P1)

a) $m\dot{v} = F_g - \beta v$

b) se $v = \frac{F_g}{\beta}$ temos $\dot{v} = 0$, logo o corpo não acelera mais

c)

$$\begin{cases} \frac{d}{dt}\left[(M+m)v\right] = Ku - (M+m)g_L \\ -u = \dot{m} \end{cases} \Leftrightarrow \begin{cases} \dot{m}v + (M+m)\dot{v} = -(M+m)g_L + Ku \\ \dot{m} = -u \end{cases}$$

P2)

a) $\dot{x} = Kx(M-x)$

$0 = Kx(M-x) \Leftrightarrow \begin{cases} x = 0 \\ x = M \end{cases}$

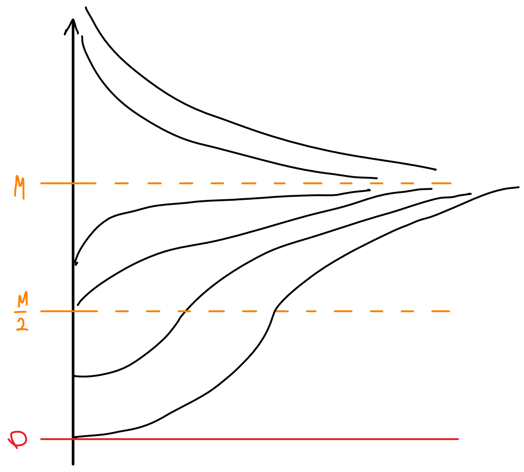
		0		$\frac{M}{2}$		M	
x	ND	[illegible]	↗ ⋮ ∪	[illegible]	↗ ⋮ ∩	[illegible]	↘ ⋮ ∪
$\dot{x}$	ND	0	+	[illegible]	+	0	−
$\ddot{x}$	ND	0	+	0	−	0	+

$$\ddot{x} = \frac{d}{dt}(KMx - Kx^2)$$

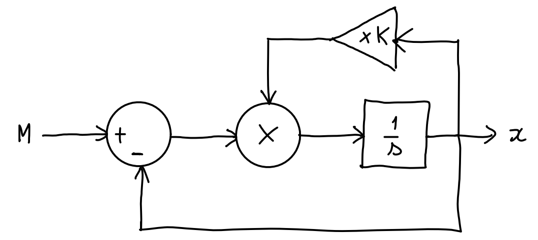
$$= KM\dot{x} - 2Kx\dot{x} =$$

$$= K^2x(M-2x)(M-x)$$

$$\ddot{x} = 0 \begin{cases} x = \frac{M}{2} \\ x = M \\ x = 0 \end{cases}$$



b) $\dot{x} = Kx(M-x) \Leftrightarrow x = \int Kx(M-x)$



P3) G_1: 2 polos imaginários com Re. < 0 $\quad \omega = 1$

↳ Oscila estável $\quad 2\omega\xi = 0.8 \Rightarrow \xi = 0.4$ $\quad$ D

G_2: $\omega = \sqrt{2}$

$2\omega\xi = 3 \Rightarrow \xi > 1 \Rightarrow$ Sobre-amortecido $\Rightarrow$ B

G_3: $NP = NZ + 1 \Rightarrow$ Derivada descontínua na origem $\Rightarrow$ A

G_4: $\omega = 1$

$2\omega\xi = 0.3 \Rightarrow \xi = 0.15$ $\quad$ Menos atenuado $\Rightarrow$ F

G_5: tem zeros $\Rightarrow$ C

G_6: $\omega = \sqrt{0.16}$

$2\omega\xi = 0.32 \Rightarrow \xi = 0.4$ $\quad$ Menor freq. $\Rightarrow$ E

P4) a) $\ddot{y} + 3\dot{y} + 2y = u$

G_1:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

G_2:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 5 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

G_3:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 5 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix}$$

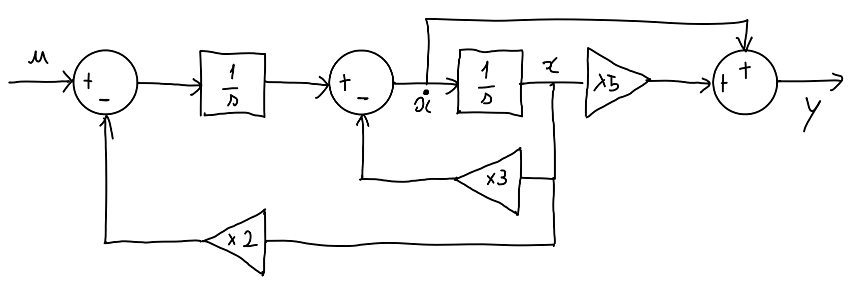
b) $\ddot{y} = u - 3\dot{y} - 2y$

↓

$\dot{y} = -3y + \int u - 2y$

↓

$y = \int -3y + \int u - 2y$



P5)

$$J\dot{\omega} = T_{cc} - \beta_1 \omega - F_1 r$$

$$F_2 = m\ddot{x}_2 = -K(x_2 - x_1) - \beta_2 \dot{x}_2$$

$$F_1 = K(x_1 - x_2) + \beta_2 \dot{x}_2$$

$$\dot{x}_1 = \omega r$$

$$T_{cc} = K u$$

P5A)

a)

$$m_1 \ddot{z}_1 = -K_1 z_1 - \beta_1 \dot{z}_1 - K_3(z_1 - z_2) - \beta_3(\dot{z}_1 - \dot{z}_2) + u$$

$$m_2 \ddot{z}_2 = -K_2 z_2 - \beta_2 \dot{z}_2 - K_3(z_2 - z_1) - \beta_3(\dot{z}_2 - \dot{z}_1)$$

b)

$$x_1 = z_1$$
$$x_2 = \dot{z}_1 = \dot{x}_1$$

$$x_3 = z_2$$
$$x_4 = \dot{z}_2 = \dot{x}_2$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{K_1+K_3}{m_1} & -\frac{\beta_1+\beta_3}{m_1} & \frac{K_3}{m_1} & \frac{\beta_3}{m_1} \\ 0 & 0 & 0 & 1 \\ \frac{K_3}{m_2} & \frac{\beta_3}{m_2} & -\frac{K_2+K_3}{m_2} & -\frac{\beta_2+\beta_3}{m_2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m_1} \\ 0 \\ 0 \end{bmatrix} u$$