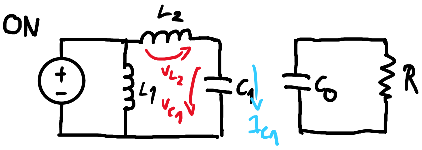
1) The DC/DC converter shown in Fig. 1 operates in CCM. The switches S1 and S2 are driven synchronously (both ON or OFF). All the components are ideal: losses may be ignored.

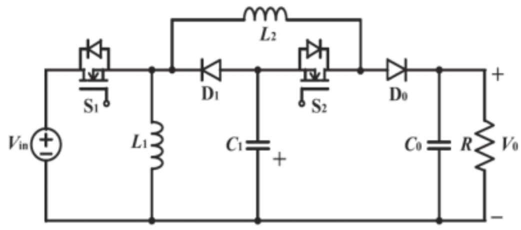
a) Derive the DC voltage transfer function V_O/V_{IN} as a function of D and sketch its plot.

ON



$$V_{in} \cdot D \cdot T_s + V_{C1}(1-D) \cdot T_s = 0$$

$$(V_{in} - V_{C1})D - (V_0 - V_{C1})(1-D) = 0$$



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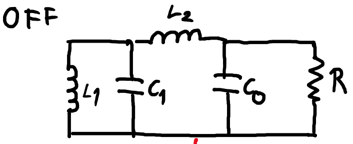
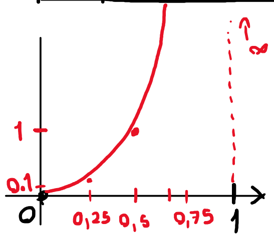


$$V_{C1} = -\frac{V_{in} D}{1-D}$$

$$\left(V_{in} + \frac{V_{in} D}{1-D}\right) D - \left(V_0 + \frac{V_{in} D}{1-D}\right)(1-D) = 0$$

$$V_{in}\left(D + \frac{D^2}{1-D}\right) - V_0(1-D) - V_{in} D = 0 \Rightarrow \frac{V_0}{V_{in}} = \frac{D^2}{(1-D)^2}$$



b) Derive expressions for the average currents I_{L1}, I_{L2} as a function of D and I_O.

$$P_{in} = P_{out} \Leftrightarrow V_i \cdot I_i = V_0 \cdot I_0 \Rightarrow I_{in} = \frac{D^2}{(1-D)^2} I_0$$

$$I_{C0} \cdot D = (I_{L2} - I_0)(1-D) \Leftrightarrow I_0 D + I_0(1-D) = I_{L2}(1-D) \Rightarrow I_{L2} = \frac{I_0}{1-D}$$

$$I_{in} = (I_{L1} + I_{L2}) D \Rightarrow \frac{D^2}{(1-D)^2} I_0 = I_{L1} D + I_0 \frac{D}{1-D} \Leftrightarrow \frac{2D-1}{(1-D)^2} I_0 = I_{L1}$$

Assuming V_{IN} = 1.5V, V_O = 48V, P_O = 10W, f_{sw} = 300 kHz:

c) Select L_1 and L_2 such that peak-to-peak current ripple is less than 20% of the average current.

$$\frac{V_0}{V_{in}} = \frac{D^2}{(1-D)^2} \Rightarrow D = \frac{\sqrt{V_0/V_{in}}}{1 + \sqrt{V_0/V_{in}}} = 0.85$$

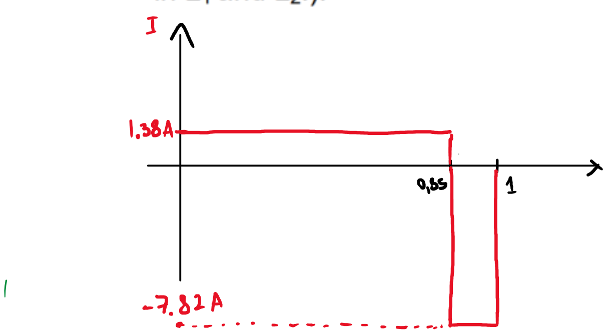
$I_0 = 0.208A \Rightarrow I_{L1} = 6.471A$ $\quad V_{C1} = -8.5V$

$\hookrightarrow I_{L2} = 1.387A$

$$\frac{V}{L_1} = \frac{\Delta i}{\Delta t} \Rightarrow \frac{V_{in} \cdot D \cdot 1/f_{sw}}{L_1} < 0.2 I_{L1} \Leftrightarrow L_1 > \frac{V_{in} \cdot D \cdot 1/f_{sw}}{0.2 I_{L1}} \Leftrightarrow L_1 > 3.3\mu H$$

$$\frac{V}{L_2} = \frac{\Delta i}{\Delta t} \Rightarrow \frac{(V_{in} - V_{C1}) D \, 1/f_{sw}}{0.2 I_{L2}} < L_2 \Leftrightarrow L_2 > 102.1\mu H$$

d) Sketch a plot of the current flowing through C_1 as a function of time (neglect the current ripple in L_1 and L_2.).



For on $I_{C1} = I_{L2}$

For off

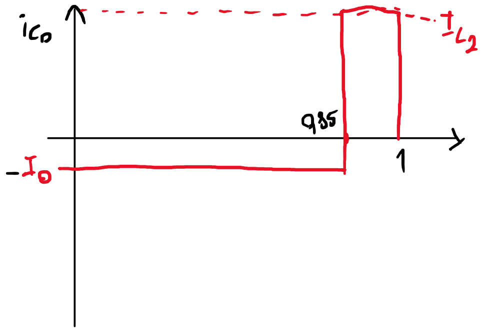
$$I_{C1} = -\frac{0{,}85 \times 1.38}{1 - 0{,}85} = -7.82A!$$

e) Select C_1 to ensure that the peak-to-peak voltage ripple is less than 0.3V.

$$\frac{i}{C} = \frac{\Delta V}{\Delta t} \Rightarrow \Delta V < \frac{i \Delta t}{C} \Rightarrow C < \frac{i \Delta t}{\Delta V} \qquad I_{L_2} = 1.387\,A$$

$$\frac{I_{L_2} \cdot D \cdot 1/f_{sw}}{0.3} \geq C \Leftrightarrow C \leq 13.1\,\mu F$$

f) Calculate the peak-to-peak output voltage ripple (C = 910 μF, ESR = 25 mΩ).



$I_{L_2} - I_0 = 1.179\,A$

$I_0 = 0.208$

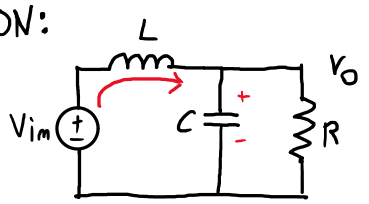
$$\Delta V = \frac{I_0 \cdot D \cdot 1/f_{sw}}{C} + ESR \cdot (I_{L_2}) =$$

$$= 0.65\,mV + 34.7\,mV = 35.35\,mV$$

2) The DC/DC converter shown in Fig. 2 operates in CCM. The switches S1 and S2 are driven synchronously (both ON or OFF). All the components are ideal: losses may be ignored. Assume V_{IN} = 12V, D = 0.63, R= 2Ω f_{sw} = 300 kHz.

a) Derive the DC voltage transfer function V_O/V_{IN} as a function of D and sketch its plot.

ON:



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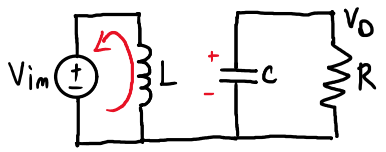


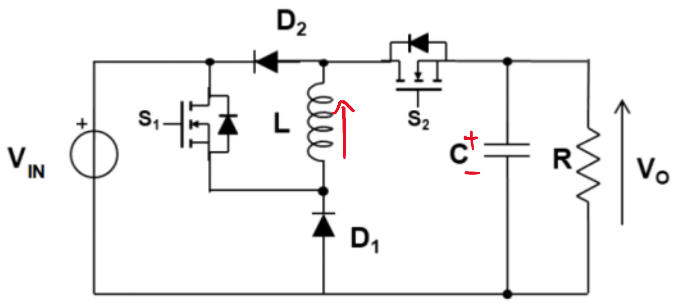
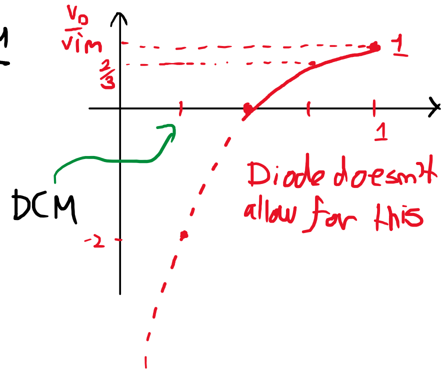


Fig. 2

$$(V_{im}-V_o)DT_s - V_{im}(1-D)T_s = 0 \Leftrightarrow (V_{im}-V_o)D = V_{im}(1-D) \Leftrightarrow$$

$$\Leftrightarrow V_{im}D - V_{im}(1-D) = V_o D \Leftrightarrow V_{im}(D-1+D) = V_o D \Leftrightarrow$$

$$\Leftrightarrow \frac{V_o}{V_{im}} = \frac{2D-1}{D}$$



$D = 0.63$

$$\frac{V_o}{V_{im}} = 0.413$$

$V_o = 0.413 \cdot V_{im} = 4.95\,V$

$I_o = \frac{V_o}{R} = 2.475\,A$

b) Derive an expression for the conditions under which this converter operates in CCM. Express your result in the form K > K_{crit}(D), and give an expression for K_{crit} (D).

$$\Delta i = \frac{V}{L}\Delta t = \frac{V_{im}}{L}(1-D)T_s \Rightarrow I_{LB} = \frac{V_{im}(1-D)T_s}{2L} = \frac{V_o D(1-D)T_s}{2L(2D-1)}$$

Charge balance: $(I_L - I_o)DT_s - I_o(1-D)T_s = 0$

$$I_L D = \cancel{I_o D} + I_o - \cancel{I_o D} \Rightarrow I_L D = I_o$$

CCM if $I_L > I_{LB} \Rightarrow \frac{I_o}{D} > I_{LB}$

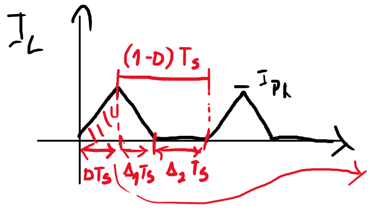
$$\frac{I_o}{D} > \frac{V_o D(1-D)T_s}{2L(2D-1)} \Leftrightarrow \frac{2L}{R_o T_s} > \frac{D^2(1-D)}{2D-1}$$

$$K_{crit}(D) = \frac{D^2(1-D)}{2D-1}$$

c) Calculate the minimum inductance value such to ensure CCM operation.

$$L_{min} = \frac{R_o T_s D^2(1-D)}{2(2D-1)} = 1.88\,\mu H$$

d) Assume L< L_{min}. Derive an expression for the DC voltage transfer function V_O/V_{IN} of the converter operating in DCM.



$$\frac{V_{in}-V_O}{L}\cdot DT_s = \frac{V_{in}\Delta_1 T_s}{L} \iff \Delta_1 = \frac{V_{in}-V_O}{V_{in}} D = D\cdot\left(1-\frac{V_O}{V_{in}}\right)$$

$$\text{Since } \bar{I}_C = 0 \text{ then} \Rightarrow \bar{I}_O = \bar{I}_{S2} = \frac{I_{PK}\cdot D \cancel{T_s}}{2\cancel{T_s}} = \frac{V_{in}\Delta_1 T_s}{2L}\cdot D$$

$$\frac{V_O}{R} = I_O = \frac{V_{in}T_s D^2}{2L}\left(1-\frac{V_O}{V_{in}}\right) \Rightarrow \frac{V_O}{V_{in}} = \frac{D^2}{K}\cdot\frac{1}{\left(1+D^2/K\right)}$$

e) Calculate the output voltage V_O assuming L=0.5 μH.

$L = 0{,}5\,\mu H$

$K = \frac{2L}{RT_s}$

$$V_O = \frac{D^2}{K}\cdot\frac{1}{(1+D^2/K)}\cdot V_{in} = 8.7\,V$$