1.

t $\quad$ $t+\Delta t$

$f_0(t)$ $\quad$ $f_0(t+\Delta t)$

$f_0(t+\Delta t) = f_0(t) +$ canais que abriram em Δt $-$ canais que fecharam em Δt $\quad$ e $f_c = 1 - f_0$

$$f_0(t+\Delta t) = f_0(t) + K^+ f_c(t)\Delta t - K^- f_0(t)\Delta t$$

$$\frac{f_0(t+\Delta t) - f_0(t)}{\Delta t} = K^+(1-f_0) - K^- f_0$$

$\Delta t \to 0$

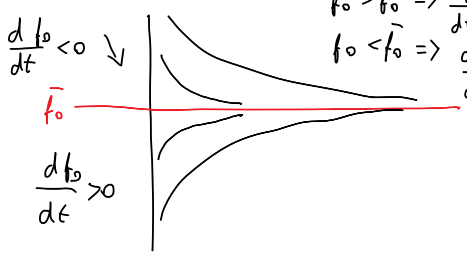
$$\frac{df_0}{dt} = -(K^+ + K^-) f_0 + K^+$$

b) Ponto de equilíbrio

$$\frac{df_0}{dt} = 0 \Rightarrow -(K^+ + K^-) f_0 + K^+ = 0 \iff \bar{f_0} = \frac{K^+}{K^+ + K^-}$$

$f_0 > \bar{f_0} \Rightarrow \frac{df_0}{dt} < 0$

$f_0 < \bar{f_0} \Rightarrow \frac{df_0}{dt} > 0$



$$\frac{d^2}{dt^2} f_0 = -(K^+ + K^-)\frac{df_0}{dt}$$

tem sempre sinal oposto a $\frac{df_0}{dt}$

Para $f_0 > \bar{f_0}$ – $\frac{d^2 f_0}{dt^2} > 0 \to \cup$

Para $f_0 < \bar{f_0}$ – $\frac{d^2 f_0}{dt^2} < 0 \to \cap$

2.

a) t $\quad$ $\Delta t + t$

$$x(t+\Delta t) = x(t) + \Delta t\, x(t)(N - x(t))K$$

$$\frac{dx}{dt} = Kx(t)(N - x(t))$$

b) $\triangle$ Ponto Eq.

$\square$ 1ª D.

$\pentagon$ 2ª D.

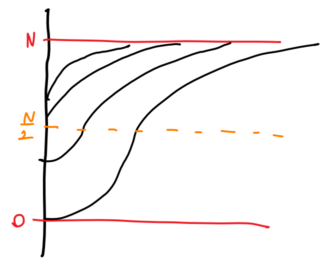
$\triangle$ – $\frac{dx}{dt} = 0 \iff Kx(t)(N - x(t)) = 0$

$x(t) = 0 \vee x(t) = N$

$\square$

		0		N	
x	ND		$\nearrow$		ND
$\frac{dx}{dt}$	ND	0	+	0	ND

Não temos agricultores negativos e não temos mais de N agricultores



$\pentagon$ $\frac{d^2x}{dt^2} = \frac{d}{dt} Kx(t)(N - x(t)) = K\left(N\frac{dx}{dt} - 2x\frac{dx}{dt}\right) = K^2 x(N-2x)(N-x)$

$\frac{d^2x}{dt^2} = 0$ $\begin{cases} x = \frac{N}{2} \\ x = N \\ x = 0 \end{cases}$

		0		$\frac{N}{2}$		N	
x	ND		$\nearrow$ ⋮ $\cup$		$\nearrow$ ⋮ $\cap$		ND
$\frac{dx}{dt}$	ND	0	+		+	0	ND
$\frac{d^2x}{dt^2}$	ND	0	+	0	−	0	ND

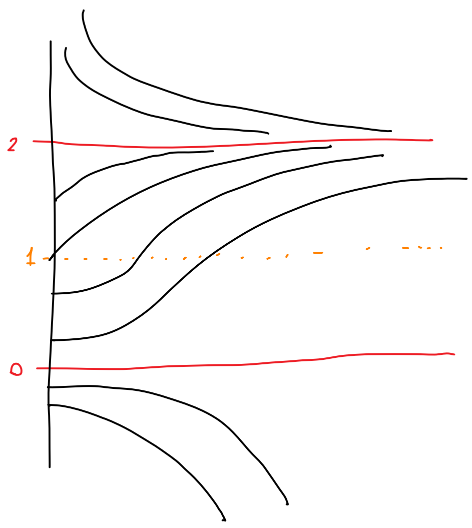
P3

a) $\frac{dx}{dt} = x(2-x)$ $\qquad$ $\frac{dx}{dt} = 0 \iff x(2-x) = 0 \to x = 0 \vee x = 2$

$$\frac{d^2x}{dt^2} = \frac{d}{dt}\left(x(2-x)\right) = \frac{dx}{dt}(2-x) - x\frac{dx}{dt} = x(2-x)(2-2x) \Rightarrow 2x(2-x)(1-x) = \frac{d^2x}{dt^2}$$

$x = 0$
$x = 2$
$x = 1$

		0		1		2	
x	$\searrow$ ⋮ $\cap$		$\nearrow$ ⋮ $\cup$		$\nearrow$ ⋮ $\cap$		$\searrow$ ⋮ $\cup$
$\frac{dx}{dt}$	−	0	+		+	0	−
$\frac{d^2x}{dt^2}$	−	0	+	0	−	0	+



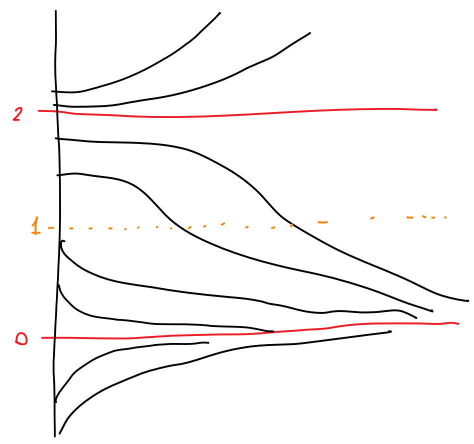
b) $\frac{dx}{dt} = -x(2-x)$

$\frac{dx}{dt} = 0 \iff -x(2-x) = 0 \rightarrow | \; x = 0 \vee x = 2$

$\frac{d^2x}{dt^2} = -\frac{dx}{dt}(2-x) + x\frac{dx}{dt} = \frac{dx}{dt}(2x-2) = -x(2-x)(2x-2) = x(2-x)(1-x)$

$x = 0$
$x = 1$
$x = 2$

		0		1		2	
x	↗ ⋮ ∩		↘ ⋮ ∪		↘ ⋮ ∩		↗ ⋮ ∪
$\frac{dx}{dt}$	+	0	−		−	0	+
$\frac{d^2x}{dt^2}$	−	0	+	0	−	0	+



c) $\frac{dx}{dt} = x(2-x)(4-x)$

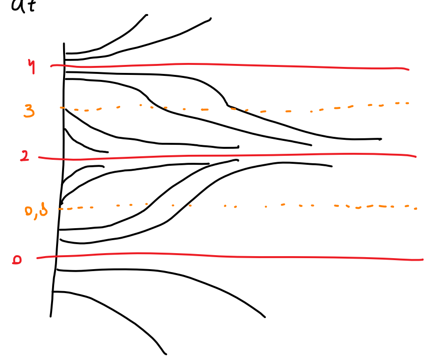
$\frac{dx}{dt} = 0 \iff x(2-x)(4-x) = 0 \rightarrow \left| \begin{array}{l} x = 0 \\ x = 2 \\ x = 4 \end{array} \right.$

$\frac{d^2x}{dt^2} = \frac{dx}{dt}(2-x)(4-x) - \frac{dx}{dt}x(4-x) - \frac{dx}{dt}x(2-x) =$

$= \frac{dx}{dt}\left[3x^2 - 12x + 8\right]$

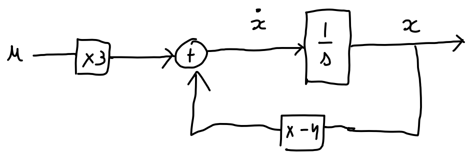
$\frac{dx}{dt}(3x^2 - 12x + 8) = \frac{d^2x}{dt^2} \Rightarrow \left| \begin{array}{l} x = 0 \\ x = 2 \\ x = 4 \\ x \approx 0.8 \\ x \approx 3 \end{array} \right.$

		0		0.8		2		3		4	
x	↘ ⋮ ∩		↗ ⋮ ∪		↗ ⋮ ∩		↘ ⋮ ∪		↘ ⋮ ∩		↗ ⋮ ∪
$\frac{dx}{dt}$	−	0	+		+	0	−		−	0	+
$\frac{d^2x}{dt^2}$	−	0	+	0	−	0	+	0	−	0	+



PS.

a) $\frac{dx}{dt} = -4x + 3u$



b)

$\dot{x}_1 = x_2$

$\dot{x}_2 = -x_1 - 0.5x_2 + u$