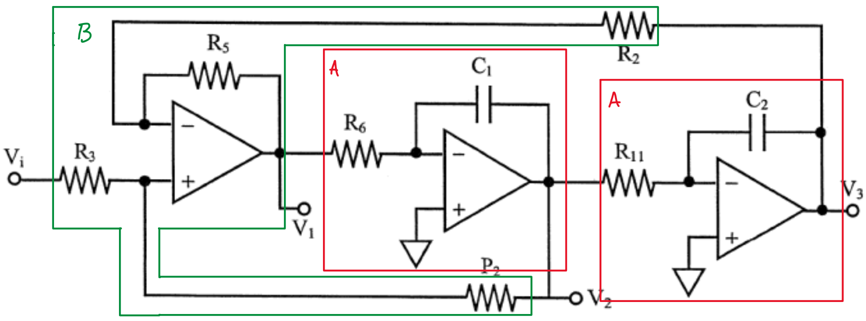
$T_3 = \frac{V_3}{V_i}$

$$V_3 = -\frac{1}{sT}V_2 = \frac{1}{s^2T^2}V_1 = \frac{1}{s^2T^2}\left(KV_i + \frac{1}{Q}V_2 - V_3\right) = \frac{1}{s^2T^2}\left(KV_i - \frac{sT}{Q}V_3 - V_3\right) \Leftrightarrow$$

$$\Leftrightarrow V_3\left(s^2T^2 + \frac{sT}{Q} + 1\right) = KV_i \Leftrightarrow$$

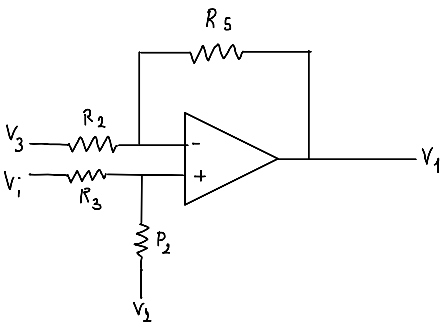
$$\Leftrightarrow T_3 = \frac{V_3}{V_i} = \frac{K}{s^2T^2+\frac{sT}{Q}+1} = \frac{\omega_p^2 K}{s^2+\frac{\omega_p}{Q}s+\omega_p^2} \Rightarrow \text{Passa Baixo}$$

1.5.2)



A → Integrador

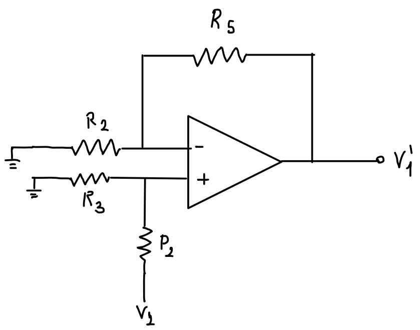
B → Somador (Subtractor)



$\frac{V_1}{V_i} = ?$

Teorema da Sobreposição

$-V_i = 0;\ V_3 = 0;\ V_2 \neq 0 \Rightarrow$

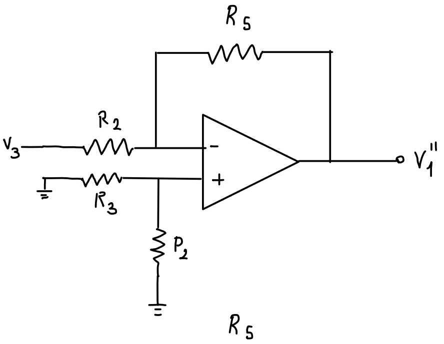


$$V_+ = V_- = \frac{R_3}{R_3+P_2}V_2$$

$$V_- = V_1'\frac{R_2}{R_5+R_2}$$

$$\Rightarrow \frac{R_3}{R_3+P_2}V_2 = \frac{R_2}{R_5+R_2}V_1' \Rightarrow V_1' = \frac{R_3(R_5+R_2)}{R_2(R_3+P_2)}V_2$$

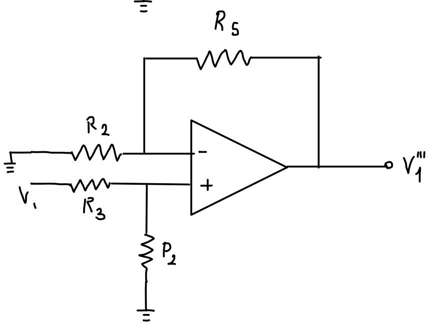
$-V_i = 0;\ V_3 \neq 0;\ V_2 = 0 \Rightarrow$



$V_+ = V_- = 0$

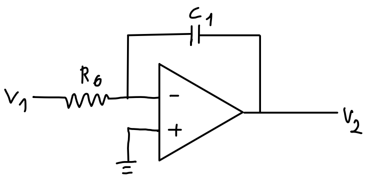
$$\frac{V_3 - V_-}{R_2} = \frac{V_- - V_1''}{R_5} \Leftrightarrow \frac{V_3}{R_2}R_5 = -V_1'' \Leftrightarrow V_1'' = -\frac{R_5}{R_2}V_3$$

$-V_i \neq 0;\ V_3 = 0;\ V_2 = 0 \Rightarrow$

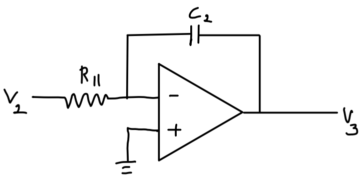


$$\begin{cases} V_+ = V_- = V_i \dfrac{P_2}{R_3 + P_2} \\ V_- = V_1''' \dfrac{R_2}{R_5 + R_2} \end{cases} \Rightarrow V_i \frac{P_2}{R_3 + P_2} = V_1''' \frac{R_2}{R_5 + R_2} \Leftrightarrow V_1''' = V_i \frac{P_2}{R_3 + P_2} \cdot \frac{R_5 + R_2}{R_2}$$

$$\Rightarrow V_1 = V_1' + V_1'' + V_1''' = \frac{R_3(R_5 + R_2)}{R_2(R_3 + P_2)}V_2 - \frac{R_5}{R_2}V_3 + V_i \frac{P_2}{R_3 + P_2}\frac{R_5 + R_2}{R_2} = \frac{R_5 + R_2}{R_3 + P_2}\left(\frac{R_3}{R_2}V_2 + \frac{P_2}{R_2}V_i\right) - \frac{R_5}{R_2}V_3$$



$$\frac{V_1 - 0}{R_6} = \frac{0 - V_2}{\frac{1}{sC_1}} \Leftrightarrow V_2 = -\frac{V_1}{sC_1R_6} \qquad \alpha = C_1R_6$$



$$\Rightarrow V_3 = -\frac{V_2}{sC_2R_{11}} = \frac{V_1}{s^2C_2C_1R_{11}R_6} \qquad \beta = C_2R_{11}$$

$$V_1 = \frac{R_5 + R_2}{R_3 + P_2}\left(-\frac{R_3}{R_2}\frac{V_1}{sC_1R_6} + \frac{P_2}{R_2}V_i\right) - \frac{R_5}{R_2}\frac{V_1}{s^2C_2C_1R_{11}R_6} \Leftrightarrow$$

$$\Leftrightarrow V_1 + \frac{R_5 + R_2}{R_3 + P_2}\frac{R_3}{R_2}\frac{\alpha}{s}V_1 + \frac{R_5}{R_2}\frac{\alpha\beta}{s^2}V_1 = \frac{R_5 + R_2}{R_3 + P_2}\frac{P_2}{R_2}V_i \Leftrightarrow$$

$$\Leftrightarrow \frac{V_1}{V_i} = \frac{\frac{R_5 + R_2}{R_3 + P_2}\frac{P_2}{R_2}}{1 + \frac{R_5 + R_2}{R_3 + P_2}\frac{R_3}{R_2}\frac{\alpha}{s} + \frac{R_5}{R_2}\frac{\alpha\beta}{s^2}} = \frac{\frac{R_5 + R_2}{R_3 + P_2}\frac{P_2}{R_2}s^2}{s^2 + \frac{R_5 + R_2}{R_3 + P_2}\frac{R_3}{R_2}\alpha\, s + \frac{R_5}{R_2}\alpha\beta} \quad \text{e Como} \quad \frac{V_1}{V_i} = \frac{Ks^2}{s^2 + \frac{\omega_p}{Q}s + \omega_p^2}$$

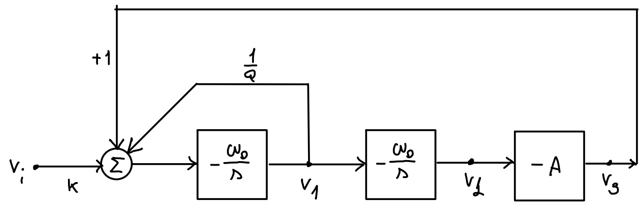
Temos: $\omega_0^2 = \frac{R_5}{R_2}\alpha\beta \Rightarrow \omega_0 = \sqrt{\frac{R_5}{R_2}\frac{1}{C_2C_1R_{11}R_6}}$ //

$$\frac{\omega_0}{Q} = \frac{R_5 + R_2}{R_3 + P_2}\frac{R_3}{R_2}\alpha \Leftrightarrow Q = \omega_0 \frac{R_3 + P_2}{R_5 + R_2}\frac{R_2}{R_3}C_1R_6 = \frac{R_3 + P_2}{R_5 + R_2}\cdot\frac{R_2}{R_3}\cdot\sqrt{\frac{R_5}{R_2}\frac{C_1R_6}{C_2R_{11}}} \quad //$$

$$k = \frac{R_5 + R_2}{R_3 + P_2}\frac{P_2}{R_2} \quad //$$

Com $R_2 = R_5$ e $C_2R_{11} = C_1R_6$: $\omega_0 = \frac{1}{R_6C_1}$ $\quad k = \frac{2P_2}{R_3 + P_2}$ $\quad Q = \frac{R_3 + P_2}{2R_3} = \frac{1}{2}\left(1 + \frac{P_2}{R_3}\right)$

1.6.1)



$$T_1: \quad V_1 = -\frac{\omega_0}{s}\left(\frac{V_1}{Q} + V_3 + kV_i\right) \qquad \begin{cases} V_3 = -AV_2 \\ V_2 = -\frac{\omega_0}{s} V_1 \end{cases} \Rightarrow \quad V_3 = A\frac{\omega_0}{s} V_1$$

$$V_1 + \frac{\omega_0}{sQ} V_1 + \frac{\omega_0}{s}\cdot\frac{\omega_0}{s}\cdot A \cdot V_1 = -\frac{\omega_0}{s} k V_i$$

$$T_1 = \frac{V_1}{V_i} = \frac{-\frac{\omega_0}{s}k}{1 + \frac{\omega_0}{sQ} + \frac{\omega_0^2}{s^2}A} = \frac{-\omega_0 K s}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2 A} \Rightarrow \text{Passa Banda}$$

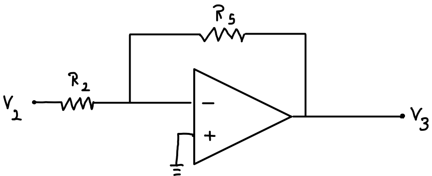
$$T_2: \quad V_2 = -\frac{\omega_0}{s} V_1$$

$$T_2 = \frac{V_2}{V_i} = -\frac{\omega_0}{s}\frac{V_1}{V_i} = -\frac{\omega_0}{s} T_1 = \frac{\omega_0^2 K}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2 A} \Rightarrow \text{Passa Baixo}$$

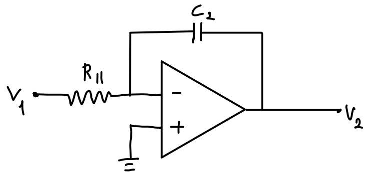
$$T_3: \quad V_3 = -AV_2$$

$$T_3 = \frac{V_3}{V_i} = -A\,T_2 = \frac{-A\,\omega_0^2 K}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2 A} \Rightarrow \text{Passa Baixo}$$

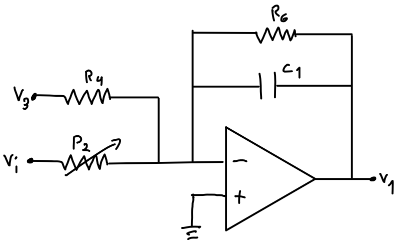
$\Rightarrow$ Temos que ter $A = 1$ //



$$\frac{V_2 - V_-}{R_2} = \frac{V_- - V_3}{R_5} \Rightarrow V_3 = -V_2\frac{R_5}{R_2} \Rightarrow A = \frac{R_5}{R_2} \Rightarrow R_5 = R_2$$



$$\Rightarrow V_2 = -\frac{V_1}{s\,C_2 R_{11}} \Rightarrow V_3 = -V_2 = \frac{V_1}{s\,C_2 R_{11}}$$



$$\frac{V_3}{R_4} + \frac{V_i}{P_2} = -\frac{V_1}{R_6 /\!/ C_1} \Leftrightarrow \frac{V_1}{sC_2 R_{11} R_4} + \frac{V_i}{P_2} = -\frac{V_1}{R_6}(sC_1R_6 + 1) \qquad R_6 /\!/ C_1 = \frac{1}{sC_1 + \frac{1}{R_6}} = \frac{R_6}{sC_1R_6 + 1}$$

$$V_1\left(\frac{1}{sC_2R_{11}R_4} + \frac{sC_1R_6+1}{R_6}\right) = -\frac{V_i}{P_2}$$

$$T_1 = \frac{V_1}{V_i} = \frac{-\frac{1}{P_2}}{\frac{1}{sC_2R_{11}R_4} + \frac{sC_1R_6+1}{R_6}} = \frac{-s\frac{1}{C_1P_2}}{s^2 + s\frac{1}{C_1R_6} + \frac{1}{C_2R_{11}R_4C_1}} \Rightarrow \begin{cases} \omega_0 = \sqrt{\frac{1}{C_2R_{11}R_4C_1}} \\ Q = C_1R_6\omega_0 = R_6\sqrt{\frac{C_1}{C_2R_{11}R_4}} \\ K = \frac{1}{P_2C_1\omega_0} = \frac{1}{P_2}\sqrt{\frac{C_2R_{11}R_4}{C_1}} \end{cases}$$

Considerando $A \neq 1$

$$\omega_0 = \sqrt{\frac{R_5}{R_2C_2R_{11}R_4C_1}} \qquad Q = R_6\sqrt{\frac{R_5C_1}{R_2C_2R_{11}R_4}} \qquad K = \frac{1}{P_2}\sqrt{\frac{R_2C_2R_{11}R_4}{R_5C_1}}$$

1.6.2)

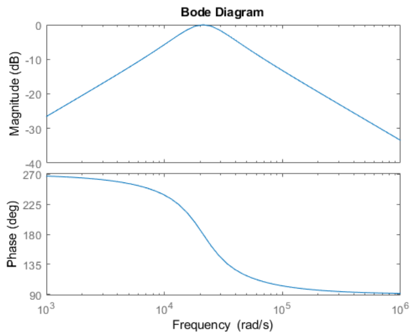
$$\omega_0 = \sqrt{\frac{R_5}{R_2 C_2 R_{11} R_4 C_1}} = \sqrt{\frac{100K}{100K \cdot 4{,}7m \cdot 10K \cdot 10K \cdot 4{,}7m}} = \frac{1}{4{,}7m \cdot 10K} = 21.28\ kHz$$

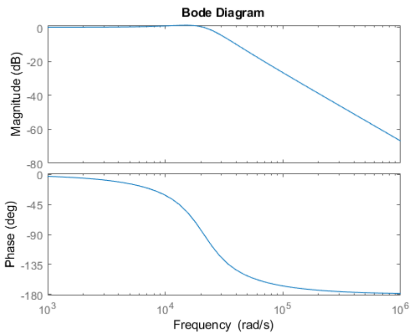
$$Q = R_6 \sqrt{\frac{R_5 C_1}{R_2 C_2 R_{11} R_4}} = C_1 R_6 \omega_0 = \frac{R_6}{R_4} = 1$$

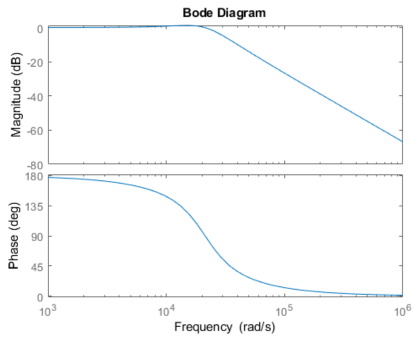
$$K = \frac{1}{P_2}\sqrt{\frac{R_2 C_2 R_{11} R_4}{R_5 C_1}} = \frac{1}{P_2 C_1 \omega_0} = 1$$

Logo

$$T_1 = \frac{-\omega_0 s}{s^2 + s\,\omega_0 + \omega_0^2} \qquad T_2 = \frac{\omega_0^2}{s^2 + s\,\omega_0 + \omega_0^2} \qquad T_3 = \frac{-\omega_0^2}{s^2 + s\,\omega_0 + \omega_0^2}$$







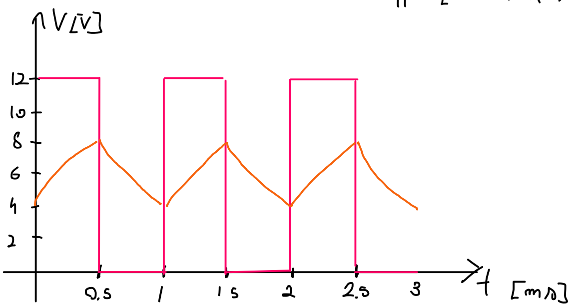
2.5.1)

$$t_L = C_1(R_D + R_2)\ln(2) \approx C_1 R_2 \ln(2)$$

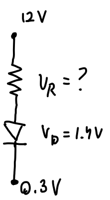
$$t_H = C_1(R_1 + R_2)\ln(2)$$

$$D = \frac{t_H}{t_H + t_L} \cdot 100 = \frac{R_1 + R_2}{R_1 + 2R_2} \cdot 100 = 54.8\,\%$$

$$f = \frac{1}{t_H + t_L} = \frac{1}{\ln(2)\cdot(R_1 + 2R_2)C_1} = 930\ Hz$$



2.5.2) Max ⇒ $V_M = 0.3\,V$



$12 - 0.3 = V_R + 1.4 \Leftrightarrow 10.3\,V = V_R$

$I_R = I_D = \frac{V_R}{R} = 68.67\,mA$

I_{Max} { $D = 100\% = 0.1\,A$; $D = 1\% = 1\,A$ } $\approx I_{Max} = 0.5\,A = 500\,mA \Rightarrow D = 50\%$

Agora é necessário saber a corrente de sink do V_M, para sabermos quanta corrente dá para puxar. De notar que o valor máximo apresentado na datasheet é 50 mA. Logo não devemos conseguir baixar sem danificar o diodo ou o 555.

2.6.1)

$$\underset{n=2}{BP} \rightsquigarrow \underset{n=1}{LP}$$

$A_p = 3\,dB$ $\qquad$ $A_p = 3\,dB$

$f_0 = 1\,kHz$

$B = 250\,Hz$

$$\left(\begin{array}{l} \omega_0^2 = \omega_{p_1} \cdot \omega_{p_2} \\ B = \omega_{p_1} - \omega_{p_2} \end{array} \Rightarrow \begin{array}{l} f_{p_2} = 882.78\,Hz \\ f_{p_1} = 1132.78\,Hz \end{array}\right)$$

$A_p = 10 \log(1+\varepsilon^2) \Rightarrow \varepsilon = 1$

Temos $n = 1$ e $\varepsilon = 1$

$$T(S) = \frac{1}{S+1}\bigg|_{S = \sqrt[n]{\varepsilon}\left(\frac{s^2+\omega_0^2}{Bs}\right)} \Rightarrow T(s) = \frac{Bs}{s^2 + Bs + \omega_0^2}$$

$T(s = 0 + j\omega_0) = 1$ Logo $|K| = 10^{\frac{24}{20}} = 10^{1.2} = 15.85$

$$\text{Logo } T(s) = \pm \frac{KBs}{s^2 + Bs + \omega_0^2} = \pm \frac{24812s}{s^2 + 1571s + 39.5 \times 10^6} \quad //$$

2.6.2) Implementar como um filtro LC em escada duplamente terminado para reduzir a sensibilidade.

Filtrar mais os harmonicos, ou seja implementar um filtro passa baixo de 2ª ordem, ao revés de um passa banda!