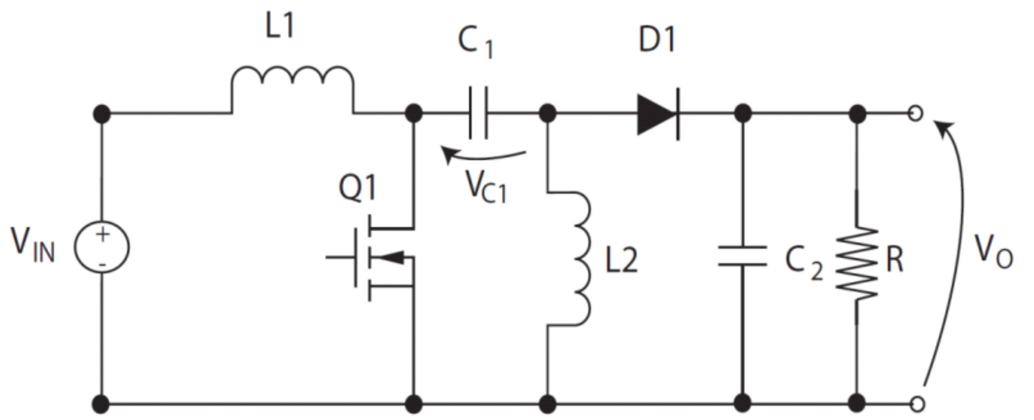
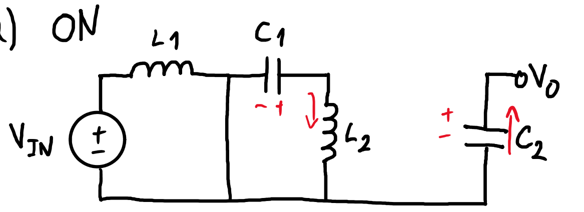


1) The DC-DC converter shown in Figure 1 operates in CCM.
a) Derive the DC voltage transfer function V_O/V_{IN} as a function of the duty cycle D (ignore the voltage ripple across the capacitor C_1).

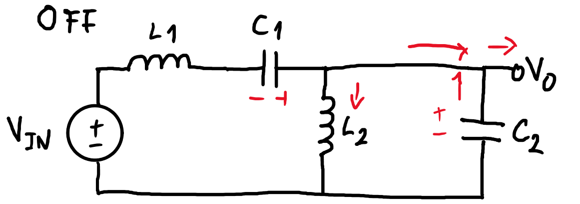
a) ON



$V_{L1ON} = V_{in}$

$V_{L2ON} = -V_{C1}$

OFF



$V_{L1OFF} = V_{in} - V_{C1} - V_o$

$V_{L2OFF} = V_o$

$L_1 \Rightarrow V_{in} \cdot D \cdot T_S + (V_{in} - V_{C1} - V_o)(1-D) \cdot T_S = 0 \quad \Leftrightarrow \begin{cases} V_{in} \cdot D + (V_{in} - V_{C1} - V_o)(1-D) \\ \frac{V_o(1-D)}{D} = V_{C1} \end{cases}$

$L_2 \Rightarrow -V_{C1} \cdot D \cdot T_S + V_o(1-D) T_S = 0$

$\Leftrightarrow \begin{cases} V_{in} D + \left(V_{in} - \frac{V_o(1-D)}{D} - V_o\right)(1-D) = 0 \\ - \end{cases}$

$\rightsquigarrow V_{in} - V_o(1-D)\left(\frac{1-D}{D} + 1\right) = 0 \Leftrightarrow$

$\Leftrightarrow V_{in} = V_o(1-D) \cdot \frac{1}{D} \Leftrightarrow \frac{V_o}{V_{in}} = \frac{D}{1-D}$

Assuming V_{IN} = 12V, V_O = 5V, $L_1 = L_2$ = 500 µH, C_1 = 4.7 µF, P_O = 10 W, f_{SW} = 250kHz:
b) Calculate the average current in the inductors L_1 and L_2.

$V_{in} = 12$ and $V_o = 5 \Rightarrow D = \frac{V_o}{V_o + V_{in}} = 0.294$

$P_o = 10W \Rightarrow I_o = 2A \quad \Rightarrow \frac{I_{in}}{I_o} = \frac{V_o}{V_{in}} = \frac{D}{1-D} \rightsquigarrow I_{in} = 0.833A$

Amp sec balance

$I_{L_1} = I_{in} = 0.833A$

$I_{C_2ON} = I_o \quad I_{C_2off} = I_o - I_{in} - I_{L2}$

$I_{C_1ON} = -I_{L_2} \quad I_{C_1oN} = I_{in} \quad \rightsquigarrow -I_{L_2} \cdot D \cdot T_S + I_{in}(1-D) T_S = 0 \Rightarrow I_{L_2} = \frac{I_{in}(1-D)}{D} = I_o = 2A$

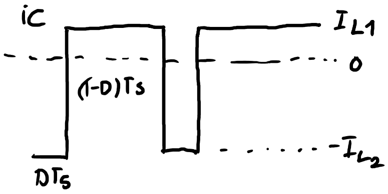
c) Calculate the peak-to-peak current ripple in L_1 and L_2.

$V = L \frac{\Delta i}{\Delta t}$

$V_{L1ON} = V_{in} \Rightarrow \Delta i_{L1} = \frac{V_{in}}{L_1} \cdot D \cdot \frac{1}{f_s} =$ 28.24 mA

$V_{L2OFF} = V_0 \Rightarrow \Delta i_{L_2} = \left| \frac{V_o}{L_2} \cdot (1-D) \cdot \frac{1}{f_s} \right| =$ 28.24 mA

d) Calculate the peak-to-peak voltage ripple across C_1.



$\Delta V_{C1} = \frac{I_{L2} D T_s}{C_1} =$ 0.5V

If the ripple from L_1 and L_2 is Considered: $\Delta V_{C1} = 500\,mV + 3\,mV = 503\,mV$

Current in C_1 → Triangle ⇒ Voltage → weird Sine

↗ $\Delta V = \frac{1}{C} \Delta Q$!

$i = C \frac{dv}{dt} \Rightarrow \Delta V = \frac{1}{C} \int i\,dt \longrightarrow \Delta V = \frac{1}{C} \cdot \underbrace{\frac{T_s}{2} \cdot \frac{\Delta i_L}{2} \cdot \frac{1}{2}}_{\Delta Q}$ ↳ $\Delta V = \frac{\Delta i_L \cdot T_s}{8C} =$ 3mV

↳ Area of the current

e) Calculate the peak-to-peak output voltage ripple (C = 470 µF, ESR = 25 mΩ).

$I_{C2ON} = I_O \quad I_{C2off} = I_o - I_{in} - I_{L2} = -I_{in}$

$V_{ESR_{ON}} = ESR \cdot I_O \quad V_{ESR_{OFF}} = -ESR \cdot I_{in} \Rightarrow \Delta V_{ESR} = ESR(I_{in} + I_O)$

$\Delta V_{C2} = \frac{I_O D \cdot T_s}{C} + \Delta ESR = \frac{I_O D \cdot T_s}{C} + ESR(I_{in} + I_O) \Rightarrow 76\,mV$