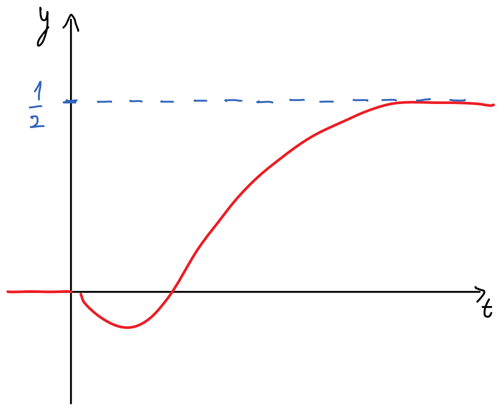
P1) a)

$G(0) = \frac{1}{2}$

1 Zero e 2 Polos $\Rightarrow$ Resposta contínua
Derivada descontínua salto finito

$\lim_{s\to\infty} G(s) = 0$ $\quad \lim_{s\to\infty} s\,G(s) = -1$

Polos: $s=-1$ e $s=-2$, ambos reais $\Rightarrow$ Não oscila
ambos negativos $\Rightarrow$ estável



b)

$$H(s) = \frac{1}{s^2+3s+2}$$

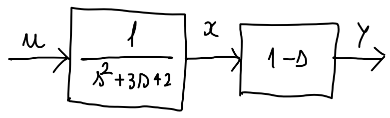
$$\ddot{y} + 3\dot{y} + 2y = u \iff \ddot{y} = u - 3\dot{y} - 2y$$

$\ddot{y} = u - 3\dot{y} - 2y$
$x_1 = y$
$x_2 = \dot{y} = \dot{x}_1$

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ -2 & -3\end{bmatrix}\cdot\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}0\\ 1\end{bmatrix}u$$

$$y = \begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}$$

c)



Logo
$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ -2 & -3\end{bmatrix}\cdot\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}0\\ 1\end{bmatrix}u$$

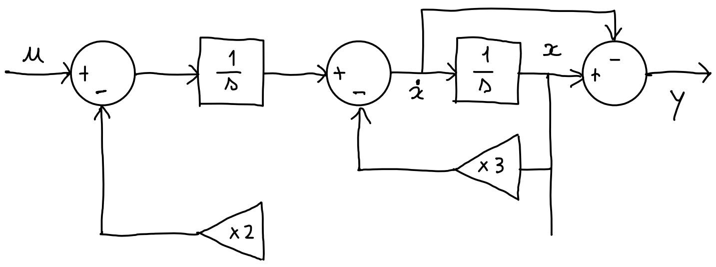
$$y = \begin{bmatrix}1 & -1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}$$

d) $\ddot{x} = u - 3\dot{x} - 2x$

$\dot{x} = -3y + \int(u - 2y)$

$x = \int\left[-3y + \int(u-2y)\right]$

$y = x - \dot{x}$



P2) a)
$$\begin{cases}-x_1 + x_2 = 0\\ x_1x_2 - 1 = 0\end{cases} \iff \begin{cases}x_2 = x_1\\ x_1x_2 = 1\end{cases} \iff \begin{cases}x_1 = 1 \vee x_2 = 1 & ①\\ x_1 = -1 \vee x_2 = -1 & ②\end{cases}$$

$\frac{\partial f_1}{\partial x_1} = -1 \qquad \frac{\partial f_2}{\partial x_1} = x_2 \qquad \begin{bmatrix}-1 & 1\\ x_2 & x_1\end{bmatrix}$

$\frac{\partial f_1}{\partial x_2} = 1 \qquad \frac{\partial f_2}{\partial x_2} = x_1$

① $\dot{x} = \begin{bmatrix}-1 & 1\\ 1 & 1\end{bmatrix}x$

② $\dot{x} = \begin{bmatrix}-1 & 1\\ -1 & -1\end{bmatrix}x$

b) $x_1 > 0$ e $x_2 > 0 \Rightarrow$ ①

$$A - \lambda I = \begin{bmatrix}-1-\lambda & 1\\ 1 & 1-\lambda\end{bmatrix}$$

$\det(A-\lambda I) = (-1-\lambda)(1-\lambda) - 1$

$(-1-\lambda)(1-\lambda) - 1 = 0 \iff -1 + \lambda - \lambda + \lambda^2 - 1 = 0 \iff \lambda = \pm\sqrt{2}$ Ponto de sela!

$\lambda_1 = \sqrt{2}$ e $\lambda_2 = -\sqrt{2}$

$(A - \lambda_1 I)\cdot V_1 = 0 \iff \begin{bmatrix}-1-\sqrt{2} & 1\\ 1 & 1-\sqrt{2}\end{bmatrix}\begin{bmatrix}V_{11}\\ V_{12}\end{bmatrix} = 0 \Rightarrow V_1 = \begin{bmatrix}1\\ 1+\sqrt{2}\end{bmatrix}$

$(A - \lambda_2 I)\cdot V_2 = 0 \iff \begin{bmatrix}-1+\sqrt{2} & 1\\ 1 & 1+\sqrt{2}\end{bmatrix}\begin{bmatrix}V_{21}\\ V_{22}\end{bmatrix} = 0 \Rightarrow V_2 = \begin{bmatrix}1\\ 1-\sqrt{2}\end{bmatrix}$

$x(t) = K_1\begin{bmatrix}1\\ 1+\sqrt{2}\end{bmatrix}e^{\sqrt{2}t} + K_2\begin{bmatrix}1\\ 1-\sqrt{2}\end{bmatrix}e^{-\sqrt{2}t}$

$x(\infty) = \begin{bmatrix}0\\ 0\end{bmatrix} \Rightarrow K_1 = 0$

Logo

$x(0) = K_2\begin{bmatrix}1\\ 1-\sqrt{2}\end{bmatrix} \rightarrow$ ~~se~~ $x(0) = (K_2, K_2(1-\sqrt{2})) \Rightarrow x(\infty) \to 0$

c) ① − Instável

② $A - \lambda I = \begin{bmatrix}-1-\lambda & 1\\ -1 & -1-\lambda\end{bmatrix}$ $\quad \det(A-\lambda I) = (-1-\lambda)^2 + 1$

$(-1-\lambda)^2 + 1 = 0 \iff -1-\lambda = \pm j \iff \lambda = -(1 \pm j) \Rightarrow$ Estável

P3)

a)

$$C_V \frac{dT_V}{dt} = q$$

$$q = \frac{1}{R}(T_L - T_V) - \frac{1}{R}(T_V - T_m)$$

Vidro: $$C_V \frac{dT_V}{dt} = \frac{1}{R}(T_L - T_V) - \frac{1}{R}(T_V - T_m)$$

Mercúrio: $$C_m \frac{dT_m}{dt} = \frac{1}{R}(T_V - T_m)$$

b)

$$\dot{T}_V = \frac{1}{RC_V}T_L - \frac{2}{RC_V}T_V + \frac{1}{RC_V}T_m$$

$$\dot{T}_m = \frac{1}{RC_m}T_V - \frac{1}{RC_m}T_m$$

$$\begin{bmatrix} \dot{T}_V \\ \dot{T}_m \end{bmatrix} = \begin{bmatrix} -\frac{2}{RC_V} & \frac{1}{RC_V} \\ \frac{1}{RC_m} & -\frac{1}{RC_m} \end{bmatrix} \begin{bmatrix} T_V \\ T_m \end{bmatrix} + \begin{bmatrix} \frac{1}{RC_V} \\ 0 \end{bmatrix} T_L$$

$$\left(-\frac{2}{RC_V} - \lambda\right)\left(-\frac{1}{RC_m} - \lambda\right) - \frac{1}{R^2 C_V C_m} = 0 \Rightarrow A\lambda^2 + B\lambda^2 + C = 0$$

Neste caso $A>0$, $B>0$, $C>0$ $\Rightarrow$ $\mathrm{Re}\{\lambda\} < 0$ Logo o sistema é estável

T_m tende para um valor!

P4) A

$x = a\cos(\omega t) + b\sin(\theta)$ $\qquad \dot{x} = -a\omega\sin(\omega t) + b\dot{\theta}\cos(\theta)$

$y = a\sin(\omega t) - b\sin(\theta)$ $\qquad \dot{y} = a\omega\cos(\omega t) + b\dot{\theta}\sin(\theta)$

$$T = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2)$$

$$U = mgy$$

$$L = T - U = \frac{1}{2}m\left(\left[-a\omega\sin(\omega t) + b\dot{\theta}\cos(\theta)\right]^2 + \left[a\omega\cos(\omega t) + b\dot{\theta}\sin(\theta)\right]^2\right) - mg\left(a\sin(\omega t) - b\sin(\theta)\right)$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) = \frac{\partial L}{\partial \theta}$$

$$\frac{\partial L}{\partial \dot{\theta}} = \frac{1}{2}m\left[2b\cos(\theta)\left(-a\omega\sin(\omega t) + b\dot{\theta}\cos(\theta)\right) + 2b\sin(\theta)\left(a\omega\cos(\omega t) + b\dot{\theta}\sin(\theta)\right)\right]$$

$$= m\left(-ab\omega\sin(\omega t)\cos(\theta) + b^2\cos^2(\theta)\dot{\theta} + ab\omega\cos(\omega t)\sin(\theta) + b^2\sin^2(\theta)\dot{\theta}\right) =$$

$$= mb^2\dot{\theta} + mab\omega\left(\sin(\theta)\cos(\omega t) - \sin(\omega t)\cos(\theta)\right)$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{\theta}} = mb^2\ddot{\theta} + mab\omega\left(\dot{\theta}\cos(\theta)\cos(\omega t) - \omega\sin(\theta)\sin(\omega t) - \omega\cos(\omega t)\cos(\theta) + \dot{\theta}\sin(\omega t)\sin(\theta)\right) =$$

$$= mb^2\ddot{\theta} + mab\omega(\dot{\theta} - \omega)\left(\cos(\theta)\cos(\omega t) + \sin(\omega t)\sin(\theta)\right)$$

$$\frac{\partial L}{\partial \theta} = \frac{1}{2}m\left[-2b\dot{\theta}\left(b\dot{\theta}\cos(\theta) - a\omega\sin(\omega t)\right)\sin(\theta) + 2b\dot{\theta}\left(b\dot{\theta}\sin(\theta) + a\omega\cos(\omega t)\right)\cos(\theta)\right] + mgb\cos(\theta) =$$

$$= mb\dot{\theta}\left[a\omega\sin(\omega t)\sin(\theta) + a\omega\cos(\omega t)\cos(\theta)\right] + mgb\cos(\theta) =$$

$$= ma\omega b\dot{\theta}\left(\sin(\omega t)\sin(\theta) + \cos(\omega t)\cos(\theta)\right) + mgb\cos(\theta) //$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) = \frac{\partial L}{\partial \theta}$$

$$mb^2\ddot{\theta} + mab\omega(\dot{\theta} - \omega)\left(\cos(\theta)\cos(\omega t) + \sin(\omega t)\sin(\theta)\right) = ma\omega b\dot{\theta}\left(\sin(\omega t)\sin(\theta) + \cos(\omega t)\cos(\theta)\right) + mgb\cos(\theta) \Leftrightarrow$$

$$\Leftrightarrow \ddot{\theta} = \frac{\left(mab\omega\dot{\theta} - mab\omega(\dot{\theta} - \omega)\right)}{mb^2} \cdot \left(\cos(\theta)\cos(\omega t) + \sin(\theta)\sin(\omega t)\right) + \frac{mgb}{mb^2}\cos(\theta) \Leftrightarrow$$

$$\Leftrightarrow \ddot{\theta} = \frac{a}{b}\omega^2\cos(\theta - \omega t) - \frac{g}{b}\sin(\theta) //$$

B

$$T = \frac{1}{2}m_2\dot{x}_2^2 + \frac{1}{2}m_1\dot{x}_1^2$$

$$U = \frac{1}{2}Kx_1^2 + \frac{1}{2}K(x_2 - x_1)^2 = Kx_1^2 + \frac{1}{2}Kx_2^2 - Kx_1x_2$$

$$L = T - U = \frac{1}{2}m_2\dot{x}_2^2 + \frac{1}{2}m_1\dot{x}_1^2 + Kx_1x_2 - Kx_1^2 - \frac{1}{2}Kx_2^2$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_1}\right) = \frac{\partial L}{\partial x_1}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_2}\right) = \frac{\partial L}{\partial x_2}$$

$\frac{\partial L}{\partial \dot{x}_1} = m_1\dot{x}_1$ $\qquad \frac{\partial L}{\partial \dot{x}_2} = m_2\dot{x}_2$

$\frac{\partial L}{\partial x_1} = Kx_2 - 2Kx_1$ $\qquad \frac{\partial L}{\partial x_2} = Kx_1 - Kx_2$

$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_1}\right) = m_1\ddot{x}_1$ $\qquad \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_2}\right) = m_2\ddot{x}_2$

$$m_1\ddot{x}_1 = Kx_2 - 2Kx_1 \Leftrightarrow \ddot{x}_1 = \frac{K}{m_1}x_2 - \frac{2K}{m_1}x_1$$

$$m_2\ddot{x}_2 = Kx_1 - Kx_2 \Leftrightarrow \ddot{x}_2 = \frac{K}{m_2}(x_1 - x_2)$$