P1)

a) $$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \\ x_3(k+1) \end{bmatrix} = \begin{bmatrix} \beta_1 & 0 & 0 \\ 1-\beta_1 & \beta_2 & 0 \\ 0 & 1-\beta_2 & \beta_3 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \mu(k)$$

b) $x_1(\infty) = \beta x_1(\infty) + \mu \iff x_1(\infty) = \frac{1}{1-\beta}\mu = 105.3$

$x_2(\infty) = (1-\beta)x_1(\infty) + \beta x_2(\infty) \iff x_2(\infty) = x_1(\infty) = \frac{1}{1-\beta}\mu = 105.3$

$x_3(\infty) = x_2(\infty)(1-\beta) + \beta x_3(\infty) \iff x_3(\infty) = x_2(\infty) = x_1(\infty) = \frac{1}{1-\beta}\mu = 105.3$

P2) a) $\begin{cases} 0 = s - d x_1 - \alpha x_1 x_2 \\ 0 = \alpha x_1 x_2 - \mu x_2 \end{cases} \iff \begin{cases} - \\ 0 = x_2(\alpha x_1 - \mu) \end{cases}$

$x_2 = 0 \wedge x_1 = \frac{s}{d} \rightarrow (500;0) \rightarrow$ Sela ①

$x_1 = \frac{\mu}{\alpha} \wedge x_2 = \frac{s}{\mu} - \frac{d}{\alpha} \rightarrow (240;21{,}67) \rightarrow$ Doente ②

b) $\frac{\partial f_1}{\partial x_1} = -d - \alpha x_2 \qquad \frac{\partial f_2}{\partial x_1} = \alpha x_2$

$\frac{\partial f_1}{\partial x_2} = -\alpha x_1 \qquad \frac{\partial f_2}{\partial x_2} = \alpha x_1 - \mu$

$A = \begin{bmatrix} -d - \alpha x_2 & -\alpha x_1 \\ \alpha x_2 & \alpha x_1 - \mu \end{bmatrix}$

① $\begin{bmatrix} -0.02 & -0.5 \\ 0 & 0.26 \end{bmatrix}$ $(-0.02-\lambda)(0.26-\lambda) = 0 \iff \lambda^2 - 0.24\lambda - 0.02 \cdot 0.26 = 0$

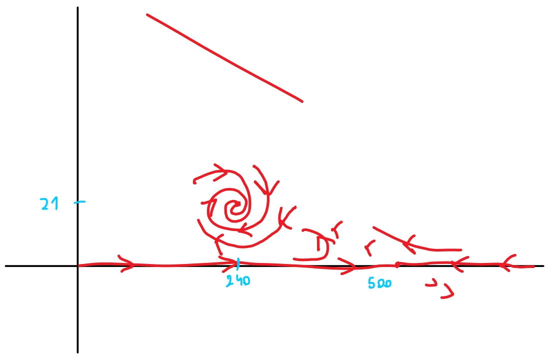
$\lambda = \frac{0.24 \pm 0.28}{2} \iff \lambda_1 = -0.02,\ \lambda_2 = 0.26$ Ponto de Sela, Instável

$V_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad V_2 = \begin{bmatrix} 1 \\ -0.56 \end{bmatrix}$

② $\begin{bmatrix} -0.042 & -0.24 \\ 0.022 & 0 \end{bmatrix}$ $(-0.042-\lambda)(-\lambda) + 0.24 \cdot 0.022 = 0 \iff \lambda^2 + 0.042\lambda + 0.0053 = 0$

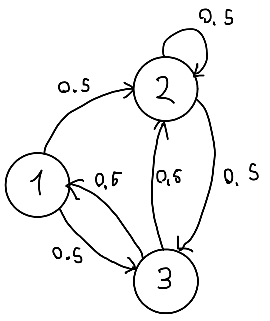
$Re\{\lambda\} > 0$ e $Im\{\lambda\} \neq 0 \Rightarrow$ Foco estável

c)



P3)

a)



$A = \begin{bmatrix} 0 & 0.5 & 0.5 \\ 0 & 0.5 & 0.5 \\ 0.5 & 0.5 & 0 \end{bmatrix}$

$P(k+1) = A^T P \iff \begin{bmatrix} P_1(k+1) \\ P_2(k+1) \\ P_3(k+1) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0.5 \\ 0.5 & 0.5 & 0.5 \\ 0.5 & 0.5 & 0 \end{bmatrix} \begin{bmatrix} P_1(k) \\ P_2(k) \\ P_3(k) \end{bmatrix}$

b) $P_1(0) = 1, \; P_2(0) = 0, \; P_3(0) = 0 \Rightarrow P(1) = A^T P(0) \Rightarrow P_1(1) = 0, \; P_2(1) = 0.5, \; P_3(1) = 0.5 \Rightarrow P(2) = A^T P(1) \Rightarrow P_1(2) = 0.25, \; P_2(2) = 0.5, \; P_3(2) = 0.25$

c) $\begin{cases} P = A^T P \\ P_1 + P_2 + P_3 = 1 \end{cases} \iff \begin{cases} P_1 = 0.5 P_3 \\ P_2 = 0.5P_1 + 0.5P_2 + 0.5P_3 \\ P_3 = 0.5P_1 + 0.5P_2 \\ P_3 = 1 - P_1 - P_2 \end{cases} \iff \begin{cases} P_1 = 0.5 - 0.5P_1 - 0.5P_2 \\ P_2 = 0.5P_1 + 0.5P_2 + 0.5 - 0.5P_1 - 0.5P_2 \\ 1 - P_1 - P_2 = 0.5P_1 + 0.5P_2 \\ P_3 = 1 - P_1 - P_2 \end{cases} \iff \begin{cases} P_1 = 1/6 \\ P_2 = 1/2 \\ P_3 = 1/3 \end{cases}$

P4)

① $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ $\lambda_1 = 1, \; \lambda_2 = -1 \Rightarrow$ Ponto de Sela

$V_1 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}, \; V_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \Rightarrow$ B

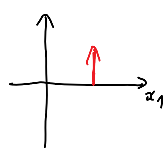
② $A = \begin{bmatrix} 0 & 1 \\ -2 & -0.6 \end{bmatrix}$ $\lambda = -3 \pm 1{,}38j$

foco estável

C

③ $A = \begin{bmatrix} 0 & -1 \\ 2 & 0 \end{bmatrix}$ $\lambda_1 = j\sqrt{2}, \; \lambda_2 = -j\sqrt{2} \Rightarrow$ Centro

$\dot{x}_2 = 2x_1 \Rightarrow \frac{\partial \dot{x}_2}{\partial x_1} = 2, \; \frac{\partial \dot{x}_2}{\partial x_2} = 0$



D

④ $A = \begin{bmatrix} -\frac{5}{3} & -\frac{4}{3} \\ \frac{4}{3} & \frac{5}{3} \end{bmatrix}$ $\lambda_1 = -1, \; \lambda_2 = 1 \Rightarrow$ Ponto de sela

A

⑤ $A = \begin{bmatrix} 0 & 1 \\ -2 & 0 \end{bmatrix}$ $\lambda = \pm j\sqrt{2} \Rightarrow$ Centro

$\dot{x}_2 = -2x_1 \Rightarrow \frac{\partial \dot{x}_2}{\partial x_1} = -2 \Rightarrow$ F

⑥ $A = \begin{bmatrix} 0 & -1 \\ 2 & -0.6 \end{bmatrix}$ E

P5)

$\downarrow \bullet M_T(t)$

$$\begin{cases} \frac{d}{dt}\left[(M+m(t))\,\dot{h}\right] = -(M+m(t))\,g_L + Ku(t) \iff \dot{m}\dot{h} + (M+m)\ddot{h} = -(M+m)\,g_L + Ku \\ \frac{dm(t)}{dt} = -u(t) \end{cases}$$

$x_1 = h$

$x_2 = \dot{h} = \dot{x}_1$

$x_3 = m$

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = \frac{1}{M+x_3}\left[-(M+x_3)\,g_L + Ku - u x_2\right] \\ \dot{x}_3 = u \end{cases}$$