P1) a) $\lim_{t\to\infty} y(t) = G(0) = \frac{1}{2}$

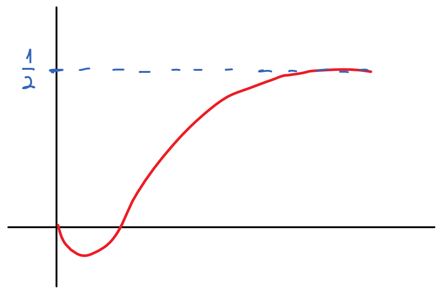
$\lim_{t\to 0} y(t) = \lim_{s\to\infty} G(s) = \lim_{s\to\infty} \frac{s\left(\frac{1}{s}-1\right)}{s\left(s+3+\frac{2}{s}\right)} = \frac{-1}{+\infty} = 0$ Vem a 0 da origem

$\lim_{t\to 0} \dot{y}(t) = \lim_{s\to\infty} s\,G(s) = \lim_{s\to\infty} \frac{s^2\left(\frac{1}{s}-1\right)}{s^2\left(1+\frac{3}{s}+\frac{2}{s^2}\right)} = -1$ Derivada negativa na origem

$G(s) = \frac{1-s}{s^2+3s+2}$ 1 zero, 2 polos $\Rightarrow$ Resposta contínua ao escalão, Derivada descontínua mas finita

$s^2+3s+2=0 \Rightarrow \begin{matrix} s=-2 \\ s=-1 \end{matrix}$ 2 Polos reais negativos $\Rightarrow$ Estável

Logo



b) $\ddot{y}+3\dot{y}+2y=u \Leftrightarrow \ddot{y} = u-3\dot{y}-2y$

$x_1 = y$

$x_2 = \dot{y} = \dot{x}_1$

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ -2 & -3\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + u\begin{bmatrix}0\\ 1\end{bmatrix}$$

$$y = [1\ 0]\,x$$

c) $$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ -2 & -3\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + u\begin{bmatrix}0\\ 1\end{bmatrix}$$

$y = (1-s)\,x \Leftrightarrow y = x - \dot{x}$

$y = [1\ \ -1]\begin{bmatrix}x_1\\ x_2\end{bmatrix}$

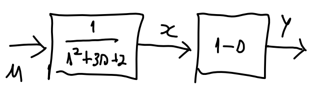
$x(1-s) = y$

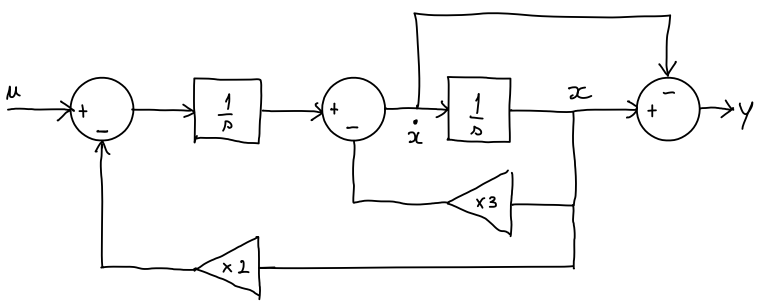
$x - \dot{x} = y$

$\dot{x} = -y + x$

$x = \int(-y + x)$

d)





$\ddot{x} + 3\dot{x} + 2x = u$

$\ddot{x} = u - 3\dot{x} - 2x$

$\dot{x} = -3x + \int u - 2x$

$x = \int\left(-3x + \int(u-2x)\right)$

P2) a) $\begin{cases} 0 = x_2 \\ 0 = -(x_1 + x_1^2) \end{cases} \Leftrightarrow$ $0 = -x_1 - x_1^2$

$0 = x_1(-1-x_1)$

$x_2 = 0 \ \vee\ x_1 = 0$ (1)

$x_2 = 0 \ \vee\ x_1 = -1$ (2)

b) $\frac{\partial f_1}{\partial x_1} = 0$ $\frac{\partial f_1}{\partial x_2} = 1$

$\frac{\partial f_2}{\partial x_1} = -1-2x_1 = -(1+2x_1)$ $\frac{\partial f_2}{\partial x_2} = 0$

$$\begin{bmatrix}0 & 1\\ -(1+2x_1) & 0\end{bmatrix}$$

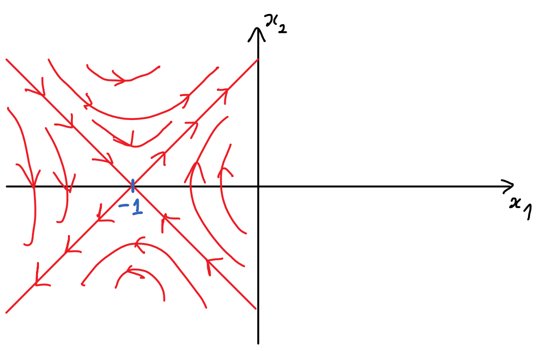
(1) $\begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix}$ $\lambda_1 = j$ (Centro)
$\lambda_2 = -j$ Não há informação

(2) $\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$ $\lambda_1 = 1$ (Ponto de sela)
$\lambda_2 = -1 \rightarrow$ Instável

c) Vetores próprios de (2)

$V_1 = \begin{bmatrix}1\\ 1\end{bmatrix}$

$V_2 = \begin{bmatrix}1\\ -1\end{bmatrix}$



P3)

$$u = RI_1 + L\dot{I}_1 + K_1\omega$$
$$J\dot{\omega} = -\beta\omega + K_T I_1$$
$$K_2\omega = L\dot{I}_2 + (R+R_L)I_2$$

$$\Rightarrow$$

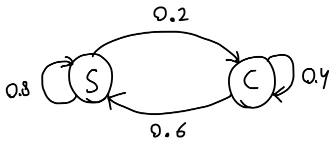
$$\dot{I}_1 = \frac{u}{L} - \frac{R}{L}I_1 - \frac{K_1}{L}\omega$$
$$\dot{\omega} = -\frac{\beta}{J}\omega + \frac{K_T}{J}I_1$$
$$\dot{I}_2 = \frac{K_2}{L}\omega - \left(\frac{R+R_L}{L}\right)I_2$$

$$\begin{bmatrix}\dot{I}_1\\ \dot{\omega}\\ \dot{I}_2\end{bmatrix} = \begin{bmatrix}-\frac{R}{L} & -\frac{K_1}{L} & 0\\ \frac{K_T}{J} & -\frac{\beta}{J} & 0\\ 0 & \frac{K_2}{L} & -\frac{R+R_L}{L}\end{bmatrix}\begin{bmatrix}I_1\\ \omega\\ I_2\end{bmatrix} + \begin{bmatrix}\frac{1}{L}\\ 0\\ 0\end{bmatrix}u$$

$$y = \begin{bmatrix}0 & 0 & R_L\end{bmatrix}\begin{bmatrix}I_1\\ \omega\\ I_2\end{bmatrix}$$

P4)

a)



	S	C
S	0.8	0.2
C	0.6	0.4

$$A = \begin{bmatrix}0.8 & 0.2\\ 0.6 & 0.4\end{bmatrix}$$

b) $A^T P = P$

$$\begin{bmatrix}0.8 & 0.6\\ 0.2 & 0.4\end{bmatrix}\begin{bmatrix}P_1\\ P_2\end{bmatrix} = \begin{bmatrix}P_1\\ P_2\end{bmatrix} \Leftrightarrow \begin{bmatrix}0.8 & 0.6\\ 0.2 & 0.4\end{bmatrix}\begin{bmatrix}P_1\\ 1-P_1\end{bmatrix} = \begin{bmatrix}P_1\\ P_2\end{bmatrix} \Leftrightarrow \begin{cases}0.8P_1 + 0.6 - 0.6P_1 = P_1\\ 0.2P_1 + 0.4 - 0.4P_1 = P_2\end{cases} \Leftrightarrow \begin{cases}P_1 = 0.75\\ P_2 = 0.25\end{cases}$$

$$P_1 + P_2 = 1 \Rightarrow P_2 = 1 - P_1$$

c) $A^m > 0$ $m = 1 \Rightarrow A > 0$ P.V. Cadeia Regular

P5)

$$L = T - U = \frac{1}{2}m(\dot{q}_1^2 + q_1^2\dot{q}_2^2) - \frac{1}{2}Kq_1^2 + mgq_1\cos(q_2)$$

$$\frac{\partial L}{\partial q_1} = mq_1\dot{q}_2^2 - Kq_1 + mg\cos(q_2) \qquad \frac{\partial L}{\partial q_2} = -mgq_1\sin(q_2)$$

$$\frac{\partial L}{\partial \dot{q}_1} = m\dot{q}_1 \qquad \frac{\partial L}{\partial \dot{q}_2} = mq_1^2\dot{q}_2$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_1}\right) = m\ddot{q}_1 \qquad \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_2}\right) = m(2q_1\dot{q}_1\dot{q}_2 + q_1^2\ddot{q}_2)$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_1}\right) = \frac{\partial L}{\partial q_1} \Leftrightarrow m\ddot{q}_1 = mq_1\dot{q}_2^2 - Kq_1 + mg\cos(q_2)$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_2}\right) = \frac{\partial L}{\partial q_2} \Leftrightarrow m(2q_1\dot{q}_1\dot{q}_2 + q_1^2\ddot{q}_2) = -mgq_1\sin(q_2) \Leftrightarrow 2q_1\dot{q}_1\dot{q}_2 + q_1^2\ddot{q}_2 + gq_1\sin(q_2)$$