P1)

a) $\begin{cases} 0 = x_1 x_2 \\ 0 = 1 - \alpha x_2 - (1+\beta x_2)x_1 \end{cases}$

$x_1 = 0$

$\hookrightarrow 0 = 1 - \alpha x_2 \Leftrightarrow x_2 = \frac{1}{\alpha}$

$x_2 = 0$

$0 = 1 - x_1 \Leftrightarrow x_1 = 1$

PE ① $x_1 = 0 \wedge x_2 = \frac{1}{\alpha}$

② $x_1 = 1 \wedge x_2 = 0$

b) $\frac{\partial \dot{x}_1}{\partial x_1} = x_2 \quad \frac{\partial \dot{x}_1}{\partial x_2} = x_1$

$\frac{\partial \dot{x}_2}{\partial x_1} = -(1+\beta x_2) \quad \frac{\partial \dot{x}_2}{\partial x_2} = -(\alpha + \beta x_1)$

$A = \begin{bmatrix} x_2 & x_1 \\ -(1+\beta x_2) & -(\alpha+\beta x_1) \end{bmatrix}$

① $A = \begin{bmatrix} \frac{1}{\alpha} & 0 \\ -(1+\frac{\beta}{\alpha}) & -\alpha \end{bmatrix}$ ② $A = \begin{bmatrix} 0 & 1 \\ -1 & -(\alpha+\beta) \end{bmatrix}$

c) ① $(\frac{1}{\alpha} - \lambda)(-\alpha - \lambda) = 0 \Leftrightarrow$

$\Leftrightarrow \lambda_1 = -\alpha \vee \lambda_2 = \frac{1}{\alpha}$

Ponto de sela

$\begin{bmatrix} \frac{1}{\alpha} - \lambda_1 & 0 \\ -(1+\frac{\beta}{\alpha}) & -\alpha - \lambda_1 \end{bmatrix} \cdot \begin{bmatrix} v_{11} \\ v_{12} \end{bmatrix} = 0$

$v_{11} \cdot (\frac{1}{\alpha} + \alpha) = 0 \Rightarrow v_{11} = 0$

$v_{12} = 1$

$v_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$\begin{bmatrix} \frac{1}{\alpha} - \lambda_2 & 0 \\ -(1+\frac{\beta}{\alpha}) & -\alpha - \lambda_2 \end{bmatrix} \cdot \begin{bmatrix} v_{21} \\ v_{22} \end{bmatrix} = 0 \Leftrightarrow$

$v_{21} = 1$

$-(1 + \frac{\beta}{\alpha}) - v_{22}(\alpha + \frac{1}{\alpha}) = 0$

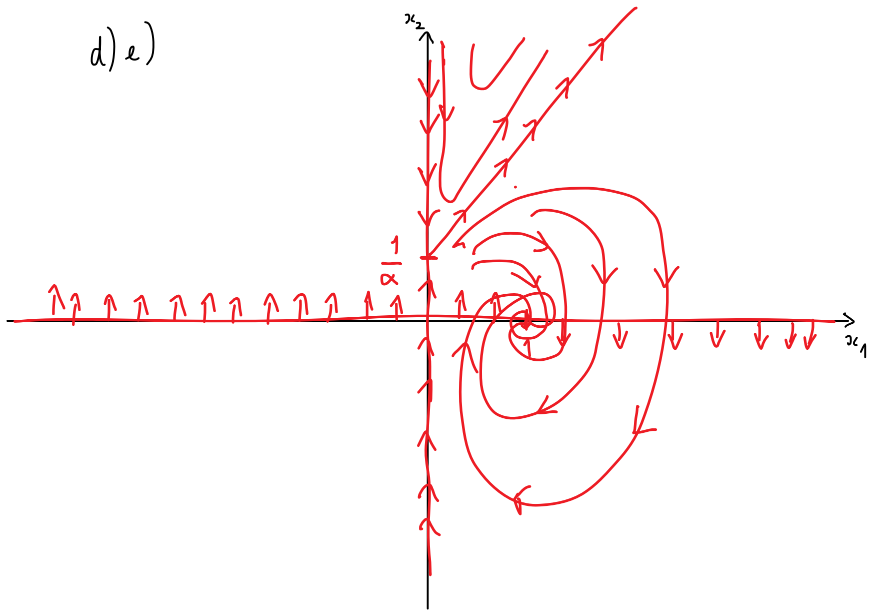
$v_{22} = \frac{1+\frac{\beta}{\alpha}}{\alpha + \frac{1}{\alpha}} = \frac{\alpha+\beta}{\alpha^2+1}$

$v_2 = \begin{bmatrix} 1 \\ \frac{\alpha+\beta}{\alpha^2+1} \end{bmatrix}$

② $\begin{bmatrix} -\lambda & 1 \\ -1 & -(\alpha+\beta)-\lambda \end{bmatrix} \rightarrow \lambda(\alpha+\beta+\lambda) + 2 = 0 \Leftrightarrow \lambda^2 + \overbrace{(\alpha+\beta)}^{<2}\lambda + 2 = 0$

$\hookrightarrow \operatorname{Re}\{\lambda\} < 0 \Rightarrow$ Foco estável

d) e)



f) I → Volta ao valor inicial

II → A intensidade irá oscilar, e no limite será tangencialmente monótono

P2) a) $A = \begin{bmatrix} 0 & -a \\ -b & 0 \end{bmatrix}$

$A - \lambda I = \begin{bmatrix} -\lambda & -a \\ -b & -\lambda \end{bmatrix}$

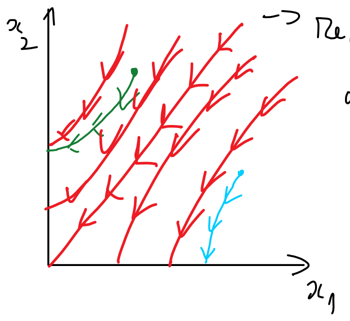
$\lambda^2 - ab = 0$

$\lambda = \pm\sqrt{ab}$

$\begin{bmatrix} -\lambda_1 & -a \\ -b & -\lambda_1 \end{bmatrix} \begin{bmatrix} v_{11} \\ v_{12} \end{bmatrix} = 0 \Leftrightarrow \sqrt{ab}\, v_{11} - a v_{12} = 0 \quad v_1 = \begin{bmatrix} 1 \\ \sqrt{\frac{b}{a}} \end{bmatrix}$

$\begin{bmatrix} -\lambda_2 & -a \\ -b & -\lambda_2 \end{bmatrix} \begin{bmatrix} v_{21} \\ v_{22} \end{bmatrix} = 0 \Leftrightarrow v_2 = \begin{bmatrix} 1 \\ -\sqrt{\frac{b}{a}} \end{bmatrix}$

b)



→ Recta de declive $\sqrt{\frac{b}{a}}$

c) • $x_2 > \sqrt{\frac{b}{a}}\, x_1$

• $x_2 < \sqrt{\frac{b}{a}}\, x_1$

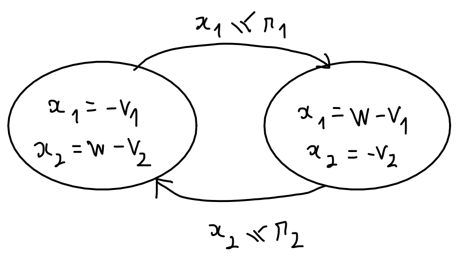
P2) a) b)

$J(\hat{a}) = \sum_{t=1}^{N} \left(y(t) - \hat{a}\, y(t-1) - u(t-1) \right)^2$

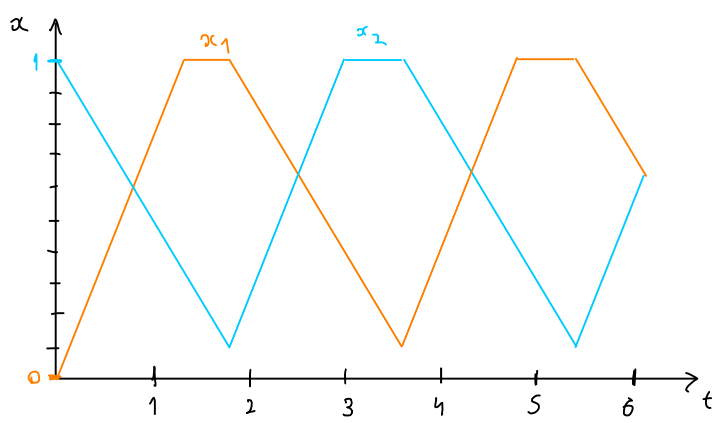
$\frac{1}{2}\frac{dJ}{d\hat{a}} = 0 \Rightarrow -\sum_{t=1}^{N} y(t-1) \cdot \left(y(t) - u(t-1) - \hat{a}\, y(t-1) \right) = 0$

$\hookrightarrow \hat{a} = \frac{\sum_{t=1}^{N} y(t-1)\left(y(t) - u(t-1) \right)}{\sum_{t=1}^{N} y^2(t-1)} = \frac{8.5}{10} = 0.85$

P3) a)



$\boxed{\begin{matrix} x_1 > 0 \\ x_2 > 0 \end{matrix}}$



P4)

a) $\frac{\partial T}{\partial t} = -\mu \, \frac{1}{\Delta} \left(T(i\Delta) - T((i-1)\Delta) \right) + \alpha R(t)$

$4\Delta F L \iff \Delta = \frac{L}{4}$

b) $\dot{T} = -\frac{\mu}{L} \left(T(0) - T(L) \right) + \alpha R(t)$

c) $\dot{T} = -\frac{\bar{\mu}}{L} \left(T(0) - T(L) \right) + \alpha R(t)$

I) $0 = -\frac{\bar{\mu}}{L} \left(T(0) - T(L) \right) + \alpha R(t) \iff T(0) - \frac{\alpha R(t) \cdot L}{\bar{\mu}} = T(L)$ //

II) ?
III)

P4A)

a) $C \dot{T}_m = \frac{1}{R} (T_L - T_m)$

b) $\dot{T}_m = 0 \Rightarrow T_m = T_L$!