P1) a) $\begin{cases} 0 = x_2 \\ 0 = x_1(1-x_1^2) - \mu x_2 \end{cases}$

PE: $x_1 = 0 \wedge x_2 = 0$ ①

$x_1 = 1 \wedge x_2 = 0$ ②

$x_1 = -1 \wedge x_2 = 0$ ③

b) $\frac{\partial f_1}{\partial x_1} = 0 \quad \frac{\partial f_1}{\partial x_2} = 1$

$\frac{\partial f_2}{\partial x_1} = 1 - 3x_1^2 \quad \frac{\partial f_2}{\partial x_2} = -\mu = -1$

$A = \begin{bmatrix} 0 & 1 \\ 1-3x_1^2 & -1 \end{bmatrix}$

① $\begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix}$ ② e ③ $\begin{bmatrix} 0 & 1 \\ -2 & -1 \end{bmatrix}$

c) ① $\det(A - \lambda I) = (-\lambda)(-\lambda - 1) - 1$

$\lambda^2 + \lambda - 1 = 0 \Leftrightarrow \lambda = \frac{-1 \pm \sqrt{5}}{2}$

Ponto de sela!

$\lambda_1 \Rightarrow (A - \lambda_1 I) V_1 = 0$

$-\lambda_1 V_{12} + V_{22} = 0$ $\Rightarrow V_1 = \begin{bmatrix} 1 \\ \frac{-1+\sqrt{5}}{2} \end{bmatrix} \approx \begin{bmatrix} 1 \\ 0{,}62 \end{bmatrix}$

$\lambda_2 \Rightarrow (A - \lambda_2 I) V_2 = 0$

$-(-\frac{1+\sqrt{5}}{2}) + V_{22} = 0$ $\Rightarrow V_2 = \begin{bmatrix} 1 \\ -\frac{1+\sqrt{5}}{2} \end{bmatrix} \approx \begin{bmatrix} 1 \\ -1{,}62 \end{bmatrix}$

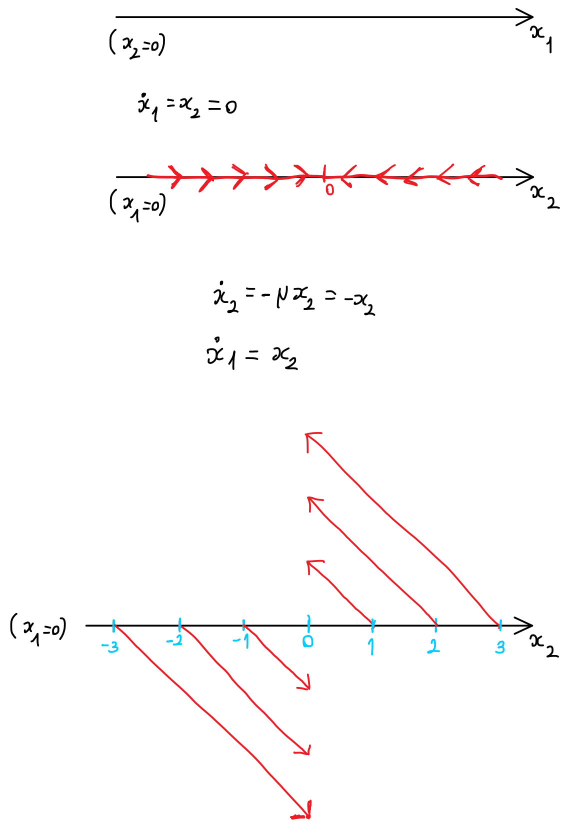
② e ③ $(-\lambda)(-\lambda - 1) + 2 = 0 \Leftrightarrow$

$\Leftrightarrow \lambda^2 + \lambda + 2 = 0 \Leftrightarrow$

$\Leftrightarrow \lambda = \frac{-1 \pm \sqrt{1-8}}{2} \Leftrightarrow \lambda = -\frac{1}{2} \pm j\sqrt{7}$

Foco estável

d)

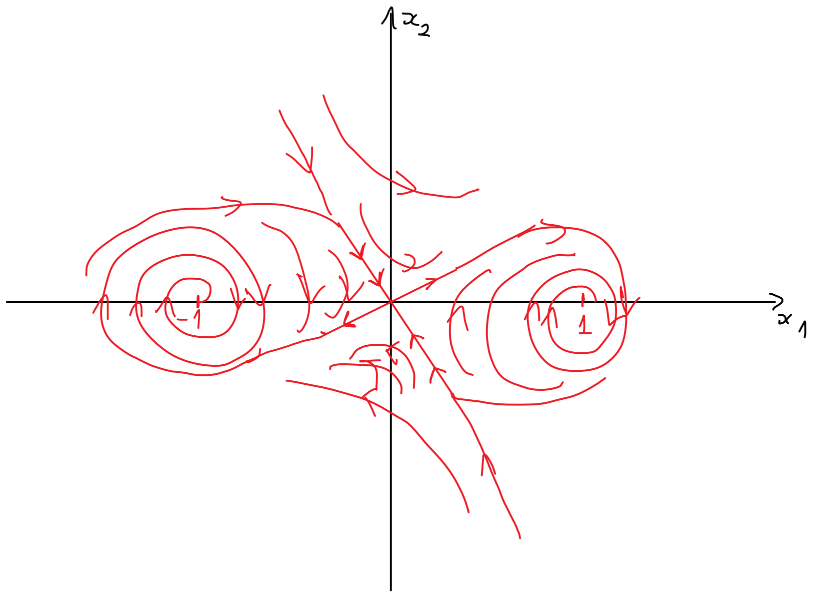


e) $\dot{x}_1 = x_2$ $\dot{x}_1' = -x_2$

$\dot{x}_2 = x_1(1-x_1^2) - \mu x_2$ $\dot{x}_2' = -x_1(1-x_1^2) + \mu x_2$ $\Rightarrow$ $\dot{x}_1' = -\dot{x}_1$, $\dot{x}_2' = -\dot{x}_2$

! Se $(x_1, x_2) \Rightarrow (\dot{x}_1, \dot{x}_2)$ então $(-x_1, -x_2) \Rightarrow (-\dot{x}_1, -\dot{x}_2)$

f)



g) $\dot{x}_1 = x_2$

$\dot{x}_2 = x_1(1-x_1^2)$

$A = \begin{bmatrix} 0 & 1 \\ 1-3x_1^2 & 0 \end{bmatrix}$ Para ② e ③ $\Rightarrow \begin{bmatrix} 0 & 1 \\ -2 & 0 \end{bmatrix}$

Centro

Temos 2 centros, e não sabemos o comportamento logo o desenho do retrato de fase não será correcto.

P1-A) $\dot{x}_1 = \alpha x_2$

$\dot{x}_2 = \beta x_1$

a) PE

① $x_2 = 0 \wedge x_1 = 0$

$\frac{\partial \dot{x}_1}{\partial x_1} = 0 \quad \frac{\partial \dot{x}_1}{\partial x_2} = \alpha$

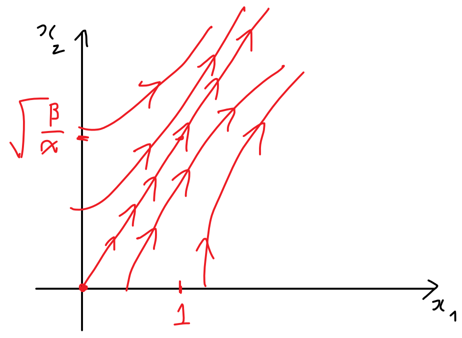
$\frac{\partial \dot{x}_2}{\partial x_1} = \beta \quad \frac{\partial \dot{x}_2}{\partial x_2} = 0$

$A = \begin{bmatrix} 0 & \alpha \\ \beta & 0 \end{bmatrix}$

$\det(A - \lambda I) = (-\lambda)(-\lambda) - \alpha\beta$

$\lambda^2 = \alpha\beta \Leftrightarrow \lambda = \pm\sqrt{\alpha\beta}$

Como $\alpha > 0$ e $\beta > 0 \Rightarrow$ Ponto de sela



$\lambda_1 = \sqrt{\alpha\beta}$

$(A - \lambda I) V_1 = 0 \Leftrightarrow -\lambda V_{11} + \alpha V_{12} = 0$

$V_{11} = 1 \quad V_{12} = \frac{\sqrt{\alpha\beta}}{\alpha} \Rightarrow V_1 = \begin{bmatrix} 1 \\ \sqrt{\frac{\beta}{\alpha}} \end{bmatrix}$

$\lambda_2 = -\sqrt{\alpha\beta}$

$V_{21} = 1 \quad V_{21} = -\sqrt{\frac{\beta}{\alpha}}$

b) Se $x_1 = 0$ e $x_2 \neq 0$

Por inspecção do retrato de fase concluimos que se vão afastar cada vez mais

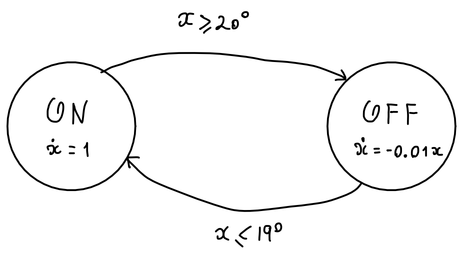
P2) a) b) $\mathcal{J}(\hat{a}) = \sum_{t=1}^{N} \left(y(t) - \hat{a}\, y(t-1) - u(t-1) \right)^2$

$\frac{1}{2} \frac{d\mathcal{J}}{d\hat{a}} = 0 \Rightarrow -\sum_{t=1}^{N} y(t-1) \cdot \left(y(t) - u(t-1) - \hat{a}\, y(t-1) \right) = 0$

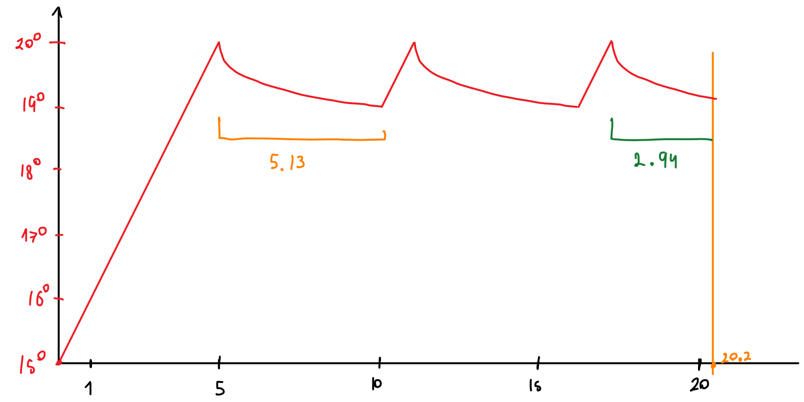
$\hat{a} = \frac{\sum_{t=1}^{N} y(t-1)\left(y(t) - u(t-1) \right)}{\sum_{t=1}^{N} y^2(t-1)} = \frac{5}{9}$

P3)

a)



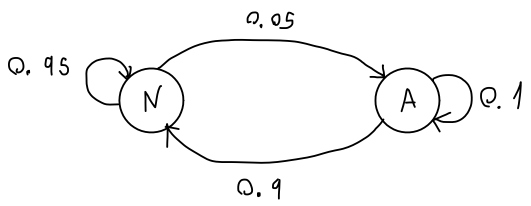
b)



$$19 = 20e^{-0.01t} \Leftrightarrow \frac{\ln\left(\frac{19}{20}\right)}{-0.01} = t \Leftrightarrow t = 5.13$$

P4)

a)



	N	A
N	0.95	0.05
A	0.9	0.1

$$\Rightarrow \quad A = \begin{bmatrix} 0.95 & 0.05 \\ 0.9 & 0.1 \end{bmatrix}$$

b) $P(k+1) = A^T P(k)$

$$\begin{bmatrix} P_N(k+1) \\ P_A(k+1) \end{bmatrix} = \begin{bmatrix} 0.95 & 0.9 \\ 0.05 & 0.1 \end{bmatrix} \cdot \begin{bmatrix} P_N(k) \\ P_A(k) \end{bmatrix}$$

c) $P_A = 1 - P_N$

$P_N = 0.95\, P_N + 0.9\,(1 - P_N) \Leftrightarrow$

$\Leftrightarrow 0.05 P_N + 0.9 P_N = 0.9 \Leftrightarrow$

$$\Leftrightarrow \begin{cases} P_N = \frac{18}{19} \\ P_A = \frac{1}{19} \end{cases}$$