1)

$G(s) = C(sI-A)^{-1}B$

$$\begin{bmatrix}1 & 2\end{bmatrix} \cdot \left(\begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -5 & -1 \\ 3 & -1 \end{bmatrix} \right)^{-1} \cdot \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$sI - A = \begin{bmatrix} s+5 & 1 \\ -3 & s+1 \end{bmatrix} \qquad \det(sI-A) = (s+5)(s+1)+3 = s^2+6s+8$$

$$\operatorname{adj}(sI-A) = \begin{bmatrix} s+1 & -1 \\ 3 & s+5 \end{bmatrix}$$

$$C \operatorname{adj}(sI-A)\cdot B = \begin{bmatrix}1 & 2\end{bmatrix} \begin{bmatrix} s+1 & -1 \\ 3 & s+5 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix}1 & 2\end{bmatrix} \begin{bmatrix} s \\ s+8 \end{bmatrix} = 3s+16$$

$$G(s) = \frac{C \operatorname{adj}(sI-A)\cdot B}{\det(sI-A)} = \frac{3s+16}{s^2+6s+8} \,//$$

2)

$$\dddot{y} + 9\ddot{y} + 26\dot{y} + 24y = 24u$$

$x_1 = y$

$x_2 = \dot{y} = \dot{x}_1$

$x_3 = \ddot{y} = \dot{x}_2$

$$\Rightarrow \begin{cases} \dot{x}_3 + 9x_3 + 26x_2 + 24x_1 = 24u \\ x_2 = \dot{x}_1 \\ x_3 = \dot{x}_2 \end{cases}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -24 & -26 & -9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 24 \end{bmatrix} u$$

$$x_1 = y \Rightarrow y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$[Z] = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix} [x]$$

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix} A \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}^{-1} \cdot \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} + \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 24 \end{bmatrix} u \Rightarrow \begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \end{bmatrix} = \begin{bmatrix} 0 & 2 & 0 \\ 0 & 0 & 1/3 \\ -36 & -78 & -9 \end{bmatrix} \cdot \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 72 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}^{-1} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} \qquad y = \begin{bmatrix} \frac{1}{2} & 0 & 0 \end{bmatrix} [Z]$$

c)

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -24 & -26 & -9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 24 \end{bmatrix} u$$ | Dinamico mantem

$$x_1 = y \Rightarrow y = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$ | não muda a saída

d) Mesma história do Z ...

P3.)

$$\frac{2}{(s+1)(s+2)} - (s+3)$$

s^2+3s+2

$\ddot{y} + 3\dot{y} + 2y = 2u$

$x_1 = y$

$x_2 = \dot{y} = \dot{x}_1$

$$\begin{cases} \dot{x}_2 + 3x_2 + 2x_1 = 2u \\ x_2 = \dot{x}_1 \end{cases}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 2 \end{bmatrix} u$$

$$y = \begin{bmatrix} 3 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

P4)

$$\dddot{y} + 4\ddot{y} + 5\dot{y} = \ddot{u} + 3\dot{u} + 2u \qquad \dot{x}_3 + 4x_3 + 5x_2 = 1$$

$y = x_1$ $\qquad x_2 = \dot{x}_1$

$\dot{y} = x_2 = \dot{x}_1$ $\qquad x_3 = \dot{x}_2$

$\ddot{y} = x_3 = \dot{x}_2$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -5 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

agora coloco o $\ddot{u} + 3\dot{u} + 2u \longrightarrow y = \begin{bmatrix} 2 & 3 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

$y = 2x_1 + 3x_2 + x_3$

$y = 2y + 3\dot{y} + \ddot{y}$

P5)

$s^4 + 9s^3 + 25s + 26 = 1$

$\ddddot{y} + 9\dddot{y} + 25\dot{y} + 26y = 1$

$y = x_1$

$\dot{y} = \dot{x}_1 = x_2$

$\ddot{y} = \dot{x}_2 = x_3$

$\dddot{y} = \dot{x}_3 = x_4$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -9 & 0 & -25 & -26 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 3 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

P6) $(sI-A) = \begin{bmatrix} s & -1 & 0 \\ 0 & s & -1 \\ 1 & 2 & s+3 \end{bmatrix}$

$C\,adj(sI-A)\,B = 10s^2 + 30s + 20$

$det(sI-A) = s^3 + 3s^2 + 2s + 1$

$adj(sI-A) = det(a) \cdot a^{-1} = \begin{bmatrix} s^2+3s+2 & s+3 & 1 \\ -1 & s^2+3s & s \\ -s & -2s-1 & s^2 \end{bmatrix}$

$G(s) = \dfrac{10s^2+30s+20}{s^3+3s^2+2s+1} = \dfrac{10(s^2+3s+2)}{s^3+3s^2+2s+1}$

P7)

$x_2(s+1) = 1$

$\dot{x}_2 = 1 - x_2$

$x_2 = \frac{1}{s+1} \cdot u$

$x_1 = \frac{\alpha}{s} x_2$

$y = x_2 + x_1$

$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & \alpha \\ 0 & -1 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$

$y = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

P8) $(sI-A) = \begin{bmatrix} s & -1 \\ \alpha & s-1 \end{bmatrix}$

$det(sI-A) = \alpha + s^2 - s$

$adj(sI-A) = det(sI-A)(sI-A)^{-1} = \begin{bmatrix} s-1 & 1 \\ -\alpha & s \end{bmatrix}$

$G(s) = \dfrac{det(sI-A)}{C\,adj(sI-A)\,B} = \dfrac{\beta(1+s\gamma)}{s^2 - s + \alpha}$

$C\,adj(sI-A)\,B = \beta + \beta s \gamma$

P9)

$x_2 = \frac{\alpha-1}{s+1} u \Rightarrow x_2 s + x_2 = (\alpha-1)u$

$\dot{x}_2 = (\alpha-1)u - x_2$

$\dot{x}_1 = u + x_2$

$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ \alpha-1 \end{bmatrix} u$

$x_1 = \frac{1}{s}\left(u \frac{\alpha-1}{s+1} + u \right) \Leftrightarrow$

$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

$\dfrac{Y}{U} = \dfrac{1}{s}\left(\dfrac{\alpha-1+s+1}{s+1} \right) = \dfrac{\alpha+s}{s(s+1)}$

P10)

$u \cdot \frac{\alpha-1}{s+1} = x_2$

$\dot{x}_1 = x_2$

$\Leftrightarrow$

$u \cdot (\alpha-1) - x_2 = \dot{x}_2$

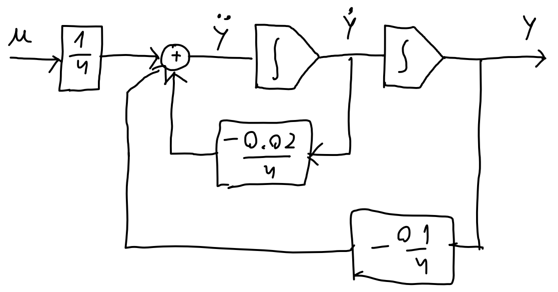
$\dot{x}_1 = x_2$

$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \alpha-1 \end{bmatrix} u$

$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

P11) $4\ddot{y} = -0.1y - 0.02\dot{y} + u$

$\ddot{y} = \frac{u}{4} - \frac{0.02}{4}\dot{y} - \frac{0.1}{4} y$



P12) $y(0) = y(1) = 1$

$k=2 - y(2) = y(1) + y(0) = 2$

$k=3 - y(3) = y(2) + y(1) = 3$

$k=4 - y(4) = y(3) + y(2) = 5$

$k=5 - y(5) = y(4) + y(3) = 8$

$k=6 - y(6) = y(5) + y(4) = 13$

P13) $1 - \frac{1}{2}z^{-1} + z^{-2} = 2z^{-10} + z^{-11}$

$\hookrightarrow z^{11} - \frac{1}{2}z^{10} + z^9 = 2z + 1$

$\dfrac{2z+1}{z^{11} - \frac{1}{2}z^{10} + z^9}$