P1) a) $0 = 0{,}0087x(4-x)$

PE ① $x = 0$

② $x = 4$

b)

c) $\dot{x} = \alpha x(4-x)$

$\ddot{x} = \frac{d}{dt}\left(4\alpha x - \alpha x^2\right) =$

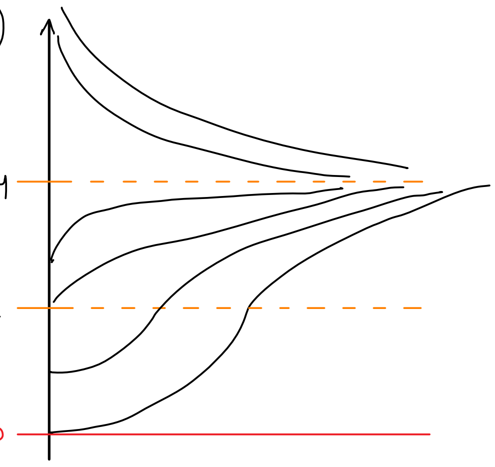
$= 4\alpha\dot{x} - \alpha \cdot 2 \cdot \dot{x} \cdot x =$

$= 2\alpha\dot{x}(2-x) = 2\alpha^2 x(4-x)(2-x)$

$x = 0$
$x = 2$
$x = 4$

		0			2			4		
x	ND	~~	↗	∪	~~	↗	∩	~~	↘	∪
$\dot{x}$	ND	0	+		~~	+		0	−	
$\ddot{x}$	ND	0	+		0	−		0	+	

d)



e) $x(t+\Delta t) = x(t) + \Delta t\, \dot{x}(t) \cdot K$

Se $\Delta t = 20$ ↳ $2x(t)$ $\Rightarrow K = \frac{1}{20}$ $\dot{x} = \frac{1}{20}x(t)$

P2) a)

$Y \cdot s \cdot (s+0.5) = U(1-s)$

$X(s \cdot (s+0.5)) = U$ $Y = X(1-s)$

$\ddot{x} = u - \dot{x}\,0.5$

$x_1 = x$

$x_2 = \dot{x} = \dot{x}_1$

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ 0 & -0.5\end{bmatrix} + \begin{bmatrix}0\\ 1\end{bmatrix} u$$

$$y = \begin{bmatrix}1 & -1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}$$

b) $y = x - \dot{x}$

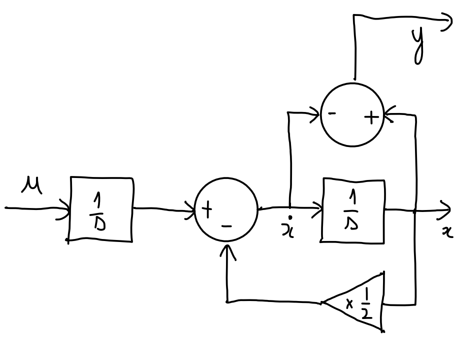
$\ddot{x} = u - 0.5\dot{x}$

↓

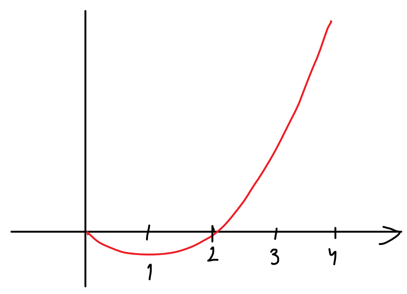
$\dot{x} = -0.5x + \int u$

↓

$x = \int\left(-0.5x + \int u\right)$



c)



P3) a) $m_2\ddot{z}_2 = -\beta\dot{z}_2 - K(z_2 - z_1)$

$m_1\ddot{z}_1 = -\beta\dot{z}_1 - K(z_1 - z_2) + u$

$\dot{x}_2 = -\frac{\beta}{m_1}x_2 - \frac{K}{m_1}x_1 + \frac{K}{m_1}x_3 + \frac{1}{m}u$

$\dot{x}_4 = -\frac{\beta}{m_2}x_4 - \frac{K}{m_2}x_3 + \frac{K}{m_2}x_1$

b) $x_1 = z_1$

$x_2 = \dot{z}_1 = \dot{x}_1$

$x_3 = z_2$

$x_4 = \dot{z}_2 = \dot{x}_3$

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\end{bmatrix} = \begin{bmatrix}0 & 1 & 0 & 0\\ -\frac{K}{m_1} & -\frac{\beta}{m_1} & \frac{K}{m_1} & 0\\ 0 & 0 & 0 & 1\\ \frac{K}{m_2} & 0 & -\frac{K}{m_2} & -\frac{\beta}{m_2}\end{bmatrix} \cdot \begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix} + \begin{bmatrix}0\\ \frac{1}{m_1}\\ 0\\ 0\end{bmatrix} u$$

c) $L = T - U$

$T = \frac{1}{2}m_2(\dot{z}_2)^2 + \frac{1}{2}m_1(\dot{z}_1)^2$

$U = \frac{1}{2}K(z_2 - z_1)^2$

$\frac{\partial L}{\partial z_1} = K(z_2 - z_1)$

$\frac{\partial L}{\partial \dot{z}_1} = m_1\dot{z}_1$

$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{z}_1}\right) = m_1\ddot{z}_1$

1:

$m_1\ddot{z}_1 - K(z_2 - z_1) = 0$

$\frac{\partial L}{\partial z_2} = -K(z_2 - z_1)$

$\frac{\partial L}{\partial \dot{z}_2} = m_2\dot{z}_2$

$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{z}_2}\right) = m_2\ddot{z}_2$

$m_2\ddot{z}_2 + K(z_2 - z_1) = 0$

P4)

① $A = \begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$ $\lambda_1 = 1$, $\lambda_2 = -1$ $\Rightarrow$ Ponto de Sela

$v_1 = \begin{bmatrix}-1\\ 1\end{bmatrix}$, $v_2 = \begin{bmatrix}1\\ 1\end{bmatrix}$ $\Rightarrow$ B

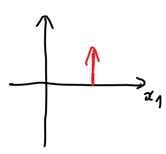
② $A = \begin{bmatrix}0 & 1\\ -2 & -0.6\end{bmatrix}$ $\lambda = -3 \pm 1{,}38j$

foco estável

C

③ $A = \begin{bmatrix}0 & -1\\ 2 & 0\end{bmatrix}$ $\lambda_1 = j\sqrt{2}$, $\lambda_2 = -j\sqrt{2}$ $\Rightarrow$ Centro

$\dot{x}_2 = 2x_1 \rightarrow \frac{\partial \dot{x}_2}{\partial x_1} = 2$, $\frac{\partial \dot{x}_2}{\partial x_2} = 0$



D

④ $A = \begin{bmatrix}-\frac{5}{3} & -\frac{4}{3}\\ \frac{4}{3} & \frac{5}{3}\end{bmatrix}$ $\lambda_1 = -1$, $\lambda_2 = 1$ $\Rightarrow$ Ponto de sela

A

⑤ $A = \begin{bmatrix}0 & 1\\ -2 & 0\end{bmatrix}$ $\lambda = \pm j\sqrt{2} \Rightarrow$ Centro

$\dot{x}_2 = -2x_1 \Rightarrow \frac{\partial \dot{x}_2}{\partial x_1} = -2 \Rightarrow$ F

⑥ $A = \begin{bmatrix}0 & -1\\ 2 & 0.6\end{bmatrix}$ E