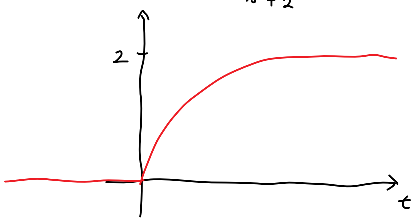
P1

a) $G_1(s) = \frac{4}{s+2}$




$G_1(0) = 2 = \lim_{t\to\infty} f(t)$

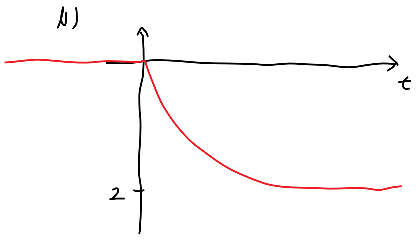
$\lim_{t\to 0} f(t) = \lim_{s\to\infty} \frac{4}{s+2} = 0$

$\lim_{t\to 0} \dot f(t) = \lim_{s\to\infty} s\,\frac{4}{s+2} = 4$

Nº pólos > Nº zeros → resposta contínua

Nº pólos = nº zeros +1 → derivada descontínua/finita

b)

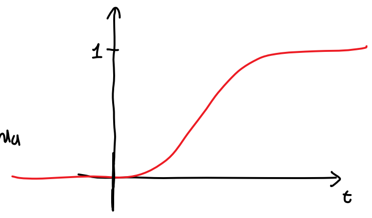



$G_3(s) = -\frac{4}{s+2}$  $G_1(0) = -2$

Pólo real estável

Nº pólos > Nº zeros ⇒ Resposta contínua

Nº pólos > Nº zeros +1 ⇒ derivada contínua




$\lim_{t\to\infty} y(t) = G(0) = 1$

$\lim_{t\to 0} y(t) = \lim_{s\to+\infty} G(s) = 0$

$\lim_{t\to 0} \dot y(t) = \lim_{s\to+\infty} s\,G(s) = 0$

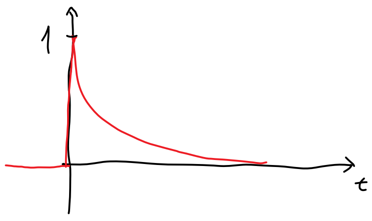
2 Pólos reais ⇒ estável

d)

$G(0) = 0$  A resposta é descontínua

$\lim_{s\to\infty} \frac{s}{s+2} = 1$  A derivada é infinita

$\lim_{s\to+\infty} = 1$

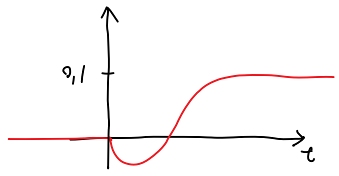



e) $G_5(s) = \frac{1-s}{(s+2)(s+5)}$  derivada descontínua

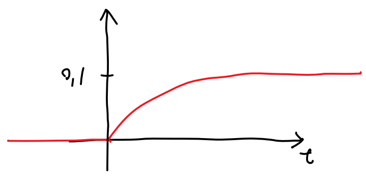
$G(0) = \frac{1}{10}$

$\lim_{s\to\infty} G(s) = 0$

$\lim_{s\to\infty} s\,G(s) = \left.\frac{-1+\frac{1}{s}}{(1+\frac{2}{s})(1+\frac{5}{s})}\right|_{s=\infty} = -1$




f)




P2

a) $G(0) = -1$

b)

pólos ⇒ $p_1 = -3, \dots$ ; $p_2 = 0,7\dots$ ⇒ sistema instável! ⇒ → $-\infty$

P3

a) $c_2^{\gamma}(t) = c_{50}^{\gamma}$

$r(t) = \frac{100\, c_{50}}{c_{50} + c_{50}} = 50\%$

b)

$\bar{r} = 100 \frac{c_{50}}{c_{50} + c_e}$

$c_e = \left(100 \frac{c_{50}^{\gamma}}{\bar{r}} - c_{50}^{\gamma}\right)^{\frac{1}{\gamma}}$

$\lim_{s\to 0} \lambda \left(\frac{d_1}{s+\lambda_1} + \frac{d_2}{s+\lambda_2}\right) u = c_e$

$\left(\frac{d_1}{\lambda_1} + \frac{d_2}{\lambda_2}\right) u = c_e$

P4

a) - $G_4$  d) - $G_1$  g) - $G_2$

b) - $G_7$  e) - $G_8$  h) - $G_6$

c) - $G_3$  f) - $G_5$

$G_1 \Rightarrow \omega = 1$  $2\xi\omega = 0,1 \Rightarrow \xi = 0,05$

$G_2 \Rightarrow \omega = 4$  $2\xi\omega = 2 \Rightarrow \xi = 0,25$

$G_3 \Rightarrow \omega = 1$  $2\xi\omega = 0.5 \Rightarrow \xi = 0,25$

$G_5 \Rightarrow \omega = 4$  $2\xi\omega = 0,4 \Rightarrow \xi = 0,05$

$\underbrace{G_4,\ G_6,\ G_8}_{\text{zeros}}$  $G_7 \to 2$ pólos reais!

P5

A - $G_2$  D - $G_3$

B - $G_4$  E - $G_6$

C - $G_5$  F - $G_1$

$G_1$: $\frac{0.25}{s^2 + 0.3s + 0.25}$  $\omega = \sqrt{0.25} = 0.5$  $2\omega\xi = 0.3 \Rightarrow \xi = 0.3$

$G_4$: zero de fase não mínima  derivada na origem $-0.6$

$G_3$: $\frac{1}{s^2 + 0.6s + 1}$  $\omega = 1$  $2\omega\xi = 0.6 \Rightarrow \xi = 0.3$

$G_5$: zero de fase não mínima  derivada na origem $-2$

$G_2$: 3 pólos e 0 zeros ⇒ derivada contínua

$G_6$: não oscila e não tem zeros

P6

$G(0) = 1$ ⇒ $G_1$ X  $G_3$ X  $G_5$ ✓

$G_2$ ✓  $G_4$ ✓

$G_4$: $s^2 + 2s + 1 = 0 \Rightarrow s: \{-1, -1\}$

2 pólos reais X

$\lim_{+\infty} s\,G(s) = 0$ ⇒ $G_2$ ✓  $G_4$ ✓  $G_5$ X

É o $G_2$

P7

$\dot{x} = A\,x(t)$

a) $A v_i = \lambda_i v_i$

$\det(A - \lambda I) = 0$

$\det \begin{bmatrix} -2{,}5-\lambda & 0{,}5 \\ 0{,}5 & -2{,}5-\lambda \end{bmatrix} = (-2{,}5-\lambda)^2 - 0{,}5^2 = 6 + 5\lambda + \lambda^2$

$\lambda^2 + 5\lambda + 6 = 0 \Rightarrow \lambda = \dfrac{-5 \pm \sqrt{25 - 4\times 6}}{2} = \dfrac{-5 \pm 1}{2} = \left| \begin{array}{l} \lambda_1 = -3 \\ \lambda = -2 \end{array}\right.$

$\begin{bmatrix} -2.5 & 0.5 \\ 0.5 & -2.5 \end{bmatrix} \begin{bmatrix} v_{1.1} \\ v_{1.2} \end{bmatrix} = \begin{bmatrix} -3v_{1.1} \\ -3v_{1.2} \end{bmatrix}$ $\quad -2.5v_1 + 0.5v_2 = -3v_1 \quad v_{1,1} = 1$

$0.5v_1 - 2.5v_2 = -3v_2 \quad v_{1,2} = -1$

$\lambda_1 = -3 \quad v_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$\begin{bmatrix} -2.5 & 0.5 \\ 0.5 & -2.5 \end{bmatrix} \begin{bmatrix} v_{2.1} \\ v_{2.2} \end{bmatrix} = \begin{bmatrix} -2v_{2.1} \\ -2v_{2.2} \end{bmatrix}$ $\quad -2.5v_1 + 0.5v_2 = -2v_2 \quad v_{2,1} = 1$, $v_{2,2} = 1$

$\lambda_2 = -2 \quad v_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ $\quad \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = k_1 \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{-3t} + k_2 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{-2t}$ $\quad \begin{bmatrix} 2 \\ 1 \end{bmatrix} = k_1 \begin{bmatrix} 1 \\ -1 \end{bmatrix} + k_2 \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$k_1 + k_2 = 2$, $-k_1 + k_2 = 1 \Rightarrow k_2 = \frac{3}{2}$, $k_1 = \frac{1}{2}$ $\quad \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{-3t} + \frac{3}{2} \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{-2t}$

b) $k_1 \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{-15} + k_2 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{-10} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ $\quad k_1 e^{-15} + k_2 e^{-10} = 2$, $-k_1 e^{-15} + k_2 e^{-10} = 1$ $\quad k_2 = \frac{3}{2} e^{10}$, $k_1 = \frac{1}{2} e^{15}$

$\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \frac{1}{2} e^{15} \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{-3t} + \frac{3}{2} e^{10} \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{-2t}$

8

$A = \begin{bmatrix} -1.25 & 0.75 \\ 2.25 & 0.25 \end{bmatrix}$ $\quad \det\begin{pmatrix} -1.25-\lambda & 0.75 \\ 2.25 & 0.25-\lambda \end{pmatrix} =$

a)

$(-1.25-\lambda)(0.25-\lambda) - 0.75 \cdot 2.25 = 0$ $\quad$ 2 polos reais mas sem planos

$\lambda^2 + \lambda - 2 = 0 \quad \left| \begin{array}{l} \lambda_1 = -2 \\ \lambda_2 = 1 \end{array}\right.$

$-1.25 v_a + 0.75 v_b = -2v_a$

$-1.25 v_a + 0.75 v_b = v_a$

$\lambda_1 = -2 \Rightarrow v_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ $\quad x(0) = \alpha \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

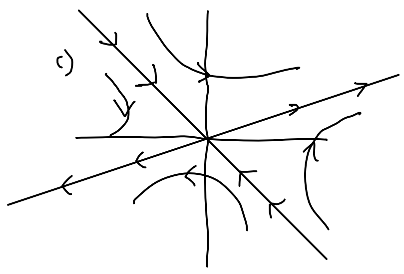
$\lambda_2 = 1 \Rightarrow v_2 = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$

b) $x(t) = k_1 \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{-2t} + k_2 \begin{bmatrix} 1 \\ 3 \end{bmatrix} e^{t}$

$k_1 = 0 \Rightarrow x(t) = k_2 \begin{bmatrix} 1 \\ 3 \end{bmatrix} e^{t}$

$x(0) = \beta \begin{bmatrix} 1 \\ 3 \end{bmatrix}$

c)



9). $A = \begin{bmatrix} 1 & 1 \\ 4 & 1 \end{bmatrix}$

$\begin{vmatrix} 1-\lambda & 1 \\ 4 & 1-\lambda \end{vmatrix} = (1-\lambda)^2 - 4 = 1 - 2\lambda + \lambda^2 - 4 = \lambda^2 - 2\lambda - 3$

$\lambda^2 - 2\lambda - 3 = 0 \rightarrow \lambda = \dfrac{2 \pm \sqrt{4 + 4\times 3}}{2} = \dfrac{2 \pm 4}{2} = \begin{array}{l} \lambda_1 = 3 \\ \lambda_2 = -1 \end{array}$ $\quad \Rightarrow v_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ $\quad \Rightarrow v_2 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$

$A v_1 = \lambda_1 v_1 \quad v_{11} + v_{12} = 3v_{11}$

$A v_2 = \lambda_2 v_2 \quad v_{21} + v_{22} = -v_{21}$

$[x(t)] = k_1 \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{3t} + k_2 \begin{bmatrix} 1 \\ -2 \end{bmatrix} e^{-t}$

$x(0) = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ $\quad \left| \begin{array}{l} 2 = k_1 + k_2 \\ 1 = 2k_1 - 2k_2 \end{array}\right. \Rightarrow \left| \begin{array}{l} 4 = 2k_1 + 2k_2 \\ 1 = 2k_1 - 2k_2 \end{array}\right. \Rightarrow \left| \begin{array}{l} \frac{3}{4} = k_2 \\ \frac{5}{4} = k_1 \end{array}\right.$

$\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \frac{5}{4} \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{3t} + \frac{3}{4} \begin{bmatrix} 1 \\ -2 \end{bmatrix} e^{-t}$

$\begin{vmatrix} -\lambda & 1 \\ 1 & -\lambda \end{vmatrix} = \lambda^2 - 1 = 0$ $\quad \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{matrix} v_{11} \\ v_{12} \end{matrix} = \begin{matrix} v_{11} \\ v_{12} \end{matrix}$ $\quad \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{matrix} v_{11} \\ v_{12} \end{matrix} = \begin{matrix} -v_{11} \\ -v_{12} \end{matrix}$

$\lambda_1 = 1 \quad \lambda_2 = -1$ $\quad v_{12} = v_{11}$, $v_{11} = v_{12}$ $\quad v_{12} = v_{11}$, $-v_{11} = v_{12}$

$v_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad v_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$x(t) = k_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{t} + k_2 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{-t}$ $\quad 2 = k_1 + k_2$, $1 = k_1 - k_2$ $\quad k_1 = \frac{3}{2}$, $\frac{1}{2} = k_2$ $\quad x(t) = \frac{3}{2} \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{t} + \frac{1}{2} \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{-t}$

$\begin{bmatrix} 2 \\ 1 \end{bmatrix} = k_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} + k_2 \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$A = \begin{bmatrix} 0 & 1 \\ -1 & -3 \end{bmatrix}$

$\det(A - \lambda I) = 0 \Leftrightarrow \begin{vmatrix} -\lambda & 1 \\ -1 & -3-\lambda \end{vmatrix} = 0$

$\lambda(\lambda + 3) + 1 = 0 \Leftrightarrow \lambda^2 + 3\lambda + 1 = 0$

$\lambda = \dfrac{-3 \pm \sqrt{9-4}}{2} = \left| \begin{array}{l} \lambda_1 = \frac{-3+\sqrt{5}}{2} \\ \lambda_2 = -\frac{3+\sqrt{5}}{2} \end{array}\right.$ $\rightarrow v_1 = \begin{bmatrix} 1 \\ \frac{-3+\sqrt{5}}{2} \end{bmatrix}$ $\quad v_2 = \begin{bmatrix} 1 \\ -\frac{3+\sqrt{5}}{2} \end{bmatrix}$

$\begin{pmatrix} 0 & 1 \\ -1 & -3 \end{pmatrix} \begin{pmatrix} v_{11} \\ v_{12} \end{pmatrix} = \lambda_1 \begin{pmatrix} v_{11} \\ v_{12} \end{pmatrix}$

$0 \cdot v_{11} + v_{12} = \lambda_1 v_{11}$

$-v_{11} - 3v_{12} = \lambda_1 v_{12}$

$\begin{bmatrix} 2 \\ 1 \end{bmatrix} = k_1 \begin{bmatrix} 1 \\ \frac{-3+\sqrt{5}}{2} \end{bmatrix} + k_2 \begin{bmatrix} 1 \\ -\frac{3+\sqrt{5}}{2} \end{bmatrix}$

$2 \cdot \frac{3+\sqrt{5}}{2} = k_1 \frac{3+\sqrt{5}}{2} + k_2 \frac{3+\sqrt{5}}{2}$

$2 = k_1 + k_2$

$1 = k_1 \frac{-3+\sqrt{5}}{2} - k_2 \frac{3+\sqrt{5}}{2}$ $\quad \frac{4+\sqrt{5}}{3+\sqrt{5}} = k_1$

$x(t) = \frac{4+\sqrt{5}}{3+\sqrt{5}} \begin{bmatrix} 1 \\ \frac{-3+\sqrt{5}}{2} \end{bmatrix} e^{\frac{-3+\sqrt{5}}{2}t} + \left(2 - \frac{4+\sqrt{5}}{3+\sqrt{5}}\right) \begin{bmatrix} 1 \\ \frac{-(3+\sqrt{5})}{2} \end{bmatrix} e^{-\frac{3+\sqrt{5}}{2}t}$

P(0) $\dot{x} = \begin{bmatrix} 1 & -2 \\ 0 & -1 \end{bmatrix} x$

a) $\begin{vmatrix} 1-\lambda & -2 \\ 0 & -(1+\lambda) \end{vmatrix} = -(1+\lambda)(1-\lambda) = -(1-2\lambda+\lambda^2)$

$\lambda_1 = 1$
$\lambda_2 = -1$

$A V_1 = \lambda_1 V_1$ $\quad A V_1 = -V_1$

$\begin{bmatrix} 1 & -2 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} V_{11} \\ V_{12} \end{bmatrix} = \begin{matrix} V_{11} \\ V_{12} \end{matrix}$ $\quad 1 - 2V_{12} = -1$

$V_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $\quad V_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ $\quad 1 - 2V_{12} = 1$

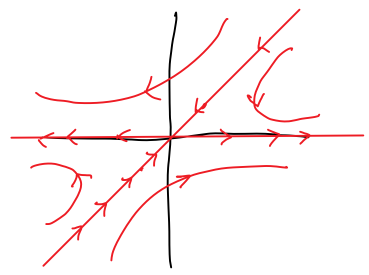
b) $\begin{bmatrix} 2 \\ -0.5 \end{bmatrix} = K_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + K_2 \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ $\quad K_2 = -0.5$

$2 + 0.5 = K_1$

$[x] = 2.5 \begin{bmatrix} 1 \\ 0 \end{bmatrix} e^t - 0.5 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{-t}$

c) $x(t) = \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{-t}$

d)



d) $A_1 - D - T_3$
$A_2 - A - T_4$
$A_3 - C - T_2$
$A_4 - B - T_1$

P(2) $A_1$ A

$A_2$ $\begin{vmatrix} -\lambda & -1 \\ 2 & -\lambda \end{vmatrix} = 0 \iff \lambda^2 + 2 = 0 \Rightarrow D$

$\lambda = \sqrt{-2}$

$A_3$ $\begin{vmatrix} -\lambda & 1 \\ -2 & -0.6-\lambda \end{vmatrix} = 0 \iff \lambda(0.6+\lambda) + 2 = 0 \Rightarrow \lambda^2 + 0.6\lambda + 2 = 0$

C

$A_4$ $\begin{vmatrix} -\lambda & 1 \\ 1 & -\lambda \end{vmatrix} = 0 \iff \lambda^2 - 1 = 0 \Rightarrow \lambda = 1 \Rightarrow B$
ou
$\lambda = -1$

P(3) $\frac{dx_1}{dt} = Kx_2 - \alpha x_1 + g$

$\frac{dx_2}{dt} = \ell x_1 - \beta x_2 + h$

a) $0 = Kx_2 - \alpha x_1 + g$
$0 = \ell x_1 - \beta x_2 + h$

$x_2 = \frac{\ell g + h\alpha}{\alpha\beta - K\ell}$ $\quad x_1 = \frac{K}{\alpha} \cdot \frac{\ell g + h\alpha}{\alpha\beta + K\ell} + \frac{g}{\alpha}$

b) $\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & K \\ \ell & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

c) $\begin{matrix} -\alpha & K \\ \ell & -\beta \end{matrix}$

$x_2 = \frac{h + \ell x_1}{\beta} = \frac{\ell g + h\alpha}{\alpha\beta - K\ell}$

$K \cdot \frac{h + \ell x_1}{\beta} - \alpha x_1 + g = 0$

$\frac{Kh}{\beta} + \frac{K\ell x_1}{\beta} - \alpha x_1 = -g$

$x_1 \left( \frac{K\ell}{\beta} - \alpha \right) = -g - \frac{Kh}{\beta}$

$x_1 = \frac{-g - \frac{Kh}{\beta}}{\frac{K\ell}{\beta} - \alpha}$

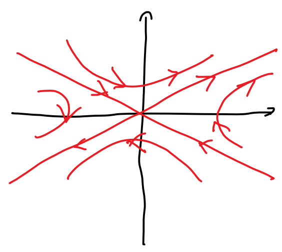
$\begin{vmatrix} -\lambda & K \\ \ell & -\lambda \end{vmatrix} = \lambda^2 - K\ell$

$\lambda^2 - K\ell = 0 \Rightarrow \lambda = \pm\sqrt{K\ell}$

$K V_{12} = +\sqrt{K\ell}\, V_{11}$ $\quad V_1 = \begin{bmatrix} 1 \\ \frac{\sqrt{K\ell}}{K} \end{bmatrix}$ $\quad V_2 = \begin{bmatrix} 1 \\ -\frac{\sqrt{K\ell}}{K} \end{bmatrix}$

$A V_1 = \lambda_1 V_1$

$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = K_1 \begin{bmatrix} 1 \\ \sqrt{\frac{\ell}{K}} \end{bmatrix} e^{\sqrt{K\ell}\, t} + K_2 \begin{bmatrix} 1 \\ -\sqrt{\frac{\ell}{K}} \end{bmatrix} e^{-\sqrt{K\ell}\, t}$



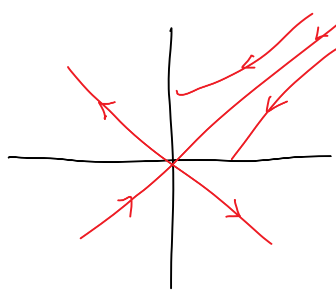
P(4) $\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & -\alpha \\ -b & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ $\quad \begin{vmatrix} -\lambda & -\alpha \\ -b & -\lambda \end{vmatrix}$ $\quad \lambda^2 - b\alpha = 0$

$\lambda = \pm\sqrt{b\alpha}$

$\lambda_1 = \sqrt{b\alpha}$ $\quad A V_1 = \lambda_1 V_1$ $\quad -\alpha V_{12} = \sqrt{b\alpha}\, V_{11} \rightarrow V_1 = \begin{bmatrix} 1 \\ \frac{\sqrt{b\alpha}}{-\alpha} \end{bmatrix}$

$\lambda_2 = -\sqrt{b\alpha}$ $\quad A V_2 = \lambda_2 V_2$ $\quad -\alpha V_{22} = -\sqrt{b\alpha}\, V_{21} \rightarrow$ $\quad V_2 = \begin{bmatrix} 1 \\ \sqrt{\frac{b}{\alpha}} \end{bmatrix}$

$x_2 = \sqrt{\frac{b}{\alpha}}\, x_1$



$\begin{matrix} x_1 \\ x_2 \end{matrix} = K_1 \begin{bmatrix} 1 \\ \frac{\sqrt{b\alpha}}{-\alpha} \end{bmatrix} e^{\sqrt{b\alpha}\, t} + K_2 \begin{bmatrix} 1 \\ \sqrt{\frac{b}{\alpha}} \end{bmatrix} e^{-\sqrt{b\alpha}\, t}$

P(5) $\frac{di}{dt} = -i + V$

$\frac{dV}{dt} = -i + g(u - V)$

$\begin{cases} 0 = -i + V \\ 0 = -i + g(1 - V) \end{cases}$

$\begin{cases} 0 = -i + V \\ 0 = -i + (1-V)(1-V-1)(1-V-4) \end{cases} \iff V = i$
$0 = -V + (1-V)(-V)(-3-V)$

$V = 0 \lor 0 = -1 - (1-V)(-3-V)$

$0 = -1 - (-3 - V + 3V + V^2)$

$0 = -1 + 3 - 2V - V^2$

$V^2 + 2V - 2 = 0 \rightarrow V = \frac{-2 \pm \sqrt{10}}{2}$ $\quad V_1 = -1 + \sqrt{3}$, $\quad V_2 = -1 - \sqrt{3}$

$A = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ -1 & -3V^2 - 4V + 3 \end{bmatrix}$

Ponto 1 $V = i = 0$

$\begin{vmatrix} -1-\lambda & 1 \\ -1 & 3-\lambda \end{vmatrix} = 0 \iff -(1+\lambda)(3-\lambda) + 1 = 0$ $\quad \lambda_1 = 1 - \sqrt{3}$, $\quad \lambda_2 = 1 + \sqrt{3} \mapsto$ instável

Ponto 2 $\begin{vmatrix} -1-\lambda & 1 \\ -1 & K-1.5-\lambda \end{vmatrix} = 0 \Rightarrow \lambda_{1,2}$ com parte real negativa $\Rightarrow$ estável

Ponto 3 $\rightarrow \lambda_{1,2}$ com parte real negativa $\Rightarrow$ estável

16)

$0=(1-x_2-x_1)x_1$

$0=(\frac{1}{2}-\frac{1}{4}x_1-\frac{3}{4}x_2)x_2$

$\bar{x}_1=0$, $\bar{x}_2=0$ $\quad$ $\bar{x}_1=0$, $\bar{x}_2=\frac{2}{3}$ $\quad$ $\bar{x}_1=1$, $\bar{x}_2=0$ $\quad$ $x_1=\frac{1}{2}$, $\bar{x}_2=\frac{1}{2}$

$0=(\frac{1}{2}-\frac{3}{4}x_2)x_2$

$\frac{1}{2}-\frac{3}{4}x_2=0 \Rightarrow x_2=\frac{1}{2}\cdot\frac{4}{3}=\frac{2}{3}$

$(1-x_2-x_1)=0$

$(\frac{1}{2}-\frac{3}{4}x_2-\frac{1}{4}x_1)=0$

$1-x_1=0$

a) $A=\begin{bmatrix}-2x-y+1 & -x\\ -\frac{y}{4} & -\frac{3}{2}y-\frac{x}{4}+\frac{1}{2}\end{bmatrix}$

$x=y=0$ $\quad A=\begin{bmatrix}1 & 0\\ 0 & \frac{1}{2}\end{bmatrix}$ $\qquad$ $x=1$, $y=0$ $\quad A=\begin{bmatrix}-1 & -1\\ 0 & \frac{1}{4}\end{bmatrix}$

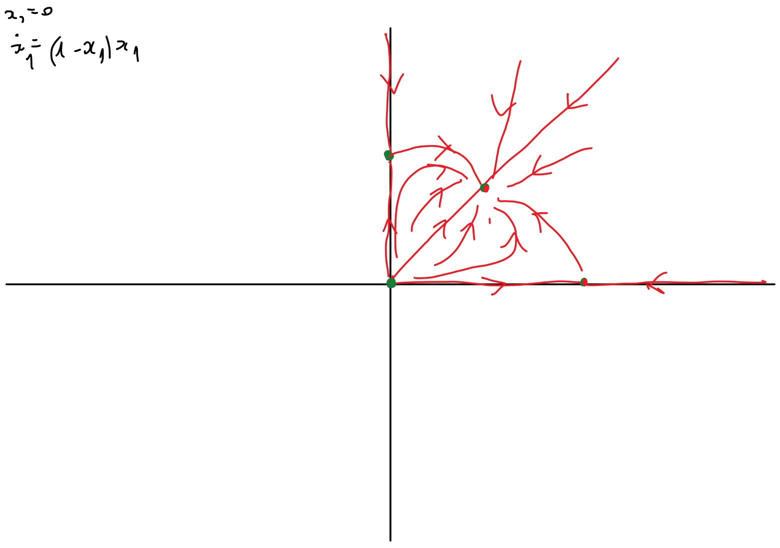
$x=0$, $y=\frac{2}{3}$ $\quad \begin{bmatrix}\frac{1}{3} & 0\\ -\frac{1}{6} & \frac{1}{2}\end{bmatrix}$ $\qquad$ $x=\frac{1}{2}$, $y=\frac{1}{2}$ $\quad A=\begin{bmatrix}-\frac{1}{2} & -\frac{1}{2}\\ -\frac{1}{8} & -\frac{3}{8}\end{bmatrix}$

$0=(1-x_2-x_1)x_1$

$0=(\frac{1}{2}-\frac{1}{4}x_1-\frac{3}{4}x_2)x_2$

$x_2=0$

$\dot{x}_1=(1-x_1)x_1$



17)

$\frac{dN}{dt}=N(1-N-P)$

$\frac{dP}{dt}=P(\frac{1}{2}-\frac{1}{4}P-\frac{3}{4}N)$

$\frac{1}{2}(1-\frac{1}{2}-\frac{1}{2})=0$ P.V.

$\frac{1}{2}(\frac{1}{2}-\frac{1}{4}\frac{1}{3}-\frac{3}{4}\frac{1}{3})=0$ P.V.

$\frac{\partial f_1}{\partial N}=1-2N-P$ $\qquad$ $\frac{\partial f_2}{\partial N}=-\frac{3}{4}P$

$\frac{\partial f_1}{\partial P}=-N$ $\qquad$ $\frac{\partial f_2}{\partial P}=\frac{1}{2}-\frac{1}{2}P-\frac{3}{4}N$

$A\Big|_{P=1/2,\ N=1/2}=\begin{bmatrix}-1/2 & -1/2\\ -3/8 & -1/8\end{bmatrix}$

$(\lambda+\frac{1}{2})(\lambda+\frac{1}{8})-\frac{3}{16}=0$

$\lambda^2+(\frac{1}{2}+\frac{1}{8})\lambda+\frac{1}{16}-\frac{3}{16}=0$

$8\lambda^2+5\lambda-1=0 \Rightarrow$ Um deles é negativo

É instável

P18)

a

$\dot{N}_1=(3-1)N_1-1(N_1+N_2)N_1$

$\dot{N}_2=(2-1)N_1-1(N_1+N_2)N_2$

$\Rightarrow \left|\begin{array}{l}\dot{N}_1=2N_1-(N_1+N_2)N_1\\ \dot{N}_2=N_2-(N_1+N_2)N_2\end{array}\right.$

$\boxed{\begin{array}{lll}N_1=0 & N_1=0 & N_1=2\\ N_2=0 & N_2=1 & N_2=0\end{array}}$ $\to$ Eq.

$\left|\begin{array}{l}0=2N_1-(N_1+N_2)N_1\\ 0=N_2-(N_1+N_2)N_2\end{array}\right.$ $\qquad$ $\left|\begin{array}{l}2N_1=(N_1+N_2)N_1\\ N_2=(N_1+N_2)N_2\end{array}\right.$ $\qquad$ $\boxed{\begin{array}{l}2=N_1+N_2\\ 1=N_1+N_2\end{array}}$

IMP

b)

$A=\begin{bmatrix}\frac{\partial f_1}{\partial N_1} & \frac{\partial f_1}{\partial N_2}\\ \frac{\partial f_2}{\partial N_1} & \frac{\partial f_2}{\partial N_2}\end{bmatrix}$

$\frac{\partial}{\partial N_1}f_1=2-2N_1-N_2$

$\frac{\partial}{\partial N_2}f_1=-N_1$

$\frac{\partial f_2}{\partial N_1}=-N_2$

$\frac{\partial f_2}{\partial N_2}=1-N_1-2N_2$

$N_1=N_2=0$

$A=\begin{bmatrix}2 & 0\\ 0 & 1\end{bmatrix}$

$\Downarrow$

$(2-\lambda)(1-\lambda)=0$

$\downarrow$

$\lambda_1=1$ e $\lambda_2=2$

Instável

$Av_1=\lambda_1 v_1 \to v_1=\begin{bmatrix}0\\1\end{bmatrix}$

$Av_2=\lambda_2 v_2 \to v_2=\begin{bmatrix}1\\0\end{bmatrix}$

$N_1=0$, $N_2=1$

$A=\begin{bmatrix}1 & 0\\ -1 & -1\end{bmatrix}$

$(1-\lambda)(-1-\lambda)=0$

$\downarrow$

$\lambda=1$ e $\lambda=-1$

Instável

$\downarrow$

$v_1=\begin{bmatrix}1\\ -\frac{1}{2}\end{bmatrix}$

$v_2=\begin{bmatrix}0\\1\end{bmatrix}$

$N_1=2$, $N_2=0$

$A=\begin{bmatrix}-2 & -2\\ 0 & -1\end{bmatrix}$

$(-2-\lambda)(-1-\lambda)=0$

$\downarrow$

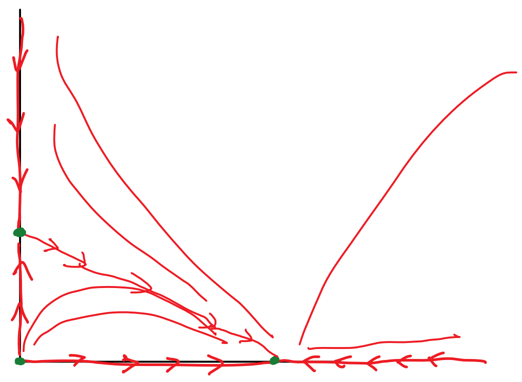
$\lambda=-1$ e $\lambda=-2$

Estável

$\downarrow$

$v_1=\begin{bmatrix}1\\0\end{bmatrix}$

$v_2=\begin{bmatrix}1\\ -\frac{1}{2}\end{bmatrix}$



1a) $0 = -0 - \alpha(0^2+0^2) \rightarrow$ 
$0 = 0 - 0(0^2+0^2) \rightarrow$ } $\mapsto$ Ponto de equilíbrio

$$\frac{\partial \dot{x}_1}{\partial x_1} = -3x_1^2 - x_2^2 \qquad \frac{\partial \dot{x}_2}{\partial x_1} = 1 - 2x_1x_2$$

$$\frac{\partial \dot{x}_1}{\partial x_2} = -2x_1x_2 - 1 \qquad \frac{\partial \dot{x}_2}{\partial x_2} = -3x_2^2 - x_1^2$$

$x_1 = 0$, $x_2 = 0$, $\lambda_2 = -1$

$$A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$\lambda^2 - 1 = 0$

$\lambda = \pm 1$

$\lambda_1 = 1 \mapsto$ é instável uwu

$A v_1 = \lambda_1 v_1 \rightarrow -v_{12} = v_{11} \rightarrow v_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$A v_2 = \lambda_2 v_2 \rightarrow -v_{22} = -v_{21} \rightarrow v_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

**P20.** Considere o modelo epidemiológico simplificado em que $x$ representa a população sã e $y$ a população infetada, e $K$ e $L$ são parâmetros positivos:

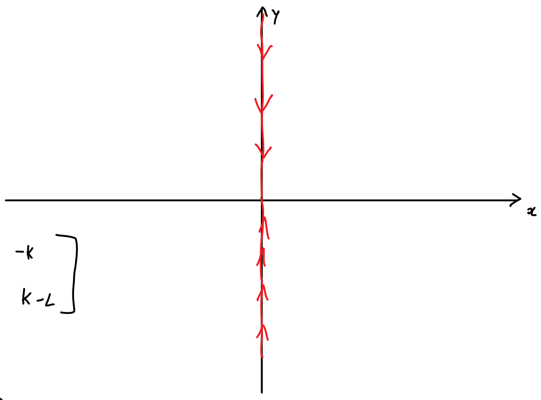
$$\begin{cases} \frac{dx}{dt} = -Kxy \\ \frac{dy}{dt} = Kxy - Ly \end{cases}$$

Recorrendo aos sinais da derivada esboce qualitativamente o retrato de fase deste sistema.

20) $\dot{x} = -Kxy \qquad 0 = -Kxy$

$\dot{y} = Kxy - Ly \qquad 0 = Kxy - Ly$

Pontos de equilíbrio: $y = 0$, $x \in \mathbb{R}$




$$A = \begin{bmatrix} -Ky & -Kx \\ Ky & Kx - L \end{bmatrix} = \begin{bmatrix} 0 & -K \\ K & K - L \end{bmatrix}$$

$y = 0$, $x = 1$

$$\begin{vmatrix} -\lambda & -K \\ K & K - L - \lambda \end{vmatrix} = 0$$

$-\lambda(K - L - \lambda) + K^2 = 0$

$\lambda^2 - (K - L)\lambda + K^2 = 0$

$$\frac{(K-L) \pm \sqrt{(K-L)^2 - 4K^2}}{2}$$

$K_0 = 40 / 35$

$K_v = 12$

?

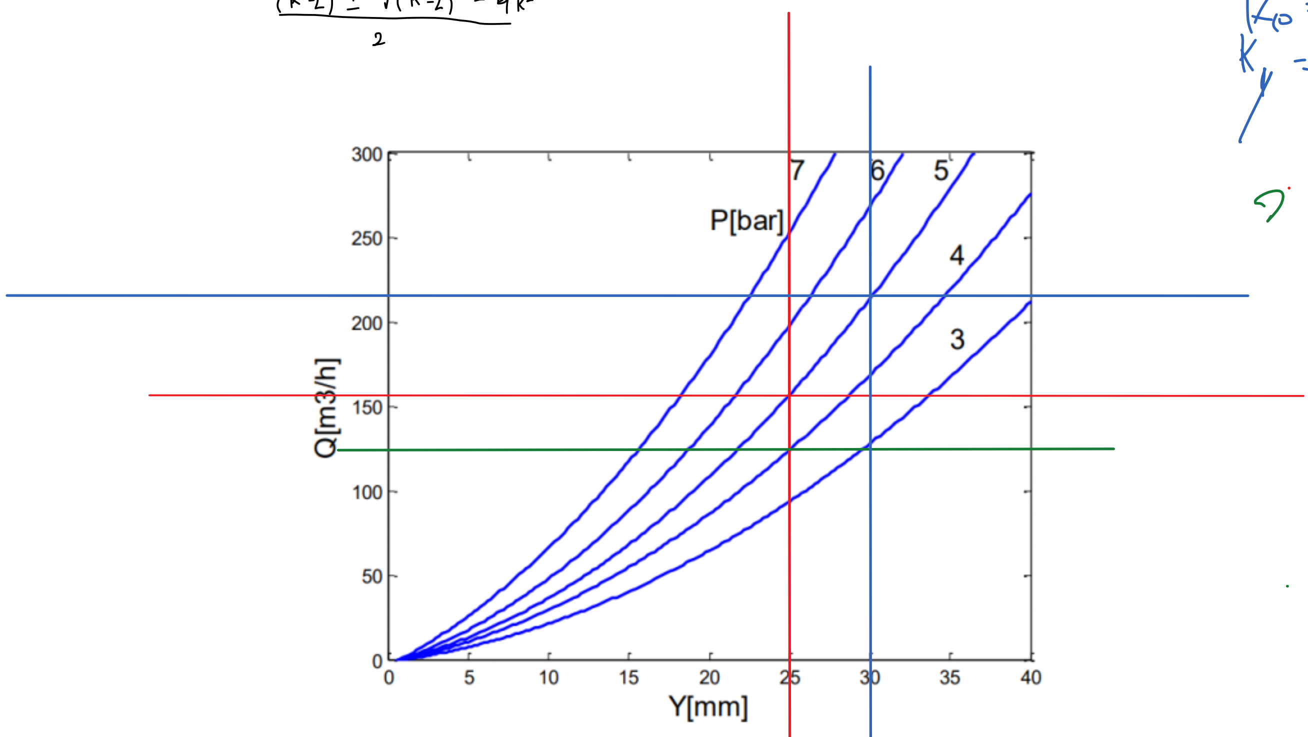



Fig. P21-1. Característica estática da válvula.