1) The current-fed push-pull converter shown in Fig. 1 is operating at f_{sw}=500 kHz (assume ideal switches, diodes and transformer).

a) Derive the DC transfer function V_O/I_{IN} assuming $n_1=n_2$.

Consider the following specifications: I_{IN} = 5A, V_O=5 V, P_O= 10 W.

b) Calculate the duty-cycle D.

c) Sketch a plot of the current i_d as a function of time.

d) Calculate the peak-to-peak output voltage ripple (C = 470 μF, ESR = 10 mΩ).

e) Derive the transfer functions $G_{oc}(s) = \frac{\tilde{v}_o}{\tilde{v}_c}$, assuming G_{PWM} (s)=1 V^{-1} (**hint**: perturb and linearize the average diode current $\langle i_d \rangle_{T_s}$). Sketch the Bode plot of $G_{oc}(s)$.

f) Determine the compensator transfer function, G_c (s), such that the loop gain has a constant slope of -20 dB / dec, crossing the 0dB axis at f_c = 50 kHz.

g) Sketch the asymptotic plot of the closed-loop output impedance.

h) Sketch the transient response to a load step of 0.5A.

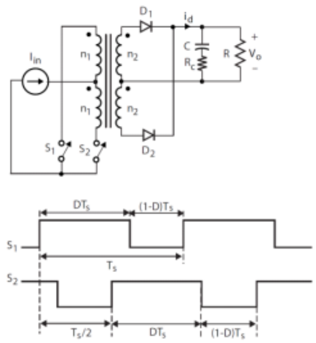
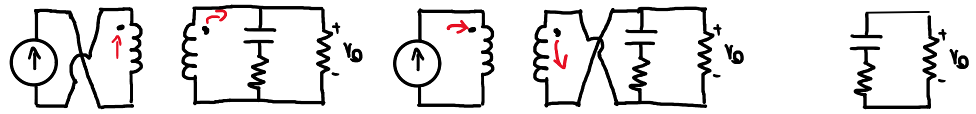


Fig. 1

a) S1 ON S2 OFF | S1 OFF S2 ON | S1 ON S2 ON

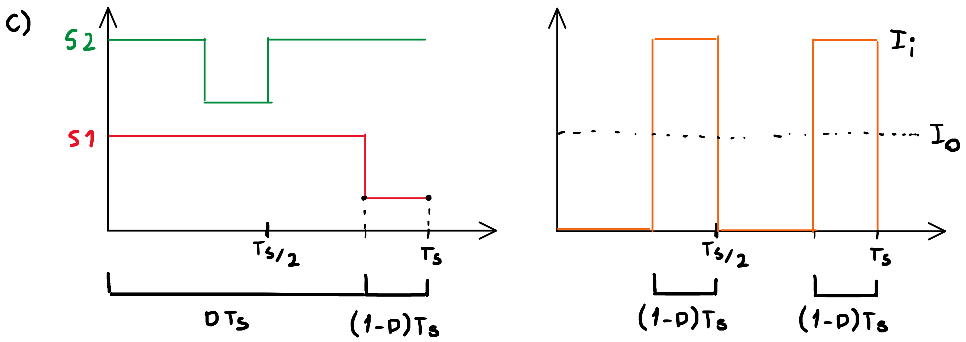


$$I_D = I_O = \frac{I_i(1-D)T_s + I_i(1-D)T_s}{T_s} \Leftrightarrow \frac{V_o}{R} = 2I_i(1-D) \Leftrightarrow \frac{V_o}{I_i} = 2R(1-D)$$

b) $I_i = 5A$, $V_o = 5V$, $P = 10W$ $\Rightarrow$ $I_o = 2A$, $R = 2.5\Omega$

$$1 - \frac{V_o}{2\cdot R\cdot I_i} = D = 0.8$$

c)



d)

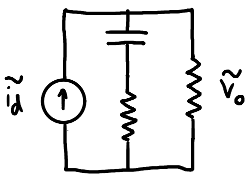
$$\Delta V_o = \Delta V_c + \Delta ESR = 52.55 mV$$

$$\Delta ESR = ESR \cdot \Delta I = 50 mV$$

$$I = C\frac{\Delta V}{\Delta t} \Rightarrow \Delta V_c = \frac{(I_{in} - I_o)(1-D)}{f_s \cdot C} = 2.55 mV$$

e) $I_d = 2I_i(1-D)$

$$I_d + \tilde{i}_d = 2(I_i + \tilde{i}_i)(1-D-\tilde{d})$$



$$\frac{\tilde{v}_o}{\tilde{i}_d} = \frac{R(1+sR_cC)}{1+sC(R+R_c)}$$

$$I_d = 2I_i(1-D)$$

$$\tilde{i}_d = 2\tilde{i}_i(1-D) - 2I_i\tilde{d}$$

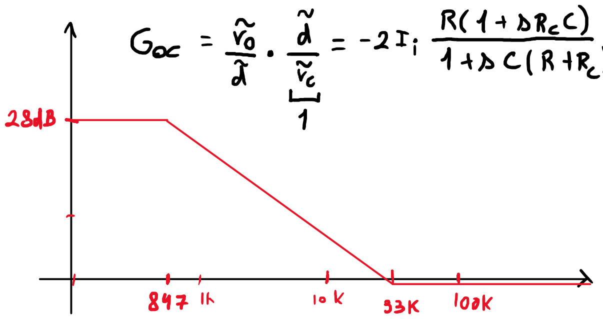
$$\frac{\tilde{i}_d}{\tilde{d}} = -2I_i \Leftrightarrow \frac{\tilde{v}_o}{\tilde{d}} = -2I_i\frac{R(1+sR_cC)}{1+sC(R+R_c)}$$

$$G_{oc} = \frac{\tilde{v}_o}{\tilde{d}} \cdot \underbrace{\frac{\tilde{d}}{\tilde{v}_c}}_{1} = -2I_i\frac{R(1+sR_cC)}{1+sC(R+R_c)}$$

$$|G_{oc}(0)| = 2I_iR = 25 \cong 28dB$$

$$\omega_z = \frac{1}{R_cC} = 212\ Krad/s = 33.9\ KHz$$

$$\omega_p = \frac{1}{C(R+R_c)} = 847\ rad/s = 135\ Hz$$



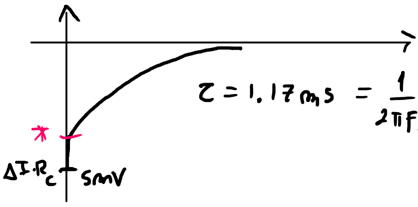
$$G_{oc} = -2I_i \frac{R(1+sR_cC)}{1+sC(R+R_c)} \qquad L = \frac{\omega_c}{s} \Rightarrow \frac{1}{1+L} = \frac{1}{1+\frac{\omega_c}{s}} = \frac{s/\omega_c}{\frac{s}{\omega_c}+1}$$

$$L = G_{oc} \cdot G_c \qquad G_c = \frac{L}{G_{oc}} = \frac{1}{2I_i} \cdot \frac{\omega_c}{s} \cdot \frac{1+sC(R+R_c)}{R(1+sR_cC)}$$

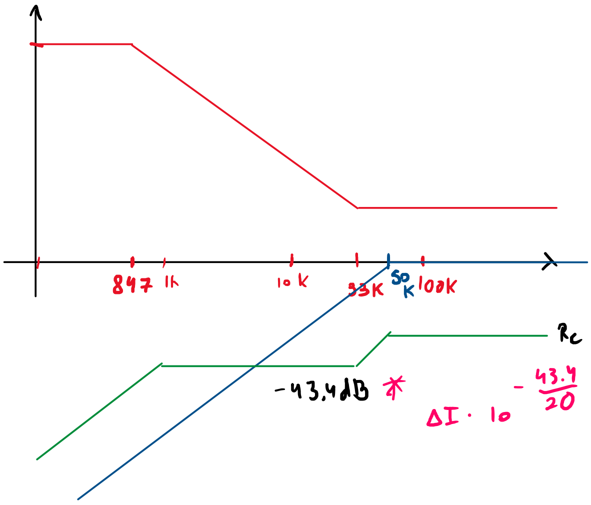
g) $$Z_{o\,cl} = \frac{Z_{o\,ol}}{1+L}$$

$$Z_o\big|_{ol} = R \,//\, \left(R_c + \frac{1}{sC}\right) = \frac{R(1+sR_cC)}{1+sC(R+R_c)}$$

h) $I_{Load} = 0.5\,A$



$$\tau = 1.17\,ms = \frac{1}{2\pi f_p}$$



2) The Cuk converter in Fig. 2 operates in CCM at f_{sw} = 250 kHz.
Consider the following specifications: V_{IN} = 12V, V_O=12 V, P_O = 60 W; L_1=54 µH; L_2=27µH; C_1=22 µF; C_2=100µF. Ripples are small.
a) Derive the DC voltage transfer function V_O/V_{IN}.
b) Calculate the average inductor currents I_{L1} and I_{L2}.
c) Calculate the peak-to-peak voltage ripple across C_1.
The output resistor is increased, pushing the converter toward the CCM/DCM boundary:
d) Sketch a plot of i_D as a function of time at the CCM/DCM boundary (that is, $i_D(T_S)=0$).
e) Calculate the boundary output power.
f) Show that the converter operates in DCM for $K < K_{crit}(D)$, where

$$K = \frac{2 \cdot L_1 /\!/ L_2}{R \cdot T_S} \quad \text{and} \quad K_{crit} = (1-D)^2$$

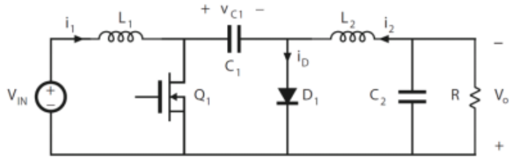


Fig. 2

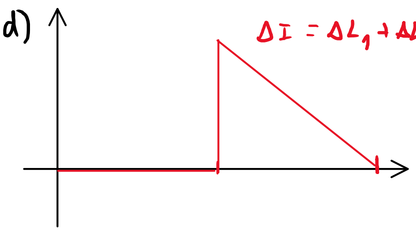
a) L_1: $V_{IN} D + (V_{IN} - V_{C1})(1-D) = 0 \Leftrightarrow V_{IN} = \frac{V_O}{D}(1-D) \Leftrightarrow \frac{V_O}{V_{IN}} = \frac{D}{1-D} \Rightarrow D = 0.5$

L_2: $(V_{C1} - V_O) D - V_O(1-D) = 0 \Leftrightarrow V_{C1} = V_O/D$

b) $\overline{I_{L1}} = I_{in} = 5A$ c) $i = C\frac{dv}{dt} \Leftrightarrow \Delta V_{C1} = \frac{I_{in}(1-D)T_s}{C_1} = 0.45V$

$\overline{I_{L2}} = I_{out} = 5A$

d)



$\Delta I = \Delta L_1 + \Delta L_2 = V_{IN} D T_S \left(\frac{1}{L_1} + \frac{1}{L_2}\right) = 1.33A$

e) $$P_o = P_{L_1} + P_{L_2} = \left(L_1 \cdot \Delta I_{L_1}^2 + L_2 \cdot \Delta I_{L_2}^2\right)\frac{f_{sw}}{2} = \left(\frac{V_{in}^2 D^2}{f_s^2 L_1} + \frac{V_{in}^2 D^2}{f_s^2 L_2}\right)\frac{f_{sw}}{2} = \frac{V_{in}^2 D^2}{f_s 2} \cdot \left(\frac{1}{L_1} + \frac{1}{L_2}\right) = 4W!$$

f) $$\frac{V_{in} D}{2f_s L_1} + \frac{V_O(1-D)}{2f_s L_2} \geq \frac{P_o}{V_O} + \frac{P_o}{V_{in}} \Leftrightarrow \frac{\cancel{V_O}(1-D)}{f_s(L_1/\!/L_2)} \geq \frac{\cancel{V_O}}{R(1-D)} \Leftrightarrow (1-D)^2 \geq \frac{2(L_1/\!/L_2)}{R\,T_S}$$