P1)

a) $Y(s)\,(s+1)(s+2) = U(s)$

$Y(s^2+3s+2) = U$

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ -2 & -3\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}0\\ 1\end{bmatrix} u$$

$\ddot{y} + 3\dot{y} + 2y = u \iff \ddot{y} = u - 3\dot{y} - 2y$

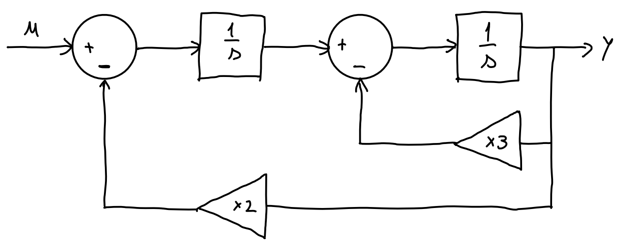
$x_1 = y$

$x_2 = \dot{y} = \dot{x}_1$

b) $\ddot{y} = u - 3\dot{y} - 2y$

$\hookrightarrow \dot{y} = -3y + \int u - 2y$

$\hookrightarrow y = \int -3y + \int u - 2y$



P2)

a) $m_1 \ddot{z}_1 = u - K(z_1 - z_2)$

$m_2 \ddot{z}_2 = -K(z_2 - z_1) - \beta \dot{z}_2$

b)

$z_1 = x_1$

$\dot{z}_1 = x_2 = \dot{x}_1$

$z_2 = x_3$

$\dot{z}_2 = x_4 = \dot{x}_3$

$\ddot{z}_1 = \frac{u}{m_1} - \frac{K}{m_1} z_1 + \frac{K}{m_1} z_2$

$\dot{x}_2 = -\frac{K}{m_1} x_1 + \frac{K}{m_1} x_3 + \frac{1}{m_1} \cdot u$

$\ddot{z}_2 = -\frac{K}{m_2} z_1 + \frac{K}{m_2} z_2 - \frac{\beta}{m_2} \dot{z}_2$

$\dot{x}_4 = +\frac{K}{m_2} x_1 - \frac{K}{m_2} x_3 - \frac{\beta}{m_2} x_4$

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\end{bmatrix} = \begin{bmatrix}0 & 1 & 0 & 0\\ -\frac{K}{m_1} & 0 & \frac{K}{m_1} & 0\\ 0 & 0 & 0 & 1\\ \frac{K}{m_2} & 0 & -\frac{K}{m_2} & -\frac{\beta}{m_2}\end{bmatrix} \cdot \begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix} + \begin{bmatrix}0\\ \frac{1}{m_1}\\ 0\\ 0\end{bmatrix} u$$

$y = [0\ 0\ 1\ 0]\ x$

c) $C = [0\ 0\ 1\ 0]$

$B = \begin{bmatrix}0\\ \frac{1}{m_1}\\ 0\\ 0\end{bmatrix}$

$$sI - A = \begin{bmatrix}s & -1 & 0 & 0\\ \frac{K}{m_1} & s & -\frac{K}{m_1} & 0\\ 0 & 0 & s & -1\\ -\frac{K}{m_2} & 0 & \frac{K}{m_2} & s+\frac{\beta}{m_2}\end{bmatrix}$$

$C\,adj(sI-A)\,B = \frac{K}{m_1 m_2}$

$\det(sI-A) = s^4 + \frac{\beta}{m_2}s^3 + \frac{K}{m_1}s^2 + \frac{K}{m_2}s^2 + \frac{\beta}{m_2} \cdot \frac{K}{m_1} s$

$$G(s) = \frac{\frac{K}{m_1 m_2}}{s^4 + \frac{\beta}{m_2}s^3 + \frac{K}{m_1}s^2 + \frac{K}{m_2}s^2 + \frac{\beta K}{m_2 m_1}s}$$

P3)

a) - G_4 d) - G_1 g) - G_2

b) - G_7 e) - G_8 h) - G_6

c) - G_3 f) - G_5

$G_1 \Rightarrow \omega = 1$ $2\xi\omega = 0,1 \Rightarrow \xi = 0,05$

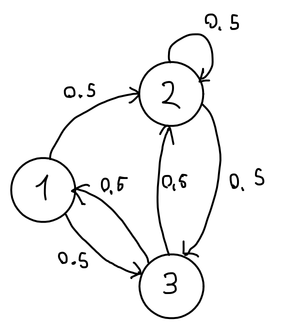
$G_2 \Rightarrow \omega = 4$ $2\xi\omega = 2 \Rightarrow \xi = 0,25$

$G_3 \Rightarrow \omega = 1$ $2\xi\omega = 0.5 \Rightarrow \xi = 0,25$

$G_5 \Rightarrow \omega = 4$ $2\xi\omega = 0,4 \Rightarrow \xi = 0,05$

G_4, G_6, G_8 ↳ 0's $G_7 \rightarrow$ 2 polos reais!

P4) a)



$$A = \begin{bmatrix}0 & 0.5 & 0.5\\ 0 & 0.5 & 0.5\\ 0.5 & 0.5 & 0\end{bmatrix}$$

$$P(k+1) = A^T P \iff \begin{bmatrix}P_1(k+1)\\ P_2(k+1)\\ P_3(k+1)\end{bmatrix} = \begin{bmatrix}0 & 0 & 0.5\\ 0.5 & 0.5 & 0.5\\ 0.5 & 0.5 & 0\end{bmatrix}\begin{bmatrix}P_1(k)\\ P_2(k)\\ P_3(k)\end{bmatrix}$$

b)

$$\begin{cases} P = A^T P \\ P_1 + P_2 + P_3 = 1\end{cases} \iff \begin{cases} P_1 = 0.5P_3 \\ P_2 = 0.5P_1 + 0.5P_2 + 0.5P_3 \\ P_3 = 0.5P_1 + 0.5P_2 \\ P_3 = 1 - P_1 - P_2\end{cases} \iff \begin{cases} P_1 = 0.5 - 0.5P_1 - 0.5P_2 \\ P_2 = 0.5P_1 + 0.5P_2 + 0.5 - 0.5P_1 - 0.5P_2 \\ 1 - P_1 - P_2 = 0.5P_1 + 0.5P_2 \\ P_3 = 1 - P_1 - P_2\end{cases} \iff \begin{cases} P_1 = 1/6 \\ P_2 = 1/2 \\ P_3 = 1/3\end{cases}$$

Ps)

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \\ x_3(k+1) \\ x_4(k+1) \end{bmatrix} = \begin{bmatrix} \beta & 0 & 0 & 0 \\ (1-\beta) & \beta & 0 & 0 \\ 0 & (1-\beta) & \beta & 0 \\ 0 & 0 & (1-\beta) & \beta \end{bmatrix} \cdot \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \\ x_4(k) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} u(k)$$

P6)

a) $\dot{x}_1 = -u(t)\frac{3}{L}(x_1 - \varphi) + \alpha R(t)$

$\dot{x}_2 = -u(t)\frac{3}{L}(x_2 - x_1) + \alpha R(t)$

$\dot{x}_3 = -u(t)\frac{3}{L}(x_3 - x_2) + \alpha R(t)$

c) ?

d) ?

e) ?

b) $x_1 = \int -u(t)\frac{3}{L}(x_1 - \varphi) + \alpha R(t)$