1) The two-phase series capacitor buck converter shown in Fig. 1 operates in CCM at $f_{sw} = 1/T_s = 1MHz$. The switch driving waveforms are illustrated in Fig. 2.

a) Derive the DC voltage transfer function V_O/V_{IN} as a function of D (assume the converter is lossless and the inductance values are matched (i.e. $L_a = L_b$)).

Assuming V_{IN} = 12V, V_O = 1.2V, P_O = 12W:

b) calculate the average currents I_{La} and I_{Lb}.

c) Select the inductance values, L_a and L_b, such that the peak-to-peak current ripple is less than 20% of the average current.

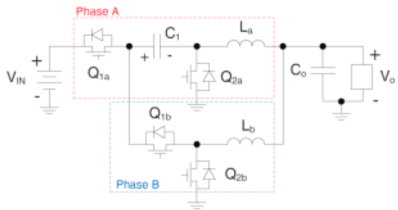
d) Sketch a plot of the current flowing through C_1 as a function of time.

e) Select C_1 to ensure that the peak-to-peak voltage ripple is less than 0.3V.

f) Sketch a plot of the current flowing through L_a and L_b.

g) Calculate the peak-to-peak current ripple in C_O.

h) Calculate the peak-to-peak output voltage ripple (C_O =47 μF, ESR=5mΩ).



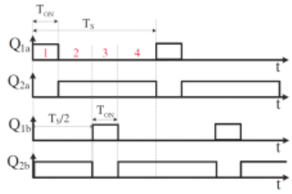
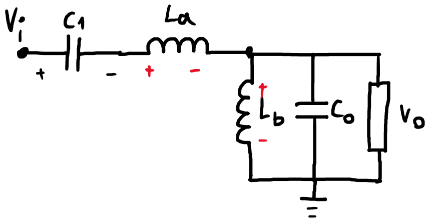
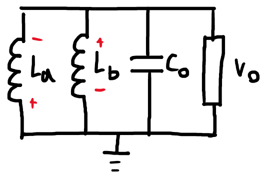


Fig. 2

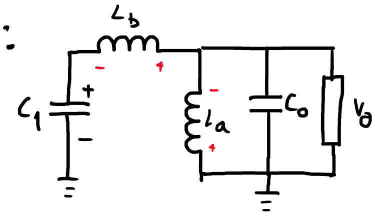
a) T1:



T2/4:



T3:



$$L_a: (V_i - V_{C_1} - V_o) D\cancel{T_s} - V_o\left(\frac{T_s}{2} - D\cancel{T_s}\right)2 - V_o D\cancel{T_s} = 0 \Leftrightarrow$$

$$\Leftrightarrow V_i D - V_{C_1} D - V_o D - V_o(1-2D) - V_o D = 0 \Leftrightarrow V_i D - V_{C_1} D - V_o = 0$$

$$L_b: V_o D\cancel{T_s} + V_o\left(\frac{T_s}{2} - D\cancel{T_s}\right)2 + (V_o - V_{C_1}) D\cancel{T_s} = 0 \Leftrightarrow$$

$$V_o D + V_o(1-2D) + V_o D = V_{C_1} D \Leftrightarrow V_{C_1} = \frac{V_o}{D}$$

$$V_i D - 2V_o = 0 \Leftrightarrow \frac{V_o}{V_{in}} = \frac{D}{2}$$

b) $\frac{V_o}{V_{in}} = \frac{1.2}{12} = 0.1 \Rightarrow D = 0.2$

$I_o + I_b = I_a \qquad I_a = \frac{I_o}{2} = 5\,A$

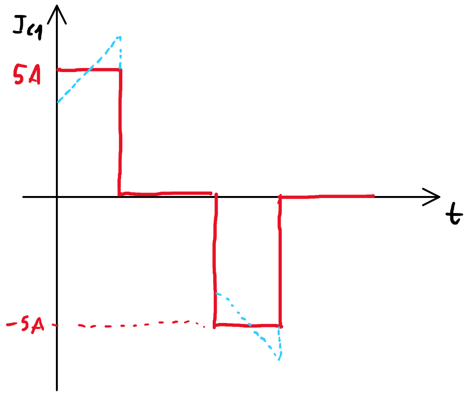
$C_1: I_a D = -I_b D \qquad I_b = -I_a = \frac{I_o}{2} = -5A$

c) $V = L\frac{\Delta i}{\Delta t}$

$I_a: L > \frac{V_o D' T_s}{0.2 I_a} \Leftrightarrow L > 9.6\,\mu H$

$I_b: L > \frac{V_o D' T_s}{0.2 I_a} \Leftrightarrow L > 9.6\,\mu H$

d)

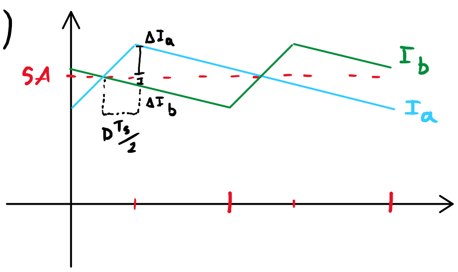


e)

$$i = C\frac{\Delta V}{\Delta t} \Leftrightarrow \Delta V = \frac{I\Delta t}{C} \Leftrightarrow C > \frac{I\Delta t}{\Delta V} \Leftrightarrow C > \frac{I D T_s}{\Delta V}$$

$C > 3.33\,\mu F$

f)



g) $\frac{\Delta i_o}{2} = \Delta I_a - \Delta I_b \Leftrightarrow \Delta i = \frac{V_o(1-2D)}{L f_s} = 0.75A$

$\Delta I_a = (V_{IN} - V_{C_1} - V_o)\frac{D}{2}T_s \cdot \frac{1}{L}$

$\Delta I_b = V_o \frac{D}{2} T_s \cdot \frac{1}{L}$

h) $\Delta V_o = \Delta V_C + \Delta V_{ESR}$

$$\Delta V_C = \frac{1}{2}\frac{T_s}{4}\cdot\frac{1}{C}\cdot\frac{\Delta I_o}{2} = \frac{\Delta I_o}{16 f_s C} = 1\,mV$$

$$\Delta V_{ESR} = \Delta I_o \cdot ESR = 3.75\,mV$$

2) The converter in Fig. 3 can be drawn in the electrically identical form shown in Fig. 4 ($q(t)=1$ when the switches are in position 1).
a) Use the circuit averaging method to derive a large-signal averaged model for this converter.
b) Perturb and linearize the averaged circuit model to obtain a small-signal ac equivalent circuit of the converter.
c) Derive the transfer function $G_{od}(s) = \frac{\hat{v}_o}{\hat{d}}$. Sketch the Bode plot of $|G_{od}(j\omega)|$.

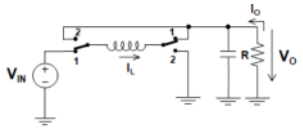
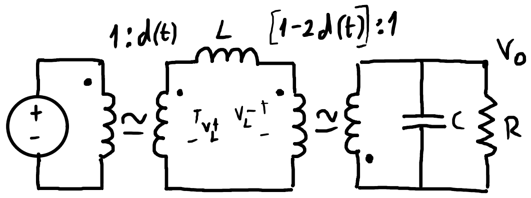


Fig. 3

Fig. 4

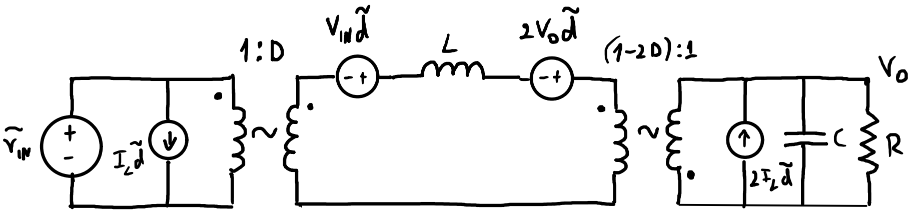
a)
$$i_{IN} = i_L\, q(t) \qquad\qquad \text{Avg.} \quad \langle i_{IN}\rangle = \langle i_L\rangle\, d(t)$$
$$V_L^+ = V_{IN}\, q(t) \qquad\qquad \langle V_L^+\rangle = \langle V_{IN}\rangle\, d(t)$$
$$V_L^- = V_o(1-2q(t)) \quad \Rightarrow \quad \langle V_L^-\rangle = \langle V_o\rangle(1-2d(t))$$
$$i_o = i_L(1-2q(t)) \qquad\qquad \langle i_o\rangle = \langle i_L\rangle(1-2d(t))$$



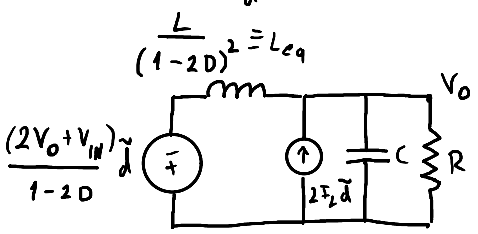
b)
$$\langle i_{IN}\rangle = \langle i_L\rangle\, d(t) \qquad\qquad I_{IN} + \tilde{i}_{IN} = (I_L + \tilde{i}_L)(D + \tilde{d})$$
$$\langle V_L^+\rangle = \langle V_{IN}\rangle\, d(t) \qquad\qquad V_L^+ + \tilde{v}_L^+ = (V_{IN} + \tilde{v}_{IN})(D + \tilde{d})$$
$$\langle V_L^-\rangle = \langle V_o\rangle(1-2d(t)) \quad \Rightarrow \quad V_L^- + \tilde{v}_L^- = (V_o + \tilde{v}_o)(1 - 2D - 2\tilde{d})$$
$$\langle i_o\rangle = \langle i_L\rangle(1-2d(t)) \qquad\qquad I_o + \tilde{i}_o = (I_L + \tilde{i}_L)(1 - 2D - 2\tilde{d})$$

DC:
$$I_{IN} = I_L D$$
$$V_L^+ = V_{IN} D$$
$$V_L^- = V_o(1-2D)$$
$$I_o = I_L(1-2D)$$

AC:
$$\tilde{i}_{IN} = I_L \tilde{d} + \tilde{i}_L D$$
$$\tilde{v}_L^+ = V_{IN}\tilde{d} + \tilde{v}_{IN} D$$
$$\tilde{v}_L^- = \tilde{v}_o(1-2D) - V_o 2\tilde{d}$$
$$\tilde{i}_o = \tilde{i}_L(1-2D) - I_L 2\tilde{d}$$



c) $G_{od}(s) = \frac{\tilde{v}_o}{\tilde{d}}$



$$\frac{\left(-\frac{2V_o+V_{IN}}{1-2D}\tilde{d} - \tilde{v}_o\right)}{L_{eq} s} + 2I_L\tilde{d} = \tilde{i}_o$$

$$\tilde{v}_o = \tilde{i}_o(R /\!/ C) = \frac{R}{1+sRC}\tilde{i}_o$$

$$\left(\frac{\left(-\frac{2V_o+V_{IN}}{1-2D}\tilde{d} - \tilde{v}_o\right)}{L_{eq} s} + 2I_L\tilde{d}\right)\cdot\frac{R}{1+sRC} = \tilde{v}_o$$

$$\tilde{d}\,\frac{2(1-2D)I_L L_{eq} s - 2V_o - V_{IN}}{(1-2D) L_{eq} s R} = \tilde{v}_o \frac{s L_{eq} + s^2 RC L_{eq} + R}{L_{eq} s}$$

$$\frac{\tilde{v}_o}{\tilde{d}} = \frac{2(1-2D)I_L L_{eq} s - 2V_o - V_{IN}}{(1-2D)\cancel{L_{eq}} s\cdot R}\cdot\frac{\cancel{L_{eq} s}}{s L_{eq} + s^2 RC L_{eq} + R} = -\frac{V_{IN}+2V_o}{(1-2D)}\cdot\frac{1 - 2 s L_{eq} I_L \frac{1-2D}{V_{IN}+2V_o}}{s^2 C L_{eq} + s\frac{L_{eq}}{R} + 1}$$

$$\frac{\tilde{v}_O}{\tilde{d}} = -\frac{V_{IN}+2V_O}{(1-2D)} \cdot \frac{1 - 2 s L_{eq} I_L \frac{1-2D}{V_{IN}+2V_O}}{s^2 C L_{eq} + s \frac{L_{eq}}{R} + 1}$$

$$V_O = V_{IN} \frac{D}{1-2D}$$

$$I_L = \frac{V_O}{R} \frac{1}{1-2D}$$

$$\frac{\tilde{v}_O}{\tilde{d}} = -\frac{V_{IN}}{(1-2D)^2} \cdot \frac{1 - s 2 L_{eq} \frac{1}{R} D}{s^2 C L_{eq} + s \frac{L_{eq}}{R} + 1}$$

$$\omega_{HRPZ} = \frac{R}{2 L_{eq} D}$$

$$\omega_o = \sqrt{\frac{1}{C L_{eq}}}$$

$$\frac{\tilde{v}_o}{\tilde{d}} = -\frac{V_{IN}}{(1-2D)^2} \cdot \frac{\left(1 - \frac{s}{\omega_{RHPZ}}\right)}{\frac{s^2}{\omega_o^2} + s \frac{2\zeta}{\omega_o} + 1}$$

$$\zeta = \frac{1}{2R} \sqrt{\frac{L_{eq}}{C}}$$