1) Fig. 1 shows a closed-loop buck converter with current-programmed mode (CPM) controller (peak current-mode). No compensation ramp is employed. The op-amp and the converter can be considered ideal. Consider the following specifications: $V_{in} = 12$ V, $V_{ref} = 5$ V, $C = 100$ µF, $L = 4$ µH, fs = 200 kHz, $I_o = V_o/R = 20$ A, $R_f = 0.1\ \Omega$.

a) Calculate the steady-state duty cycle, D.

b) Derive the transfer function $G_{oc}(s) = \frac{\hat{v}_o}{\hat{v}_c}$ (**hint**: consider a simple first-order model where the inductor is replaced by a VCCS). Sketch the Bode plot of $|G_{oc}(j\omega)|$.

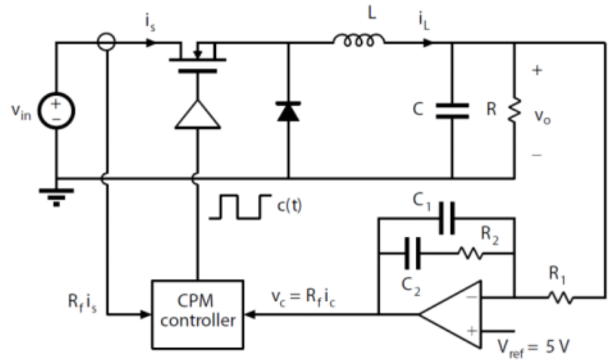
An open-loop transfer function, L(s) is desired, having a pole in the origin, a second pole at $f_{p2} = 50$ KHz and a phase-margin, $\varphi_m = 68°$.

c) Calculate the crossover frequency, f_c.

d) Derive the compensator transfer function, G_c (s). Sketch the Bode plot of $|G_c(j\omega)|$.

e) Given $R_1 = 10$ kΩ, design the compensator, i.e., select R_2, C_1, and C_2 to meet the design specifications.

f) Sketch the asymptotic plot of the closed-loop output impedance



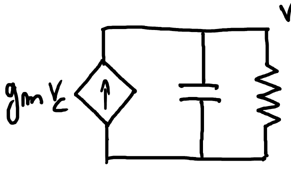
1) $(V_{IN} - V_o)D - V_o(1-D) = 0 \Leftrightarrow \frac{V_o}{V_{IN}} = D$

$V_o = V_{Ref} = 5V$

$V_{IN} = 12V$ $\Rightarrow \frac{5}{12} = D = 0.417$

$V_o = g_m V_c \cdot (R // C)$

b)



$g_m = \frac{1}{R_f}$

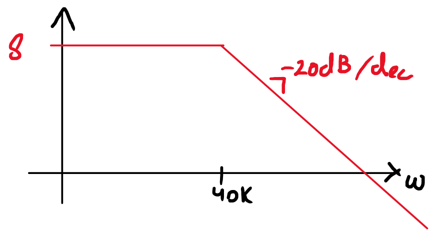
$(R//C) = \left(\frac{1}{R} + j\omega C\right)^{-1} = \frac{R}{1 + sCR}$

$G_{oc} = \frac{V_o}{V_c} = \frac{R}{R_f} \cdot \frac{1}{1 + sCR}$

$|G_{oc}(0)| = \frac{R}{R_f} = 2.5 \Rightarrow 7.96\,dB$

$\omega_p = \frac{1}{CR} = 40K\ rad/s$

$\hookrightarrow 6.37\,KHz$



c) $L(s) = G_c(s) \cdot G_{oc}(s)$

$\varphi_m = 68° = 1.1868$ rad

$L(s) = \frac{K}{s} \cdot \frac{1}{1 + s/\omega_{p_2}}$

Phase from pole at origin

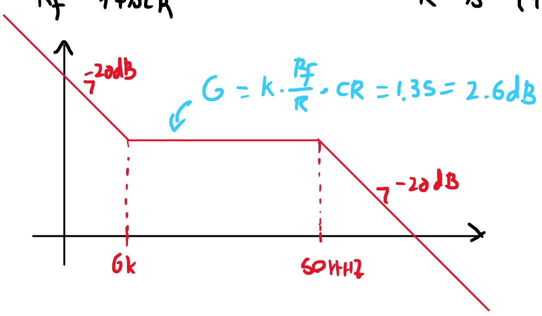
$\varphi_m = 180 + \angle[L(j\omega_c)] = -\arctan\left(\frac{\omega}{\omega_p}\right) - \frac{\pi}{2}$

$\tan\left(-\varphi_m - \frac{\pi}{2}\right) \cdot \omega_{p_2} = \omega = 20.2\,KHz$

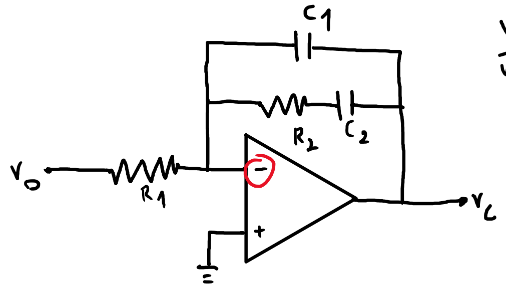
d) $1 = \left| \frac{K}{s} \cdot \frac{1}{1 + s/\omega_{p_2}} \right|$

$1 = \frac{|K|}{\left| j\omega_o - \frac{\omega_o^2}{\omega_p} \right|} \Rightarrow K = \omega_o \sqrt{1 + \frac{\omega_o^2}{\omega_p^2}} = 136 \times 10^3 = 1.36 \times 10^5 = 102.7\,dB$

$\frac{K}{s} \cdot \frac{1}{1 + s/\omega_{p2}} = \frac{R}{R_f} \cdot \frac{1}{1 + sCR} \cdot G_c \Leftrightarrow G_c = K \cdot \frac{R_f}{R} \cdot \frac{1}{s} \cdot \frac{(1 + sCR)}{(1 + s/\omega_{p_2})}$



e)



$$\frac{V_o}{V_c} = -G_c = -\frac{1}{sC_2R_1} \cdot \frac{1+sR_2C_2}{1+sR_2C_1}$$

$$G_c = K \cdot \frac{R_f}{R} \cdot \frac{1}{s} \cdot \frac{(1+sCR)}{(1+s/\omega_{p_2})}$$

$$\frac{1}{C_2R_1} = K \cdot \frac{R_f}{R} \Rightarrow C_2 = \frac{1}{R_1} \cdot \frac{1}{K} \cdot \frac{R}{R_f} = 1.85\, mF$$

$$R_2C_2 = CR \Rightarrow R_2 = \frac{CR}{C_2} = 13.5\, K\Omega$$

$$\frac{1}{\omega_{p_2}} = R_2C_1 \Rightarrow C_1 = \frac{1}{\omega_{p_2}R_2} = 236\, pF$$

f) $$Z_{oCL} = \frac{Z_{oOL}}{1+L(s)}$$

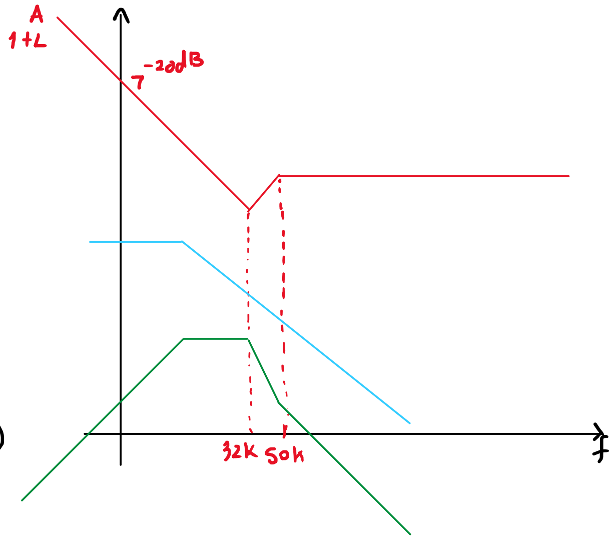
$$Z_{oOL} = \frac{R}{1+sCR}$$

$$\frac{1}{\omega_{p_2}K} = \frac{1}{\omega_0^2}$$

$$A = 1+L = 1 + \frac{K}{s} \cdot \frac{1}{1+s/\omega_{p_2}}$$

32 K

$$\frac{s\cdot(1+s/\omega_{p_2}) + K}{s\cdot(1+s/\omega_{p_2})} = \frac{\frac{s^2}{\omega_{p2}} + s + K}{s\cdot(1+s/\omega_{p2})} = K \cdot \frac{\frac{s^2}{\omega_{p2}K} + \frac{s}{K} + 1}{s\cdot(1+s/\omega_{p2})}$$



2) Fig. 2 shows a two-switch flyback converter. Switches Q1 and Q2 are gated simultaneously. Consider the following specifications: f_s=100kHz, V_{in} = 200 V, V_o =20 V, P_o = 100 W, n=0.2, L_m = 20 µH (referred to the primary side). Diodes D1 and D2 provide a path for leakage inductance current to flow after switches Q1 and Q2 are switched off. Assume that the leakage inductance, L_l is much smaller than L_m.

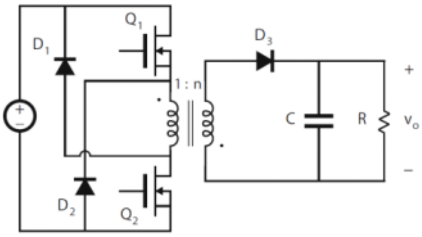
a) Check that the converter is operating in DCM.

b) Show that $\frac{V_O}{V_{IN}} = \frac{D}{\sqrt{K}}$, where $K = \frac{2 \cdot L_m}{R \cdot T_S}$ and compute the duty cycle at which the converter operates.

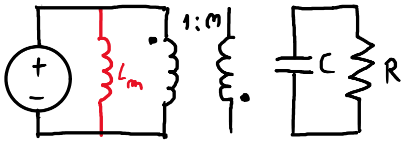
c) Sketch a plot of v_{ds} as a function of time for transistors Q1 and Q2.

d) Calculate the time needed to demagnetize the leakage inductance, assuming L_l=1 µH.

e) The conversion ratio V_o/V_{in} is independent of the turn ratio, n. Is there any advantage in increasing n? Briefly motivate your answer.



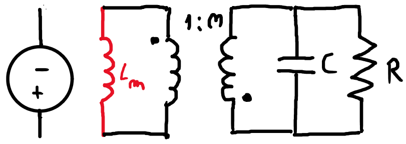
2) ON



at L_m: $V_{in} D - \frac{V_o}{m}(1-D) = 0 \Rightarrow D = \frac{1}{3}$ $\quad \frac{V_o}{V_{in}} = \frac{D}{1-D} m$

$$I_{Lmb} = \frac{\Delta I}{2} = \frac{V_{IN} D}{2 L_m f_s} = \frac{V_o (1-D)}{2 L_m f_s} \cdot \frac{1}{m}$$

OFF



$$I_o \text{ in CCM} = I_{in} \cdot \frac{1-D}{D} \cdot \frac{1}{m} \Leftrightarrow \frac{V_o}{R} = \bar{I}_L (1-D) \cdot \frac{1}{m} \Leftrightarrow \bar{I}_L = \frac{V_o}{(1-D) R} \cdot m$$

$$\bar{I}_L = \frac{I_{in}}{D}$$

$$\bar{I}_L < I_{Lmb} \Leftrightarrow \frac{I_o m}{(1-D)} < \frac{V_o (1-D)}{2 L_m f_s} \cdot \frac{1}{m} \Leftrightarrow$$

$$\Leftrightarrow I_o < \frac{V_o (1-D)^2}{2 L_m f_s} \cdot \frac{1}{m^2} \Leftrightarrow P_o < \frac{V_o^2 (1-D)^2}{2 L_m f_s} \cdot \frac{1}{m^2}$$

↳ 100 < 1000 W ! It's DCM!

b) $P_o = \frac{V_o^2}{R} = \frac{1}{2} L_m \Delta I^2 f_s$

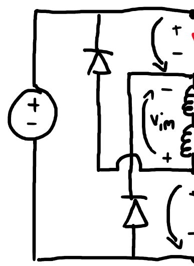
$$\Delta I = \frac{V_{IN} D}{L_m f_s} \qquad \frac{V_o^2}{R} = \frac{1}{2} L_m \left(\frac{V_{IN} D}{L_m f_s}\right)^2 f_s \Leftrightarrow \frac{V_o}{V_{IN}} = D \sqrt{\frac{R T_s}{2 L_m}} \ ! \quad D = 0.1!$$

2c) 1) L_l discharge

2) L_m discharge

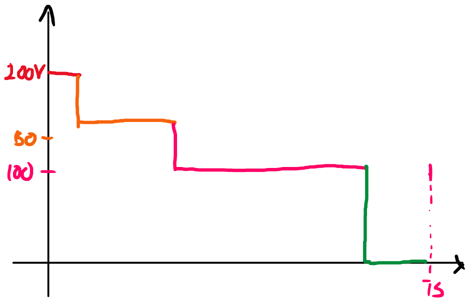
3) Rest

①

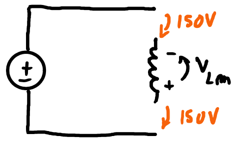


$V_{Ll} = (V_{in} - \frac{V_o}{m})$

$V_{Lm} = \frac{V_o}{m}$

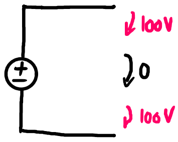


②



$V_{Lm} = \frac{V_o}{m}$

③



d) $V_{in} - \frac{V_o}{m} = L_l \frac{\Delta I}{\Delta t}$ ↳ $\Delta t = 100$ ms

$\Delta I \approx \frac{V_{in} D}{L_m f_s}$

e) Reduce core losses: $\hat{B} \propto \frac{1}{m}$

Tune L_m : $L_m \propto m$