P1.)

$$m\frac{d^2z}{dt^2} = u - \beta\frac{dz}{dt} - Kz \qquad \begin{matrix} x_1 = z \\ x_2 = \dot z \end{matrix}$$

$$\dot x_1 = x_2$$

$$\dot x_2 = -\frac{K}{m}x_1 - \frac{\beta}{m}x_2 + \frac{u}{m}$$

$$\begin{bmatrix}\dot x_1\\ \dot x_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ -\frac{K}{m} & -\frac{\beta}{m}\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}0\\ 1\end{bmatrix}u$$

$$y = [1\ 0]\begin{bmatrix}x_1\\ x_2\end{bmatrix}$$

c) $$\left(s^2 + \frac{\beta}{m}s + \frac{K}{m}\right)Z = \frac{1}{m}U$$

$$G(s) = \frac{\frac{1}{m}}{s^2 + \frac{\beta}{m}s + \frac{K}{m}}$$

d) $\frac{K}{m} = 1$, $\beta = 0$ $\Rightarrow G(s) = \frac{\frac{1}{m}}{s^2+1}$

P2)

$$J\frac{d^2\theta}{dt^2} = T - K\theta - \beta\frac{d\theta}{dt}$$

$$\frac{\frac{1}{J}}{s^2 + \frac{\beta}{J}s + \frac{K}{J}}$$

$x_1 = \theta$, $x_2 = \dot\theta$ $\qquad x_2 = \dot x_1$

$$\dot x_2 = \frac{T}{J} - \frac{K}{J}x_1 - \frac{\beta}{J}x_2$$

$$\begin{bmatrix}\dot x_1\\ \dot x_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ -\frac{K}{J} & -\frac{\beta}{J}\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}0\\ \frac{1}{J}\end{bmatrix}T$$

$$Y = [1\ 0]\begin{bmatrix}x_1\\ x_2\end{bmatrix}$$

P3) a) $$m\frac{d^2z}{dt^2} = u - \beta\frac{dz}{dt}$$

b) $x_1 = z$, $x_2 = \dot z$ $\qquad m\dot x_2 = u - \beta x_2$, $\quad x_2 = \dot x_1$

$$\begin{bmatrix}\dot x_1\\ \dot x_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ 0 & -\frac{\beta}{m}\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}0\\ \frac{1}{m}\end{bmatrix}u$$

$$Y = [1\ 0]\begin{bmatrix}x_1\\ x_2\end{bmatrix}$$

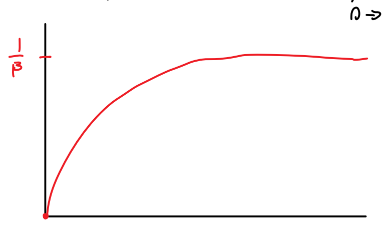
c) $$(ms^2 + \beta s)Z = U \Rightarrow \frac{1}{ms^2 + \beta s}$$

relação da velocidade:

$$\frac{1}{ms+\beta} \Rightarrow G(0) = \frac{1}{\beta}$$

$\lim_{s\to+\infty} G(s) = 0 \Rightarrow$ resposta [illegible]

$\lim_{s\to 0} s\,G(s) = \frac{1}{m} \Rightarrow$ resposta da [illegible] finita



P4)

$$m_1\frac{d^2z_1}{dt^2} = u - K(z_1 - z_2) \Rightarrow m_1\ddot z_1 = u - K(z_1 - z_2)$$

$$m_2\frac{d^2z_2}{dt^2} = -K(z_2 - z_1) - \beta\frac{dz_2}{dt} \Rightarrow m_2\ddot z_2 = -K(z_2 - z_1) - \beta\dot z_2$$

$x_1 = z_1$, $x_2 = \dot z_1$, $x_3 = z_2$, $x_4 = \dot z_2$

$$\dot x_1 = x_2$$

$$\dot x_2 = \frac{u}{m_1} - \frac{K}{m_1}(x_1 - x_3)$$

$$\dot x_3 = x_4$$

$$\dot x_4 = -\frac{K}{m_2}(x_3 - x_1) - \frac{\beta}{m}x_4$$

$$\begin{bmatrix}\dot x_1\\ \dot x_2\\ \dot x_3\\ \dot x_4\end{bmatrix} = \begin{bmatrix}0 & 1 & 0 & 0\\ -\frac{K}{m_1} & 0 & \frac{K}{m_1} & 0\\ 0 & 0 & 0 & 1\\ \frac{K}{m_2} & 0 & -\frac{K}{m_2} & -\frac{\beta}{m_2}\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix} + \begin{bmatrix}0\\ \frac{1}{m_1}\\ 0\\ 0\end{bmatrix}u$$

$$Y = [0\ 0\ 1\ 0]\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix}$$

P5)

$$m\frac{d^2z}{dt^2} = -\beta\frac{dz}{dt} - K(z - z_A)$$

$$M\frac{d^2z_A}{dt^2} = u - K(z_A - z)$$

$$u = K(z_A - z)$$

$$m\ddot z = -\beta\dot z - K(z - z_A) \Rightarrow m\ddot z = -\beta\dot z + u$$

$$(s^2m + \beta s)Z = U$$

$$\frac{1}{s^2m + \beta s}$$

P7)

$$m_1\ddot{x}_1 = -D\dot{x}_1 - k(x_1 - x_2)$$
$$m_2\ddot{x}_2 = U - k(x_2 - x_1)$$

$a = x_1$, $b = \dot{x}_1$, $c = x_2$, $d = \dot{x}_2$

$$\dot{a} = b$$
$$\dot{b} = -\frac{D}{m_1}b - \frac{K}{m_1}(a-c)$$
$$\dot{c} = d$$
$$\dot{d} = \frac{U}{m_2} - \frac{K}{m_2}(c-a)$$

$$\begin{bmatrix}\dot{a}\\ \dot{b}\\ \dot{c}\\ \dot{d}\end{bmatrix} = \begin{bmatrix}0 & 1 & 0 & 0\\ -\frac{K}{m_1} & -\frac{D}{m_1} & \frac{K}{m_1} & 0\\ 0 & 0 & 1 & 0\\ \frac{K}{m_2} & 0 & -\frac{K}{m_2} & 0\end{bmatrix}\begin{bmatrix}a\\ b\\ c\\ d\end{bmatrix} + \begin{bmatrix}0\\ 0\\ 0\\ \frac{1}{m_2}\end{bmatrix}u$$

$$y = \begin{bmatrix}0 & 0 & 1 & 0\end{bmatrix}\begin{bmatrix}a\\ b\\ c\\ d\end{bmatrix}$$

P8)

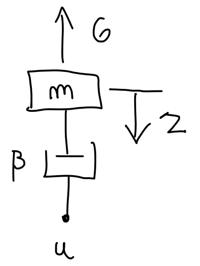
$$m_s\ddot{Z}_s = \overset{?}{F_a} - K_s(Z_s - Z_u)$$
$$m_u\ddot{Z}_u = F_a - K_s(Z_u - Z_s) - K_t(Z_u - Z_r)$$

$x_1 = Z_s$, $x_2 = \dot{Z}_s$, $x_3 = Z_u$, $x_4 = \dot{Z}_u$

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -\frac{F_a}{m_s} - \frac{K_s}{m_s}(x_1 - x_3)$$
$$\dot{x}_3 = x_4$$
$$\dot{x}_4 = \frac{F_a}{m_u} - \frac{K_s}{m_u}(x_3 - x_1) - \frac{K_t}{m_u}(x_3 - u)$$

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\end{bmatrix} = \begin{bmatrix}0 & 1 & 0 & 0\\ -\frac{K_s}{m_s} & 0 & \frac{K_s}{m_s} & 0\\ 0 & 0 & 0 & 1\\ \frac{K_s}{m_u} & 0 & -\frac{K_s}{m_u}-\frac{K_t}{m_u} & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix} + \begin{bmatrix}0\\ -\frac{F_a}{u\,m_s}\\ 0\\ \frac{F_a}{u\,m_u} + \frac{K_t}{m_u}\end{bmatrix}u$$

P9)



$$m\ddot{z} = \beta\dot{z} - F_a$$
$$m\ddot{z} = 0$$
$$\beta\dot{z} - F_a \Rightarrow \frac{F_a}{\beta} = \dot{z}$$

Velo cidad constant

P10)

$$u = L\frac{di_L}{dt} + V_C$$
$$i_L = \frac{V_c}{R} + C\frac{dV_c}{dt}$$

$$\begin{bmatrix}\dot{i}_L\\ \dot{V}_c\end{bmatrix} = \begin{bmatrix}0 & -\frac{1}{L}\\ \frac{1}{C} & -\frac{1}{CR}\end{bmatrix}\begin{bmatrix}i_L\\ V_c\end{bmatrix} + \begin{bmatrix}\frac{1}{L}\\ 0\end{bmatrix}u$$

$$\dot{i}_L = \frac{u}{L} - \frac{V_c}{L}$$
$$\dot{V}_c = \frac{i_L}{C} - \frac{V_c}{CR}$$

$$y = \begin{bmatrix}0 & \frac{1}{R}\end{bmatrix}\begin{bmatrix}i_L\\ V_c\end{bmatrix}$$

$$\frac{V_c}{R} = y$$

P11)

$$y(t) = x_2 + C\frac{dx_1}{dt} = x_2 - \frac{1}{R_1}x_1 + u$$

$$\begin{cases} u = L\frac{dx_2}{dt} + x_2R_2\\ u = x_1 + R_1C\frac{dx_1}{dt}\end{cases} \Rightarrow \begin{cases}\frac{dx_1}{dt} = -\frac{1}{R_1C}x_1 + u\\ \frac{dx_2}{dt} = -\frac{R_2}{L}x_2 + u\end{cases}$$

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}-\frac{1}{R_1C} & 0\\ 0 & -\frac{R_2}{L}\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}1\\ 1\end{bmatrix}u$$

$$y = \begin{bmatrix}-\frac{1}{R_1} & 1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + u$$

P12)

$$\frac{V_1 - V_2}{R} = C\frac{dV_2}{dt} + \frac{V_2}{R}$$
$$u = V_1$$
$$\frac{dV_1}{dt} = \frac{V_1 - V_2}{R}$$

$$\frac{u}{R} - \frac{V_2}{R} = C\dot{V}_2 + \frac{V_2}{R}$$
$$u = CR\dot{V}_2 + 2V_2$$
$$u = V_1$$
$$\dot{V}_1 = \frac{V_1}{R} - \frac{V_2}{R}$$

$$\begin{bmatrix}\dot{V}_1\\ \dot{V}_2\end{bmatrix} = \begin{bmatrix}\frac{1}{R} & -\frac{1}{R}\\ \frac{1}{CR} & \frac{2}{CR}\end{bmatrix}\begin{bmatrix}V_1\\ V_2\end{bmatrix}$$

$$y = \begin{bmatrix}0 & \frac{1}{R}\end{bmatrix}\begin{bmatrix}V_1\\ V_2\end{bmatrix}$$

P13)

$S_1 \to S_2$ (x_1K_{12}) $\quad S_1 \to S_3$ (x_1K_{13}) $\quad S_2 \to S_3$ (x_2K_{23})

$S_2 \to S_1$ (x_2K_{21}) $\quad S_3 \to S_1$ (x_3K_{31}) $\quad S_3 \to S_2$ (x_3K_{32})

$$\frac{dS_1}{dt} = K_{21}S_2 + K_{31}S_3 - (K_{12} + K_{13})S_1$$
$$\frac{dS_2}{dt} = K_{12}S_1 + K_{32}S_3 - (K_{21} + K_{23})S_1$$
$$\frac{dS_3}{dt} = K_{13}S_1 + K_{23}S_2 - (K_{31} + K_{32})S_3$$

P(4) $t \quad \Delta t$

$$f(t+\Delta t) = f(t) - K^+ f(t) + K^- f_u(t) =$$

$$= f(t) - K^+ f(t)\Delta t + K^+ (1 - f(t))\Delta t$$

$$\frac{f(t+\Delta t) - f(t)}{\Delta t} = -K^+ f_N + K^+(1-f$$

$$\frac{df}{dt} = -(K^+ + K^-) f_N + K^+$$

P(5)

$$\gamma_1 \ddot\theta_1 = T - K(\theta_1 - \theta_2) - \beta(\dot\theta_1 - \dot\theta_2)$$

$$\gamma_2 \ddot\theta_2 = -K(\theta_2 - \theta_1) - \beta(\dot\theta_2 - \dot\theta_1)$$

$x_1 = \theta_1$
$x_2 = \dot\theta_1$
$x_3 = \theta_2$
$x_4 = \dot\theta_2$

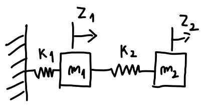
$$\begin{matrix} \dot x_1 \\ \dot x_2 \\ \dot x_3 \\ x_4 \end{matrix} = \begin{matrix} 0 & 1 & 0 & 0 \\ -\frac{K}{\gamma_1} & \frac{K}{\gamma_1} & -\frac{\beta}{\gamma_1} & \frac{\beta}{\gamma_1} \\ 0 & 0 & 0 & 1 \\ +\frac{K}{\gamma_1} & -\frac{K}{\gamma_1} & \frac{\beta}{\gamma_1} & -\frac{\beta}{\gamma_1} \end{matrix}$$

16.

$$\gamma_2 \ddot\theta_2 = T - d_2 \dot\theta_2$$

$$\frac{\gamma_2 \dot V_2}{r_2} = T - d_2 \dot V_2$$

20.)



$L = T - U$

$$T = \frac{1}{2} M_1 \dot z_1^2 + \frac{1}{2} M_2 \dot z_2^2$$

$$U = \frac{1}{2} K_1 z_1^2 + \frac{1}{2} K_2 (z_2 - z_1)^2$$

$$\frac{\partial L}{\partial z_1} - \frac{d}{dt}\frac{\partial L}{\partial \dot z_1} = 0 \iff -(K_1 z_1 + K_2 z_1 - K_2 z_2) + M_1 \ddot z_1 = 0 \iff M_1 \ddot z_1 = -K_2(z_1 - z_2) - K_1 z_1$$

$$\frac{\partial L}{\partial z_2} - \frac{d}{dt}\frac{\partial L}{\partial \dot z_2} = 0 \quad M_2 \ddot z_2 = -K_2(z_2 - z_1)$$

21.)

$$T = \frac{1}{2} m(\dot r^2 + r^2 \dot\theta^2)$$

$$U = -\frac{K}{r}$$

$$L = T - U$$

$$\frac{\partial L}{\partial r} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot r}\right) = 0$$

$$\frac{\partial L}{\partial r} = 2 r \dot\theta^2 - \frac{K}{r^2}$$

$$\frac{\partial L}{\partial \dot r} = m\dot r \rightarrow \frac{d}{dt}\frac{\partial L}{\partial \dot r} = m\ddot r$$

$$\Downarrow$$

$$m\ddot r = m r \dot\theta^2 - K\frac{\dot r}{r^2}$$

$$\frac{\partial L}{\partial \theta} - \frac{d}{dt}\frac{\partial L}{\partial \dot\theta} = 0$$

$$\frac{\partial L}{\partial \dot\theta} = m r^2 \dot\theta$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot\theta} = 2\dot r r \dot\theta m + m r^2 \ddot\theta$$

$$\Downarrow$$

$$\ddot\theta = -2\frac{\dot r}{r}\dot\theta$$

$x_1 = r \qquad x_3 = \dot r$
$x_2 = \theta \qquad x_4 = \dot\theta$

$$\dot x_1 = x_3$$

$$\dot x_2 = x_4$$

$$\dot x_3 = x_1 x_4^2 - \frac{K}{m}\frac{x_3}{x_1^2}$$

$$\dot x_4 = -2\frac{x_3}{x_1} x_4$$

a)

$$\dot c_\ell = K_{1e} c_p - K_{\ell 0} c_\ell$$

$$\dot c_p = u - (K_{1\ell} + K_{10} + K_{12}) c_p + K_{21} c_2$$

$$\dot c_2 = K_{12} c_p - (K_{21} + K_{20}) c_2$$