P1)

a) $m\ddot{y} = -K(y-u) - \beta(\dot{y}-\dot{u})$

b) $m\ddot{y} + Ky + \beta\dot{y} = Ku + \beta\dot{u}$

$m s^2 Y + \beta s Y + K Y = K U + \beta s U$

$G = \frac{Y}{U} = \frac{\beta s + K}{m s^2 + \beta s + K}$

c) $m\ddot{y} + Ky + \beta\dot{y} = Ku + \beta\dot{u}$

$x_1 = y$

$x_2 = \dot{y} = \dot{x}_1$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{K}{m} & -\frac{\beta}{m} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} u$$

$$y = \begin{bmatrix} K & \beta \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

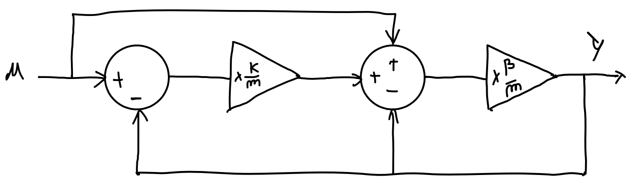
d) $\ddot{y} = \frac{K}{m}u - \frac{K}{m}y + \frac{\beta}{m}\dot{u} - \frac{\beta}{m}\dot{y}$

$\int$

$\dot{y} = \frac{\beta}{m}u - \frac{\beta}{m}y + \int\left(\frac{K}{m}u - \frac{K}{m}y\right)$

$\int$

$y = \int \frac{\beta}{m}u - \frac{\beta}{m}y + \int \frac{K}{m}u - \frac{K}{m}y \Rightarrow \frac{\beta}{m}\int\left(u - y + \frac{K}{m}\int(u-y)\right)$



P2) a) $\begin{cases} 0 = x_2 \\ 0 = -\beta x_2 - K_1 x_1 + K_2 x_1^3 \end{cases}$ $\begin{cases} x_2 = 0 \\ 0 = x_1(K_2 x_1^2 - K_1) \end{cases}$ $\begin{cases} x_2 = 0 \\ x_1 = \pm\sqrt{\frac{K_1}{K_2}} \end{cases}$

① $x_1 = 0 \wedge x_2 = 0$

② $x_1 = 2 \wedge x_2 = 0$

③ $x_1 = -2 \wedge x_2 = 0$

b) $\frac{\partial f_1}{\partial x_1} = 0$ $\quad \frac{\partial f_1}{\partial x_2} = 1$

$\frac{\partial f_2}{\partial x_1} = -K_1 + 3K_2 x_1^2$ $\quad \frac{\partial f_2}{\partial x_2} = -\beta$

$$\begin{bmatrix} 0 & 1 \\ -1+0.75x_1^2 & -0.1 \end{bmatrix}$$

① $\begin{bmatrix} 0 & 1 \\ -1 & -0.1 \end{bmatrix}$

② e ③ $\begin{bmatrix} 0 & 1 \\ 2 & -0.1 \end{bmatrix}$

c) ①

$A = \begin{bmatrix} 0 & 1 \\ -1 & -0.1 \end{bmatrix}$ $\quad A - \lambda I = \begin{bmatrix} -\lambda & 1 \\ -1 & -0.1-\lambda \end{bmatrix}$

$\det(A - \lambda I) = (-\lambda)(-0.1-\lambda) + 1 = 0.1\lambda + \lambda^2 + 1 =$

$= \lambda^2 + 0.1\lambda + 1$

$\lambda^2 + 0.1\lambda + 1 = 0 \Leftrightarrow \lambda = \frac{-0.1 \pm \sqrt{-3.99}}{2} \Leftrightarrow \begin{cases} \lambda_1 \approx e^{j92°} \\ \lambda_2 \approx e^{-j92°} \end{cases}$

② / ③

$A = \begin{bmatrix} 0 & 1 \\ 2 & -0.1 \end{bmatrix}$ $\quad A - \lambda I = \begin{bmatrix} -\lambda & 1 \\ 2 & -0.1-\lambda \end{bmatrix}$

$\det(A - \lambda I) = 0.1\lambda + \lambda^2 - 2$

$\lambda^2 + 0.1\lambda - 2 = 0 \Leftrightarrow \lambda = \frac{-0.1 \pm \sqrt{0.1^2 + 8}}{2} \Leftrightarrow \begin{cases} \lambda_1 \approx -1.47 \\ \lambda_2 \approx 1.36 \end{cases}$

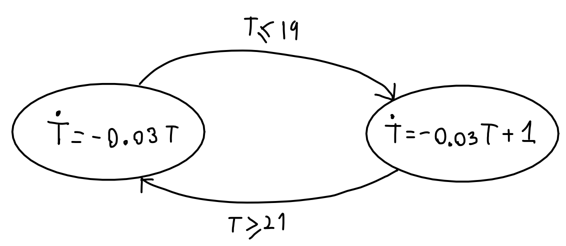
d) Em ① temos $Re\{\lambda_1\} < 0$ e $Re\{\lambda_2\} < 0$, ou seja é um foco estável

Em ② e ③ temos $Im\{\lambda_1\} = Im\{\lambda_2\} = 0$, $Re\{\lambda_1\} < 0$ e $Re\{\lambda_2\} > 0$, ou seja é um ponto de sela, e por isso instável!

P3) a) $\dot{T} = -0.03T$

b) $\dot{T} = -0.03T + 1$

c)



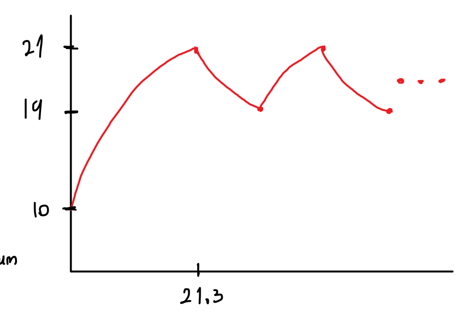
d) $T(0) = 10°C$

$\dot{T} = -0.03T + 1$

$T(t) = p \cdot e^{-0.03t} - 33.3[e^{-0.03t} - 1] =$

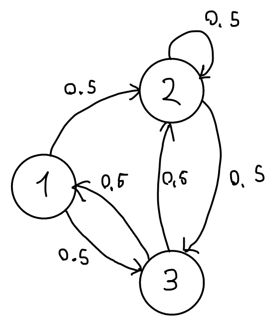
$= -23.3e^{-0.03t} + 33.3$

$T(t) = 21°C \Leftrightarrow \ln\left(\frac{21 - 33.3}{-23.3}\right) = -0.03t \Leftrightarrow t = 21.3$ min



P4)

a)



$$A = \begin{bmatrix} 0 & 0.5 & 0.5 \\ 0 & 0.5 & 0.5 \\ 0.5 & 0.5 & 0 \end{bmatrix}$$

$$P(k+1) = A^T P \iff \begin{bmatrix} P_1(k+1) \\ P_2(k+1) \\ P_3(k+1) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0.5 \\ 0.5 & 0.5 & 0.5 \\ 0.5 & 0.5 & 0 \end{bmatrix} \begin{bmatrix} P_1(k) \\ P_2(k) \\ P_3(k) \end{bmatrix}$$

b)

$$\begin{matrix} P_1(0) = 1 \\ P_2(0) = 0 \\ P_3(0) = 0 \end{matrix} \Rightarrow P(1) = A^T P(0) \Rightarrow \begin{matrix} P_1(1) = 0 \\ P_2(1) = 0.5 \\ P_3(1) = 0.5 \end{matrix} \Rightarrow P(2) = A^T P(1) \Rightarrow \begin{matrix} P_1(2) = 0.25 \\ P_2(2) = 0.5 \\ P_3(2) = 0.25 \end{matrix}$$

c)

$$\begin{cases} P = A^T P \\ P_1 + P_2 + P_3 = 1 \end{cases} \iff \begin{cases} P_1 = 0.5 P_3 \\ P_2 = 0.5 P_1 + 0.5 P_2 + 0.5 P_3 \\ P_3 = 0.5 P_1 + 0.5 P_2 \\ P_3 = 1 - P_1 - P_2 \end{cases} \iff \begin{cases} P_1 = 0.5 - 0.5 P_1 - 0.5 P_2 \\ P_2 = 0.5 P_1 + 0.5 P_2 + 0.5 - 0.5 P_1 - 0.5 P_2 \\ 1 - P_1 - P_2 = 0.5 P_1 + 0.5 P_2 \\ P_3 = 1 - P_1 - P_2 \end{cases} \iff \begin{cases} P_1 = 1/6 \\ P_2 = 1/2 \\ P_3 = 1/3 \end{cases}$$