$$y = (9 - a10)^2 + (21 - a20)^2 + (32 - a30)^2 + (38 - a40)^2 + (51 - a50)^2$$

$$\frac{1}{2}\frac{dy}{da} = 0 \Rightarrow -10(9 - a10) - 20(21 - a20) - 30(32 - a30) - 40(38 - a40) - 50(51 - a50) = 0$$

$$-9 + a10 - 2\times21 + a40 - 3\times32 + a90 - 4\times38 + a160 - 5\times51 + a250 = 0$$

$$a = \frac{9 + 2\times21 + 3\times32 + 4\times38 + 5\times51}{10 + 40 + 90 + 160 + 250}$$

P2.

$$y = (0.49 - a5^2)^2 + (1.01 - a10^2)^2 + (1.45 - a15^2)^2 + (2.05 - a20^2)^2$$

$$\frac{1}{2}\frac{dy}{da} = 0 \Rightarrow -5^2(0.49 - 5^2a) - 10^2(1.01 - a10^2) - 15^2(1.45 - a15^2) - 20^2(2.05 - a20^2)$$

$$a = \frac{0.49\times5^2 + 1.01\times10^2 + 1.45\times15^2 + 2.05\times20^2}{5^2\times5^2 + 10^2\times10^2 + 15^2\times15^2 + 20^2\times20^2}$$

P3

$$y = \sum (y - ay(t-1) - \mu(t-1))^2$$

$$\frac{1}{2}\frac{dy}{da} = 0 \Rightarrow -\sum y(t-1)(y - \mu(t-1) - ay(t-1)) = 0$$

$$a = \frac{\sum y(t-1)(y - \mu(t-1))}{\sum y^2(t-1)}$$

P4)

$$y = \sum (y + ay(t-1) - b\mu(t-1))^2$$

$$\frac{1}{2}\frac{dy}{da} = 0 \Rightarrow \sum y(t-1)(y + ay(t-1) - b\mu(t-1)) = 0$$

$$\frac{1}{2}\frac{dy}{db} = 0 \Rightarrow -\sum \mu(t-1)(y + ay(t-1) - b\mu(t-1)) = 0$$

$$a = -\frac{\sum y(t-1)(y(t) - b\mu(t-1))}{\sum y^2(t-1)}$$

$$a\sum y^2(k) = -\sum y(k)y(k+1) + b\sum y(k)\mu(k)$$

$$b = \frac{\sum \mu(t-1)(y(t) + ay(t-1))}{\sum \mu^2(t-1)}$$

$$\hat{a}30 = -1 + \hat{b}20 \qquad \hat{a} = \frac{67}{110}$$

$$\hat{b}50 = 36 + \hat{a}20 \qquad \hat{b} = \frac{53}{55}$$

$$b\sum \mu^2(k) = \sum \mu(k)y(k+1) + a\sum y(k)\mu(k)$$

P5 =

$$y = a\,y(t-1) + b\mu(t-2) + e(t)$$

$$y = \sum [y - ay(t-1) - b\mu(t-2)]^2$$

$$\frac{1}{2}\frac{\partial y}{\partial a} = 0 \Rightarrow -\sum y(t-1)[y - ay(t-1) - b\mu(t-2)] = 0 \qquad -\sum y(t-1)y(t) + a\sum y^2(t-1) + b\sum \mu(t-2)y(t-1) = 0 \Rightarrow -0.7 + \hat{a}1.4 = 0 \Leftrightarrow \hat{a} = \frac{1}{2}$$

$$\frac{1}{2}\frac{\partial y}{\partial b} = 0 \Rightarrow -\sum \mu(t-2)[y - ay(t-1) - b\mu(t-2)] = 0 \qquad -\sum \mu(t-2)y(t) + a\sum y(t-1)\mu(t-2) + b\sum \mu(t-2)^2 = 0 \Rightarrow -1 + b1 = 0 \Leftrightarrow \hat{b} = 1$$

P6)

$$y(k+1) = ay(k) + b\mu(k)$$

$$\sum [y(k+1) - ay(k) - b\mu(k)]^2 = y$$

$$\frac{1}{2}\frac{\partial y}{\partial a} = 0 \Rightarrow -\sum y(k)[y(k+1) - ay(k) - b\mu(k)] = 0$$

$$\sum y^2(k)a + \sum y(k)\mu(k)b - \sum y(k)y(k+1) = 0$$

$$\hat{a}S_1 + \hat{b}\sum y(k)\mu(k) - S_4 = 0$$

$$\frac{1}{2}\frac{\partial y}{\partial b} = 0 \quad -\sum \mu(k)[y(k+1) - ay(k) - b\mu(k)] = 0$$

$$b\sum \mu^2(k) + \sum \mu(k)y(k)a - \sum \mu(k)y(k+1) = 0$$

$$S_3 b + \sum \mu(k)y(k)\,a - S_5 = 0$$

$$\begin{cases} \hat{a}S_1 + \hat{b}S_2 - S_4 = 0 \\ \hat{b}S_3 + \hat{a}S_2 - S_5 = 0 \end{cases} \Rightarrow$$

$$\hat{a} = \frac{S_4 - \hat{b}S_2}{S_1}$$

$$\hat{b}S_3 + \frac{S_4 - \hat{b}S_2}{S_1}S_2 - S_5 = 0$$

$$\hat{b}S_3 + \frac{S_4S_2}{S_1} - \frac{\hat{b}S_2^2}{S_1} - S_5 = 0$$

$$\hat{b} = \frac{S_1S_5 - S_4S_2}{S_1S_3 - S_2^2}$$

$$\hat{a} = \frac{S_3S_1 - S_2S_5}{S_1S_3 - S_2^2}$$