P1)

$V_B = L\frac{di}{dt}$

$V_R = Ri$

$V_a = V_B + V_R - V_{\dot{x}} = Ri + L\frac{di}{dt} - K_V\dot{x}$

$$\begin{cases} V_a = Ri + L\dot{i} - K_V\dot{x} \\ m\ddot{x} = K_I i \end{cases} \qquad x_1 = i \qquad x_2 = x \qquad x_3 = \dot{x}_2 = \dot{x}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -\frac{R}{L} & 0 & \frac{K_V}{L} \\ 0 & 0 & 1 \\ \frac{K_I}{m} & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} \frac{1}{L} \\ 0 \\ 0 \end{bmatrix} V_a \qquad y = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

P3) a)

$G(s) = \frac{b}{s^2+3s+2}$

$Y(s) = G(s)\,U(s) \Leftrightarrow Y(s)(s^2+3s+2) = b\,U(s)$

G_1:

$Y(s)(s^2+3s+2) = U(s)$

$$\begin{cases} \ddot{y} + 3\dot{y} + 2y = u \\ x_1 = y \\ x_2 = \dot{y} = \dot{x}_1 \end{cases}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$y = \begin{bmatrix} 1 & 0 \end{bmatrix} x$

G_2:

$Y(s)(s^2+3s+2) = (s+5)\,U(s)$

$U(s) \rightarrow \boxed{\frac{1}{s^2+3s+2}} \xrightarrow{X(s)} \boxed{s+5} \rightarrow Y(s)$

$Y = sX + 5X \Leftrightarrow y = \dot{x} + 5x$

$\ddot{x} + 3\dot{x} + 2x = u$

$x_1 = x$

$x_2 = \dot{x}_1 = \dot{x}$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$y = \begin{bmatrix} 5 & 1 \end{bmatrix} x$

G_3: $Y(s)(s^2+3s+2) = (s^2+s+5)\,U(s)$

$y = \ddot{x} + \dot{x} + 5x$

$\ddot{x} + 3\dot{x} + 2 = 0$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$y = \begin{bmatrix} 5 & 1 \end{bmatrix} x + \begin{bmatrix} 0 & 1 \end{bmatrix} \dot{x}$

b)

$Y(s)(s^2+3s+2) = (s+5)\,U(s)$

$\ddot{y} + 3\dot{y} + 2y = \dot{u} + 5u$

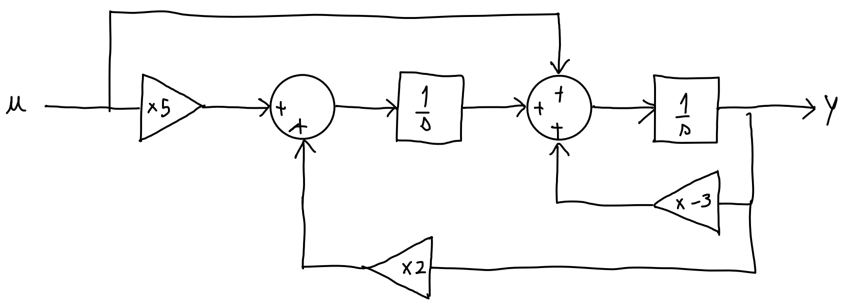
$\ddot{y} = \dot{u} + 5u - 3\dot{y} + 2y$

$\downarrow$

$\dot{y} = u - 3y + \int 5u + 2y$

$\downarrow$

$y = \int\left(u - 3y + \int(5u + 2y)\right)$



P4)

$\dot{x}_1 = s - d x_1 - \alpha x_1 x_2$

$\dot{x}_2 = \alpha x_1 x_2 - \mu x_2$

$s = 10$

$d = 0{,}02$

$\alpha = 0.001$

$\mu = 0{,}24$

PE:

1. $x_1 = 500$ e $x_2 = 0$
2. $x_1 = 240$ e $x_2 = 21.667$

a) $\dot{x}_1 = \dot{x}_2 = 0$

$$\begin{cases} 0 = s - d x_1 - \alpha x_1 x_2 \\ 0 = \alpha x_1 x_2 - \mu x_2 \end{cases}$$

$x_1 = 0 \Rightarrow$ Imp.

$x_2 = 0 \Rightarrow \{ s - d x_1 = 0 \Leftrightarrow x_1 = \frac{s}{d} = 500$

$x_1 \neq 0$ e $x_2 \neq 0$

$0 = \alpha x_1 - \mu \Leftrightarrow x_1 = \frac{\mu}{\alpha} = 240$

$x_2 = \frac{d x_1 - s}{-\alpha x_1} = 21.667$

b)

$$\begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \end{bmatrix} = \begin{bmatrix} -d - \alpha x_2 & -\alpha x_1 \\ \alpha x_2 & \alpha x_1 - \mu \end{bmatrix}$$

1. PE $\rightarrow x_1 = 500$, $x_2 = 0$ $\quad \begin{bmatrix} -0.02 & -0.5 \\ 0 & 0.26 \end{bmatrix}$

2. PE $\rightarrow x_1 = 240$, $x_2 = 21.667$ $\quad \begin{bmatrix} -0.041667 & -0.24 \\ 0.021667 & 0 \end{bmatrix}$

c) 1. $|A - \lambda I| = 0$

$A = \begin{bmatrix} -0.02 & -0.5 \\ 0 & 0.26 \end{bmatrix} \qquad A - \lambda I = \begin{bmatrix} -0.02-\lambda & -0.5 \\ 0 & 0.26-\lambda \end{bmatrix}$

$\det(A - \lambda I) = 0 \Leftrightarrow (-0.02-\lambda)(0.26-\lambda) = 0 \Leftrightarrow$

$\Leftrightarrow \lambda^2 + 0.02\lambda - 0.26\lambda - 0.02 \cdot 0.26 = 0 \Leftrightarrow \lambda^2 - 0.24\lambda - 0.0052 = 0 \Leftrightarrow \begin{cases} \lambda_1 = -0.02 \\ \lambda_2 = 0.26 \end{cases}$ Ponto de sela!

$(A - \lambda I)\,V = 0 \Leftrightarrow$ λ_1: $A V_1 = \lambda_1 V_1$ $\quad \begin{bmatrix} -0.02 & -0.5 \\ 0 & 0.26 \end{bmatrix} \begin{bmatrix} V_{11} \\ V_{12} \end{bmatrix} = \lambda_1 \begin{bmatrix} V_{11} \\ V_{12} \end{bmatrix} \Leftrightarrow V_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

λ_2: $\begin{bmatrix} -0.02 & -0.5 \\ 0 & 0.26 \end{bmatrix} \begin{bmatrix} V_{21} \\ V_{22} \end{bmatrix} = \lambda_2 \begin{bmatrix} V_{21} \\ V_{22} \end{bmatrix} \Leftrightarrow \begin{cases} V_{21} = 1 \\ -0.02 - 0.5 V_{22} = 0.26 \end{cases} \Rightarrow V_2 = \begin{bmatrix} 1 \\ -0.56 \end{bmatrix}$

c) ② $|A-\lambda I| = 0$

$$A = \begin{bmatrix} -0.041667 & -0.24 \\ 0.021667 & 0 \end{bmatrix} \qquad A-\lambda I = \begin{bmatrix} -0.041667-\lambda & -0.24 \\ 0.021667 & -\lambda \end{bmatrix}$$

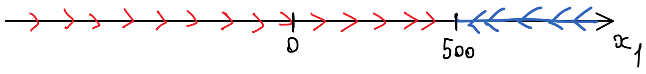
$$\det(A-\lambda I)=0 \Leftrightarrow 0.041667\lambda + \lambda^2 + 0.24 \cdot 0.021667 = 0 \Leftrightarrow$$

$$\Leftrightarrow \lambda^2 + 0.041667\lambda + 0.0052 = 0 \Leftrightarrow \lambda_1 = -0.0208333 + j0.069$$

$$\lambda_2 = -0.0208333 - j0.069$$

Foco estável

d) $\dot{x}_1 = s - d x_1$



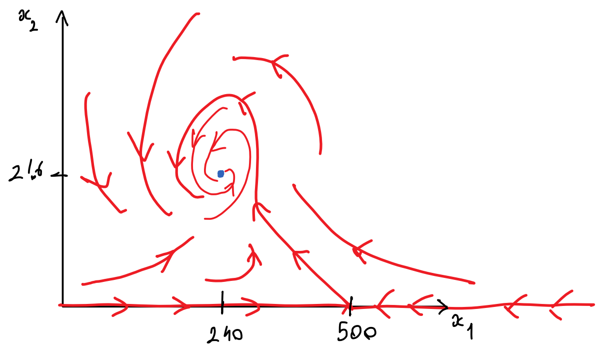
f) $\begin{cases} 0 = s - d x_1 - \alpha x_1 x_2 \\ 0 = \alpha x_1 x_2 - \mu x_2 \end{cases}$

Para $x_2 \neq 0$ e $x_1 \neq 0$

$$x_1 = \frac{\mu}{\alpha}$$

$$x_2 = \frac{s - \frac{d\mu}{\alpha}}{\mu} = \frac{s}{\mu} - \frac{d}{\alpha}$$

e)



P5) $\begin{cases} C \dfrac{dT_m}{dt} = q(t) \\ q(t) = \dfrac{1}{R}(T_L - T_m) \end{cases} \Leftrightarrow RC\dot{T}_m + T_m = T_L$

Fluxo de calor: $\dfrac{dT}{dt} = \dfrac{1}{C} q(t)$

Transferência de calor: $q(t) = \dfrac{1}{R}(T_2 - T_1)$

P6) a) $\dfrac{\partial}{\partial t} T(x,t) = -\mu(t) \dfrac{\partial}{\partial x} T(x,t) + \alpha R(t)$

$T(x,0) = g(x)$

$T(0,t) = \varphi(t)$

$L = 100$

$$\frac{\partial}{\partial t} T(L,t) = -\mu(t) \cdot \frac{T(L,t) - \overbrace{T(0,t)}^{\varphi(t)}}{L} + \alpha R(t)$$

c) Help :)

d)

e)

b) $T(L,t) = \int -\mu \cdot \dfrac{T(L,t) - \varphi(t)}{L} + \alpha R(t)$